532.542:532.55

Rotational and Axial Flow through the Gap between Eccentric Cylinders, of which the Outer One Rotates*

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We previously reported the pressure drop measurements of the flow through the annulus formed by two eccentric cylinders, with the inner cylinder rotating and the outer one stationary. This paper studies the pressure distribution around an inner cylinder and the pressure drop in the annular gap between eccentric cylinders, of which the outer one rotates and the inner one is stationary, under fully developed conditions. The relations between friction factor λ and a few parameters such as axial-flow Reynolds number Re , rotating Reynolds number R_ω , clearance ratio β and eccentricity ratio m , are investigated and compared with the previous results obtained for a rotating inner cylinder. And the effect of axial-flow Reynolds number Re on the pressure distribution is investigated at some rotating Reynolds numbers R_ω , where the rotational flow without the axial one belongs to a laminar-flow region, either of transitional or turbulent type.

1. Introduction

Considerable interest has been shown in the problem of viscous flow through the gap between eccentric cylinders with one of them rotating. Such systems are of interest in the design of forced lubricated journal bearings and various kinds of gap between shaft and sleeve of fluid machines and so on. Many studies have been conducted on the above-mentioned flow. In particular, there have been some concerning the flow between concentric rotating cylinders with the inner one rotating, and certain reports have dealt with the effect of axial flow on the instability of flow. The present authors, moreover, have earlier reported the effect of eccentricity ratio and axial flow in the case of flow between eccentric rotating cylinders where the inner one is rotating⁽¹⁾.

On the other hand, few reports have taken up the case of a rotating outer cylinder and a stationary inner cylinder⁽²⁾. And no reports seem to exist that deal with the effects of axial flow in the above case. When the inner cylinder rotates, Taylor vortices are brought about in the gap by the action of centrifugal forces. The development of these vortices will lead to an increased frictional factor and pressure distribution profile. But in the case of a rotating outer cylinder and a stationary inner cylinder, where the ac-

tion of centrifugal forces suppresses the occurrence of turbulence and stabilizes the flow, the flow will remain laminar up to a larger rotating Reynolds number. Thus, we can see great difference in the frictional factor and pressure distribution between the case of a rotating inner cylinder and the case of a rotating outer cylinder at a larger rotating Reynolds number⁽³⁾. In the case of no axial flow condition, we have already reported the experimental results of frictional torque⁽⁴⁾ and pressure distribution⁽⁵⁾ around the inner cylinder on the flow between eccentric rotating cylinders, of which the outer one rotates and the inner one is stationary. This paper investigates the frictional factor and the pressure-distribution profiles around the inner cylinder during axial and rotational flow through the gap between eccentric rotating cylinders of which the outer one rotates and the inner one is stationary.

Nomenclature

- e : eccentricity
- l : axial distance between pressure holes
- m : eccentricity ratio = e/δ
- p : pressure
- p_0 : pressure at the point of $\phi = 0$
- Δp : pressure drop between axial distance l
- r_i : radius of inner cylinder
- r_m : mean radius of annulus = $(r_0 + r_i)/2$
- r_0 : radius of outer cylinder
- Re : axial-flow Reynolds number = $\delta u/v$
- R_ω : rotating Reynolds number = $\delta U/v$
- U : characteristic velocity of rotating cylinder = $r_m \omega$
- u_m : mean axial velocity
- β : clearance ratio = δ/r_i
- δ : mean radial clearance = $r_0 - r_i$
- ϵ : Rossby number = Re/R_ω
- λ : friction factor = $4\Delta p \delta / \rho u_m^2 l$

* Received 20th November, 1972.

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ϕ : circumferential coordinate (see Fig.2)

2. Experimental apparatus and method

A general arrangement of the apparatus is shown in Fig. 1 (a). Fluid is supplied from a reservoir into the eccentric rotating cylinders by a centrifugal pump. The quantity of fluid flows is controlled by valves. An orifice is used for the devices to meter the flow rate. For only comparatively small flows, volumetric measurements of flow-rate were performed. Pressure measurements were done by means of a pressure transducer.

The arrangement of the eccentric rotating cylinders is shown in Fig. 1 (b). An inner cylinder, whose outer radius is indicated in Table 1, consists of a shaft ① inserted into a brass pipe ②. The inside radius of the outer cylinder is changed as shown in Table 1. The outer cylinder can be positioned at any eccentricity ratio from $m = 0$ to 1.0 by moving it with the bearing boxes upward and downward or right and left, by means of screws. The clearance between the cylinders is measured by a dial-gage at some circumferential positions. By the above method of arrangement the circumferential position of maximum clearance ($\phi = 0^\circ$) can be determined accurately, where a protractor and an index are used to measure the angle ϕ (Fig.2). Details of these methods are given in previous studies ⁽⁴⁾⁽⁵⁾.

