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Flow of Liquid Metals with a Transversely Applied Magnetic Field*

(3rd Report, Influence of an Electric
and a Magnetic Field on the Transition of Flows)

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It is well recognized that turbulent pulsations in an electrically conducting fluid are suppressed and a velocity profile is made uniform due to the action of a transverse magnetic field.

Murgatroyd measured the pressure gradient during the flow of mercury in a rectangular channel of 15:1 aspect ratio in the presence of a transverse magnetic field and suggested that the critical condition for the transition from turbulent to laminar flow was $R_e/M = 900$.

However, MHD devices such as conduction electromagnetic pumps used in metallurgical and chemical industries are placed in a relatively short magnetic field and therefore Murgatroyd's criterion for the transition of flows seems not to be applicable due to the end effect.

This paper is an experimental study on the influence of an electric and a magnetic field on the transition of flows in a kind of conduction electromagnetic pump. The experiment suggests that in this device, the critical Reynolds number is lower than that based on Murgatroyd's criterion owing to the end effect.

1. Introduction

Since the application of magnetohydrodynamics was proposed as the means for transport or measuring of liquid metals, the investigation and the development connected with that field have been advanced and there are devices already put to practical use. The magnetohydrodynamic (MHD) pump is the object of the investigation on the application of magnetohydrodynamics to metallurgy or casting industry. Therefore it seems very important to investigate the behavior of flow in the MHD pump from a point of view of obtaining the data for its development or estimating its performance.

It was reported by Murgatroyd⁽¹⁾ and many other investigators⁽²⁾⁻⁽⁴⁾ that the critical Reynolds number from laminar to turbulent flow increased due to the turbulence suppression effect of a magnetic field and the Hartmann effect of a transverse magnetic field. Murgatroyd⁽¹⁾ conducted an experiment on mercury flows in a rectangular channel of 15:1 aspect ratio under a transverse magnetic field and elucidated that at $M/R_e > 1/900$ ($R_e = 4au_m/\nu$ = Reynolds number, $M = \sqrt{\sigma/\eta} aB_0$ = Hartmann number, a = half-height of channel, u_m = mean velocity, ν = kinematic viscosity, σ = electric conductivity, η = viscosity, B_0 = magnetic flux), laminar flow was maintained regardless of the value of Reynolds number. Grove⁽²⁾ carried out an experiment on mercury flows in a circular pipe under a parallel magnetic field and showed that the critical Reynolds number generally in-

creased except very high Reynolds numbers.

On the other hand, Stuart⁽³⁾ and Lock⁽⁴⁾ investigated theoretically. Stuart studied the stability of the flow between parallel planes under a parallel magnetic field and found that the critical Reynolds number increased with an increase of the intensity of the field. Lock performed the same analysis as Stuart for the flow under a transverse magnetic field and reported that the transverse magnetic field very much influenced the stability of such a flow.

The above-stated experimental and analytical results are, however, relative to the flows with a transverse or a parallel magnetic field applied over a very wide range in the flow-direction. On the contrary, the conduction electromagnetic pump which is used in industry is placed in the field of a relatively short and finite length and the electric current is sent in liquid metal through the electrodes welded to opposite sides of the channel in order to intensify the effect of the electromagnetic force. Therefore, Murgatroyd's result on the transition of flows can not be applied.

The present report is an experimental investigation of the influence of an electric and a magnetic field on the transition of liquid metal flowing in a kind of the MHD pump required practically in casting or metallurgy industry. In the experiment, mercury is run in the rectangular channel of 12.3:1 aspect ratio under a transverse magnetic field of relatively short length and the electric current is sent in liquid metal through the electrodes installed on the side walls. From the experimental results, it is clear that the critical Reynolds number decreases with an increase of the electric current flowing in mercury.

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2. Experimental apparatus

The experimental set-up is shown in Fig.1. The apparatus consists of a mercury reservoir, a flow-rate regulation valve, a rectangular channel, an electromagnet, a manometer and an electric power supply.

In order to keep a constant head during measurements, the mercury reservoir is a large plastic container which has dimensions of 600mm×600mm×50mm (high). The weight of mercury used per measurement is about 3 kg and the corresponding variation of the head is about 0.6 mm. As the height h from the channel to the mercury surface in the mercury reservoir (see Fig.1) is about 800 mm, the rate of variation of the head per measurement is about 0.08 % and therefore the variation of the head can be neglected. The mercury surface is always covered with a thin layer of water in order to prevent the mercury from vaporizing. The regulation of flow-rate is carried out with a stainless steel needle valve and the regulation volume corresponds to the Reynolds number in the range 0 ~ 4000. The piping laid from the mercury reservoir to the channel is made of plastic pipes and vinyl tubes.

Fig. 1

Fig. 2

A detailed figure of the channel is shown in Fig.2(a),(b). The channel is made of plastics except the electrodes and is constructed by sticking two plastic plates together. The electrodes are copper plates which have dimensions of 120mm(long)×42mm (wide)×1.3mm(thick) and constitute a part of the side walls. The cross-section of the channel is 16mm×1.3mm (cross sectional area = 20.8mm², aspect ratio = 12.3:1) and six static pressure holes of 0.8mm in diameter are drilled on the top wall of the channel. These static pressure holes are connected to the manometer through stainless pipes and vinyl tubes.

As the distance between two points for measuring the pressure difference is short, an inclined type manometer employing two kinds of liquid as shown in Fig.3 is used for measurement of the pressure difference. This manometer consists of plastic pipes (diameter $D=30.35$ mm) and glass pipes (diameter $d=3.90$ mm). A part of the plastic pipes is always filled with mercury and the remaining part of them and a part of the glass pipes are filled with water. Therefore, we can calculate the pressure difference with the following equation by reading the difference in water surfaces between the two glass pipes:

$$p_1 - p_2 = \left(\frac{d}{D} \right)^2 (\rho_H - \rho_W) + (\rho_W - \rho_A) \sin \theta \Delta h g$$

where p is the pressure; D , d are respectively the diameter of the plastic and the glass pipe; ρ_H , ρ_W , ρ_A are respectively the densities of mercury, water and air; θ is an angle of inclination of the manometer; Δh is the difference of the column of water, g is an acceleration of gravity. Though the angle of inclination of the manometer can be changed continuously from 0° to 90° , the angle $\theta=31^\circ 46'$ is used for the region of high velocity and $\theta=6^\circ 25'$ for low velocity. Consequently, the magnifying power for $\theta=31^\circ 46'$ is about 18.5 and one for $\theta=6^\circ 25'$ is about 42.5. A small amount of a neutral cleaning material is dissolved in water in the glass pipes in order to bring water in the pipes into contact with the walls of the pipes smoothly and form smooth water surfaces.

A direct current generator is used as the power supply and it can supply the maximum current $i_{\max}=3$ A (magnetic flux $B_0=0.65$ Wb/m²) to the electromagnet and $i_{\max}=25$ A to the electrodes respectively. The rectangular pole tip dimension of the electromagnet is 80mm(wide)×120mm(long) and the electromagnet air gap is 22.4mm. The distribution of magnetic flux in the

Fig. 3

stream direction measured with a gauss meter is shown in Fig.4. The central section 75mm in the region of the applied magnetic field is used for measuring the pressure difference between two points and therefore the magnetic flux in the section used for the measurement is regarded to be uniform.

Mercury has some defects: (1) it is easily oxidized; (2) it reacts with other metals and makes amalgam; (3) it is harmful for the human body. However, mercury is used in the experiment for the following reasons: (1) it is liquid at normal temperature; (2) its electric conductivity is rather high; (3) it is easily available; (4) the comparison with many old experiments on mercury flows is possible.

3. Experimental procedure

In the experiment, efforts to remove a stain of mercury by a purifier using dilute hydrochloric acid were always made. Mercury purified is stored in the mercury reservoir and the necessary flow-rate is obtained with the needle valve for flow-rate regulation. After the electric current is sent to the electromagnet and the electrodes, the pressure difference between two points and the flow-rate are measured. A weighing method is adopted for the measurement of the flow-rate.

4. Experimental results and discussion

The co-ordinates as shown in Fig.5 are used in the arrangement of the results. The nondimensional pressure gradient λ obtained from the pressure difference between two points 75mm away for the zero-field is shown in Fig.6. Here

$$\lambda = \frac{\Delta p a}{(1/2) \rho u_m^2 l}$$

is defined and in consequence, the theoretical solution⁽⁵⁾ of the laminar flow in the rectangular channel of 12.3:1 aspect ratio is expressed as below:

$$\lambda = \frac{6.3}{R}$$

where $R (= u_m a / \nu)$ denotes the Reynolds number. In the figure, the solid line for the laminar region indicates the above relation and the experimental results agree well with the theoretical solution. In addition, the experimental results for the turbulent region also agree with the

straight line for the gradient of $-1/4$ and therefore the experimental apparatus seems to be satisfactory.

The results for only a magnetic field are shown in Fig.7 and they correspond to the Hartmann numbers $M (= \sqrt{\sigma/\eta} B_0 a) = 0, 4.4, 7.7$ and 9.2 respectively. In the figure, the experimental data with the mark "1" denote that the water surface in the manometer has oscillated during the measurement. In addition, the mark "+" shows the point where the experimental data begin to separate from the straight line in the laminar region and that point is considered to be the beginning point of the transition. It is recognized from this result that the point beginning to separate from the straight line agrees nearly with the start point of oscillation of the water surface in the manometer and therefore the observation on oscillation of the water surface is considered to be the effective means for judging the transition from laminar to turbulent flow. It is found from Fig.7 that under only a magnetic field, the critical Reynolds number R_c increases with an increase of Hartmann number M as mentioned by Murgatroyd. The critical Reynolds number obtained from Fig.7 and that calculated

Fig. 5

Fig. 6

Fig. 4

Fig. 7

ed on Murgatroyd's criterion for the transition $R_c = 225M$ are shown in Table 1. The present experimental result gives generally a lower critical Reynolds number. This seems to be caused by the difference in the channel aspect ratio and in the intensity of turbulent pulsations in the fluid flowing into the magnetic field but both results agree qualitatively.