(a) Experimental apparatus

(b) Experimental apparatus
Fig. 1

Measurements of pressure distributions around the inner cylinder were done by making the inner, ball-bearing supported cylinder rotate. Two pressure holes, No. I and No. II, were provided at the axially different positions of the same circumferential angle ϕ , as shown in Fig. 1 (b). In these measurements great attention was paid to the determination of axial positions of the above holes in order to get data in fully developed condition. To assure that pressure distributions and pressure drops were not influenced by the end effects, the pressure distribution profiles were measured at hole positions Nos. I and II, respectively. It was ascertained that the same pressure distribution profiles could be obtained at both holes, so that the difference in pressure at either always took the same value at any circumferential position ϕ . Moreover, it is a necessary condition that any axial flow does not affect the pressure-distribution profiles in the laminar flow, as seen in Navier-Stokes' equations. Therefore it was confirmed that the pressure distribution profile with no axial flow agreed with the pressure distribution profile with axial flow at the same rotating Reynolds number R_w , which belongs to the laminar flow region.

After many trials in which the location of the pressure hole was changed, a pressure hole, No. I, and another pressure hole, No. II, were set at a locus 195 mm away from the inlet and 105 mm away from the outlet, respectively.

Although the hole positions would normally have to be altered for various values of eccentricity ratios m , clearance ratios β and rotating Reynolds numbers R_w , the same two pressure holes were used throughout the measurements.

Circumferential means of the pressure differences between No. I and No. II were used as pressure drop Δp . The experimental data showed less than two percent divergence from normal value, but results obtained were comparatively accurate where both clearance ratio and eccentricity ratio were small.

Fig. 2

Table 1

	Inner rad. of out. cyl. r_o , mm	Outer rad. of inner cyl. r_i , mm	Clearance δ , mm	Clearance ratio $\beta = \delta/r_o$	Radius ratio r_o/r_i
A	32.044	31.589	0.455	0.0142	1.0144
B	29.772	28.998	0.774	0.0260	1.0267
C	32.039	30.586	1.453	0.0453	1.0473

The working fluids used in these experiments were water and 32.7 percent glycerine-water solution. The temperature of working fluid was measured by a thermocouple installed on the wall of inner cylinder.

3. Friction factor

Friction factor λ is not influenced

by the rotation of the outer cylinder in the laminar flow region, as seen in Navier-Stokes' equations, and is independent of the rotating Reynolds number R_ω . Then the value of λ in the rotational and axial-flow, which belongs to the laminar flow region,

equals the value of λ for the stationary eccentric cylinders. At small values of clearance ratio β , the value of λ for the laminar flow is given by the following well-known equation:

$$\lambda = 48 / \{ R_e (1 + 3m^2/2) \} \quad (1)$$

In the turbulent flow, the value of λ for the stationary eccentric cylinders, λ_β is given by (1)/(6)

(a) $\beta = 0.0142$, $m = 0$

(d) $\beta = 0.0142$, $m = 0.5$

(b) $\beta = 0.0142$, $m = 0.25$

(e) $\beta = 0.0260$, $m = 0.25$

(c) $\beta = 0.0142$, $m = 0.375$

(f) $\beta = 0.0260$, $m = 0.5$

Fig. 3 Relationship between friction factor λ and axial-flow Reynolds number Re

$$\lambda_s = 0.26 R_e^{0.24} \left(\frac{1}{\pi} \int_0^\pi (1+m^2 \cos \theta)^{3/1.76} d\theta \right)^{-1.76} \quad (2)$$

Figs. 3 (a)-(f) show the relationship between λ and R_e at various R_ω . The flow probably will be laminar at R_e and R_ω , where the experimental value of λ falls on the theoretical line given by Eq. (1). Take $R_\omega = 1000$, for example; the flow will be laminar for small R_e , but turbulent as R_e increases. These figures show that the flow becomes turbulent with an increase of R_e or with an increase of R_ω , and that the critical axial-flow Reynolds number R_{ec} decreases as R_ω and m increase.

In the transient region we find a notable decrease of λ . In the case of $\beta = 0.0142$ and $m = 0$, for example, the experimental value of λ at $R_\omega = 2000$ is greater than λ given by Eq. (1) in the range of $60 < R_e < 300$, and runs in parallel to a line of Eq. (1) with an increase of R_e . But the experimental value of λ suddenly decreases over $R_e = 300$ and is in close agreement with Eq. (1) within the range of $R_e = 400-900$. For $R_e > 900$, the value of λ is greater than the value given by Eq. (1), and

(a) $m = 0.25$ (b) $m = 0.5$

Fig. 4 Relationship between friction factor λ and rotating Reynolds number R_ω

the flow is turbulent. (Such phenomena also have been reported by Y. Yamada and T. Watanabe⁽²⁾ who measured the pressure drop and the frictional torque in concentric rotating cylinders with the outer one rotating.) The present authors previously obtained the critical value of rotating Reynolds number $R_{\omega k} = 1400^{(4)}$ for the only rotational flow with no axial flow, so that the rotational flow at $R_\omega = 2000$ can be seen to still remain in a transient state. When the rotational flow with no axial flow remains in a transient state, it can be considered that the turbulence will be stabilized by a moderate quantity of axial flow and that the flow becomes laminar. But the flow will not return to the perfectly laminar type, as seen in the coefficient of viscous frictional moment $C_M^{(2)}$.

The above-mentioned phenomena also occurred at $R_\omega = 2000$ for $m = 0.25$ and 0.375 , but did not occur for $m = 0.5$ and 0.75 . For $m = 0.25$ the values of λ are in close agreement with Eq. (1) in the range of $R_e = 400-600$ at $R_\omega = 2000$. For $m = 0.375$ the values of λ do not at all agree with Eq. (1) at $R_\omega = 2000$, but tend to be in close agreement with Eq. (1) near $R_e = 500$ at $R_\omega = 1500$. These values of R_ω , $R_\omega = 2000$ and 1500 , are a little greater than the critical rotating Reynolds numbers $R_{\omega k}$ for $m = 0.25$ and 0.375 , as previously obtained in measurements of viscous frictional moment⁽⁴⁾. These interesting phenomena, which occur in the transient region with an increase of R_e , can be observed in the pressure distribution too. They probably amount to a kind of stabilization of the turbulent boundary layer. For $\beta = 0.0142$ the above-mentioned stabilization of turbulent flow does not occur at any R_ω over 2500 , and the value of λ decreases in parallel with a line of Eq. (1) in the range of small R_e . In the range of very great R_e , $\lambda - R_e$ curve at any R_ω tends to approach asymptotically the $\lambda - R_e$ curve given by Eq. (2) for the eccentric stationary cylinders with an increase of R_e . For $\beta = 0.0260$ and 0.0453 , the stabilization of turbulent flow can not be observed at any R_ω for the relationship between λ and R_e . For $\beta = 0.0260$ and $m = 0.25$ or 0.5 ,

Fig. 5 Relationship between friction factor λ and eccentricity ratio m

which are shown in Figs. 3 (e) and (f) respectively, the value of λ at $R_{\omega} \geq 2000$ decreases nearly in parallel with Eq. (1), and tends to approach asymptotically the $\lambda - R_e$ curve given by Eq. (2) with an increase of R_e . For $\beta = 0.0260$ and 0.0453 , the stabilization of turbulent flow can not be observed at any R_{ω} in the relationship between λ and R_e .

Fig. 4 shows the relationship between λ and R_{ω} in the condition of $R_e = \text{const.}$ for $m = 0.25$ and 0.5 , and also the relationship between λ and R_{ω} , which we previously obtained in the case of a rotating inner cylinder and a stationary outer cylinder¹¹. In the case of a rotating inner cylinder the value of λ suddenly increases from the value given by Eq. (1) at the critical rotating Reynolds number $R_{\omega c}$ for $R_e = 250, 500$ and 1000 of $m = 0.25$, because Taylor vortices occur. The value of λ in the case of a rotating outer cylinder also suddenly increases for $R_e = 250$ as well as 500 at the radius ratio r_0/r_1 of 1.0144 ($\beta = 0.0142$). But this tendency in the relationship between λ and R_{ω} in the case of a rotating outer cylinder relates to the stabilization effects of axial flow. At

(a) $\beta = 0.0142, m = 0.25$

(b) $\beta = 0.0142, m = 0 - 0.75$

(c) Effect of clearance ratio β on λ/λ_s

Fig. 6 Relationship between λ/λ_s and R_{ω}/R_e

the other radius ratio the value of λ increases slowly with augmentation of R_{ω} . For $m = 0.5$, the value of λ slowly increases with R_{ω} for every value of R_e . This tendency can be seen both with a rotating outer cylinder and a rotating inner cylinder. However, the former value is smaller than the latter throughout the range of R_e and R_{ω} , and is remarkably different from the latter near-critical rotating Reynolds number $R_{\omega c}$, especially in the case of small R_e . The difference of λ in either case will depend on the effects of centrifugal forces. That is, the action of centrifugal forces causes the development of Taylor vortices and tends to promote the development of disturbance when the inner cylinder rotates. But when the outer cylinder rotates, the centrifugal forces exert an inhibitive action on the disturbance.

Fig. 5 shows the relationship between λ and m in the case of $R_e = 2500$, where the flow remains in a state of turbulence at every R_{ω} . The value of λ decreases with an increase of m .

Fig. 6 illustrates the relationship between λ/λ_s and R_{ω}/R_e , where λ_s is the value of λ given by Eq. (2) and R_{ω}/R_e equals the inverse of Rossby number ϵ . Experimental data used in this figure belong to the large value of R_{ω} , where the flow is turbulent. The data fall into a single curved line for $m = 0.25$ and $\beta = 0.0142$, as seen in Fig. 6 (a). Similarly, the data for $m = 0, 0.25, 0.375, 0.5$ and 0.75 gather along the same curved line for $\beta = 0.0142$, as seen in Fig. 6 (b). Fig. 6 (c) shows the relationship between λ/λ_s and $1/\epsilon$ for each β . The value of λ/λ_s increases with $1/\epsilon$ for every β , and decreases with an increase of β .

Fig. 7 Pressure profiles in the laminar flow

4. Pressure distribution

The accuracy of the pressure distribution profiles seen in the following figures was confirmed as follows. The profile of pressure distribution, which was measured at the upstream pressure hole, No. I, was in good agreement with the profile, which was measured at the downstream pressure hole, No. II.

Fig. 7 compares the pressure distribution profile at $Re \approx 200$ with the profile at $Re = 0$ for $m \approx 0.5$, 0.6 and 0.75 in the case of $\beta = 0.0142$. It is demonstrated by the measured results of friction factor⁽⁴⁾ or frictional moment that the flow is laminar in the case of Fig. 7. As can be seen in the same figure, the profile of pressure distribution for $Re \approx 200$ is in close agreement with that for $Re = 0$ at each m . From this fact it can be concluded that the experimental values of pressure do not involve any influence of cylinders' end.

Figs. 8 (a) and (b) show the effects of axial flow in the case where the flow at $Re = 0$ is laminar. The experimental results of frictional moment⁽⁴⁾ or the relationship between λ and Re (see Fig. 3 (b)) would indicate that the flow for $Re = 0$ is laminar at $R_w = 1000$. In the range of small Re (e.g., $Re = 202$), the profile of pressure distribution with some axial-flow is in good agreement with one having no

(a) $\beta = 0.0142$, $m = 0.25$

(b) $\beta = 0.0142$, $m = 0.5$

Fig. 8 Effect of axial-flow Reynolds number Re on the pressure profiles at $R_w = 1000$ (The flow with no axial-flow, $Re = 0$, is laminar)

axial flow (at $Re = 0$). But in the case of large Re , e.g., $Re = 502$, both positive and negative pressures are a little greater than each pressure for $Re = 0$. And pressure fluctuation can be measured in the range of $0^\circ \leq \phi \leq 80^\circ$, because of the local development of turbulence (data points drawn by an oblique line indicate the circumferential position ϕ where pressure fluctuation could be measured). In

(a) $\beta = 0.0142$, $m = 0.25$

(b) $\beta = 0.0142$, $m = 0.5$

(c) $\beta = 0.0260$, $m = 0.5$

Fig. 9 Effect of axial-flow Reynolds number R_w on the pressure profiles at $R_w = 2000$ (The flow with no axial-flow, $Re = 0$, belongs to the transient region)

such a value of Re , the value of λ is also in agreement with Eq. (1) (see Fig. 3 (b)). For the case of $Re = 1004$ and 1493 , both positive and negative pressures increase with Re , and the value of λ also exceeds that given by Eq. (1). It can be understood that the above-mentioned flow belongs to the fully developed turbulent flow. Fig. 8 (b) (for $m = 0.5$) also shows the same tendency as when $m = 0.25$. As noted above, in the range of R_ω where the flow increased by the rotation of outer cylinder stays in the laminar flow region, the pressure distribution is not influenced by the small quantity of axial-flow and the flow is kept laminar. However, when the flow becomes turbulent with an increase of Re , both positive and negative pressures increase with Re .

Profiles of pressure distribution, in which the flow stays in the region of transition in the case of no axial-flow, are shown in Figs. 9 (a) and (b). In Fig. 9 (a) (where $\beta = 0.0142$ and $m = 0.25$), both positive and negative pressures increase slightly from $Re = 0$ to $Re = 200$, but suddenly decrease at $Re = 500$. This sharp decrease in pressure is related to the stabilization of turbulent boundary layer, which was just mentioned concerning λ . For $Re \geq 990$, where the flow is turbulent, both positive and negative pressures increase with Re . Fig. 9 (b) shows the profiles of pressure distribution for $m = 0.5$. Although the influence of stabilization in flow is not observable on λ for $m = 0.5$ in Fig. 3 (d), the pressure distribution shows a notable change at $Re = 500$. Negative pressure, especially, decreases remarkably in comparison with that for $Re = 0$. As mentioned above, a kind of stabilization of turbulent boundary layer in the case of a rotating outer cylinder will occur for $m = 0.5$ as well as $m = 0.25$. Fig. 9 (c) shows the profiles of pressure distributions for $\beta = 0.0260$, which do not evince any marked change at $Re = 500$. At Re greater than 500, negative pressure remarkably increases with Re .

The profiles of pressure distributions in Fig. 10 belong to the case of very great R_ω , in which flow was already turbulent in the condition of no axial-flow. Both positive and negative pressures increase with Re .

5. Conclusions

The following conclusions on rotational and axial-flow through the gap between eccentric cylinders, of which the outer one rotates, may be drawn from the experimental results of friction factor λ and pressure distribution.

(1) When the flow is laminar, friction factor λ depends on axial-flow Reynolds number Re only, so that λ can be given by Eq. (1) (i.e., $\lambda = 48 / \{ (1 + 3m^2/2) Re \}$). Thereby, pressure distribution $(p - p_0) / \{ (1/2) \rho U^2 \}$ is independent of Re , but depends on R_ω .

(2) When the axial and rotational flow

is fully developed in the turbulent state, friction factor λ increases with R_ω . In the range of small Re , the relationship between λ and Re is given by a line parallel to the one which gives the value of λ for the laminar flow in the eccentric stationary cylinder. In the range of great Re , λ tends to approach asymptotically the value of λ for the turbulent flow in the eccentric stationary cylinders. In the above-mentioned state of flow, λ/λ_s can be described by a function of Rossby number $\epsilon = Re/R_\omega$ for any clearance ratio β , and is independent of eccentricity ratio m , rotating Reynolds number R_ω , and axial-flow Reynolds number Re . The value of λ/λ_s increases gradually with $1/\epsilon = R_\omega/Re$, and decreases with an increase of β . The increase of both positive and negative pressures with Re or R_ω can be seen in the profiles of pressure distributions $(p - p_0) / \{ (1/2) \rho U^2 \}$ in these cases.

(3) When the flow developed by the rotation of the outer cylinder is laminar in the case of no axial-flow, the flow state changes from laminar to turbulent with an increase of Re . In this transient process, the fluctuation of pressure can be observed and negative pressure increases.

(4) When the flow with no axial-flow belongs to the region of transition, a moderate increase of Re suppresses the development of turbulence. Then the value of λ decreases and finally agrees with the theoretical value given by Eq. (1). But a greater increase of Re makes the flow turbulent and the value of λ increases again. This decrease of λ with a moderate increase of Re can be seen for an eccentricity ratio m smaller than 0.375 at $\beta = 0.0142$. In profiles of pressure distribution, negative pressure also decreases significantly. These phenomena are considered to be caused by a kind of stabilization of turbulent boundary layer. For $m = 0.5$ such phenomena do not occur in λ , but are observed in pressure distribution.

(5) The value of λ in the case where the outer cylinder rotates, is much small-

Fig. 10 Effect of axial-flow Reynolds number Re on the pressure profiles at $R_\omega = 5000$. (The flow with no axial flow, $Re = 0$, is turbulent)

er than the value of λ where the inner cylinder rotates, especially near the critical rotating Reynolds number $R_{\omega c}$. The value of λ in the former case is smaller than the value of λ in the latter case, even if both axial-flow Reynolds number Re and rotating Reynolds number R_ω are great.

References

- (1) Yamada Y., et al., Bulletin of JSME, Vol. 12, No. 53 (1969), p.1032.
 - (2) Yamada Y. and Watanabe T., Bulletin of the JSME, Vol. 16, No. 93 (1973) p.551
 - (3) Nakabayashi K., et al., Trans. Japan Soc. Mech. Engrs., Vol. 38, No. 312 (1972), p.2075.
 - (4) Yamada Y., et al., Bulletin of JSME, Vol. 12, No. 53 (1969), p.1024.
 - (5) Yamada Y. and Nakabayashi K., Bulletin of JSME, Vol. 13, No. 55 (1970), p.59.
 - (6) Tao L.N. and Donovan W.F., Trans. ASME, Vol. 77 (1955), p.1291.
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