The results on the case that the electrodes are connected with the external power supply and the electric current is sent in mercury are shown in Figs. 8, 9. In the figures, "I" denotes the sum of the electric current flowing in mercury and a positive and a negative sign of I mean respectively the decelerating action of the pump (electromagnetic force acts in the negative direction of x) and the accelerating action (it acts in the positive direction of x). It is understood that λ for the accelerating action decreases with an increase of the absolute value of I and finally takes negative values, while λ for the decelerating action increases with the absolute value of I. In the figures, the experimental data with the mark "I" denote,

Table 1

M	experimental result	Murgatroyd's criterion
4.4	850	990
7.7	1500	1700
9.2	1700	2100

Fig. 8

Fig. 9

as above-mentioned, that the water surfaces in the manometer have oscillated during the measurement. As the electric current to the electrodes and the electromagnet is supplied from the same power supply, the electric current to the electrodes for $M=9.2$ is limited and its maximum value is $I=10A$. According to the analysis of the laminar flow between parallel planes, the relation between the nondimensional pressure gradient λ for the external current I and λ_0 for the open circuit ($I=0$) is expressed by the following equation and it is in proportion to I

$$\lambda - \lambda_0 = a \rho M I / 2c \eta \sqrt{\sigma \eta} R^2$$

An example of plots of Reynolds number R against $5(\lambda - \lambda_0)/I$ obtained from the experiment is shown in Fig. 10. It is found from this figure that the pressure gradient in the laminar region increases or decreases in proportion to the electric current I and therefore the analysis is appropriate. Plotting the relation of $(\lambda - \lambda_0)$ against R on a logarithmic paper results in a straight line for the gradient of -2 in the laminar region. Therefore, it is predicted that the experimental data agree with the straight line in the laminar region but they separate from the straight line in the turbulent region. On the other hand, plotting the relation of λ against R on a logarithmic paper yields a curve and finding the point where the experimental data begin to separate from the curve is very difficult. For this reason, we define the point of transition as the point at which the experimental data begin to separate from the straight line in the figure of R against $(\lambda - \lambda_0)$. Now, we arrange the experimental data for each occasion in the same manner as above-mentioned and determine the point of transition in consideration of oscillation of the water surfaces in the manometer. These results are shown in Figs. 11 and 12. The mark "I" in the figures indicates the point of transition. The values of the critical Reynolds number R_c for each occasion are shown in Table 2 and the relation of R_c and I is denoted in Fig. 13. It is understood from Fig. 13 that the critical Reynolds number generally decreases with an increase of the absolute value of the electric current. The critical Reynolds number R_c seems to approach that for the zero-field with an increase of the absolute value of the electric current but not to be lower than that for the zero-field.

Fig. 10

The above-mentioned results are relative to the turbulent flow in the region of relatively low Reynolds numbers ($R \leq 4000$). These results seem to be consistent with the explanation of Branover et al. (7) that the effect of the magnetic field in this region acts as the turbulence suppression effect rather than the Hartmann effect and laminarization in this region is unstable in comparison with laminarization in the region of high Reynolds numbers where the Hartmann effect prevails. Accordingly, it is considered that the flow laminarized by the turbulence suppression effect is unstable and laminarization is broken down by a little disturbance.

In order to elucidate the relation between the electric current and the intensity of the magnetic field, the following arrangement is made: we take the nondimen-

sional pressure gradient λ obtained from the experiment as the ordinate and the analytical solution $\lambda_t^{(6)}$ for the laminar flow between parallel planes as the abscissa. Here, λ_t is expressed by the following equation:

$$\lambda_t = aMI/2c\nu\sqrt{\sigma\rho\nu R^2} + 2M^2 \tanh M/(M - \tanh M)R$$

The result for the Reynolds number $R=4000$ is shown in Fig. 14 but the other results show also the same trend. The dotted lines in the figure, which connect the points for the same Hartmann number together, show the trend that they separate from the straight line for the gradient of 1, that is, from the laminar solution with an increase of the absolute value of the electric current. This means that sending the electric current in the channel reduces the turbulence suppression effect of the field or produces pulsations in mercury. This phenomenon is consistent with the above-mentioned fact that electrifying in mercury reduces the critical Reynolds number. In order to express the trend found in Fig. 14 more clearly, we take $(\lambda - \lambda_t)$ for λ as the ordinate and show the result in Fig. 15. In Fig. 15 for the Reynolds number $R=4000$, the abscissa $(\lambda - \lambda_t=0)$ denotes the laminar line and therefore the distance from the abscissa is considered to be a measure of the flow being more turbulent or not.

The following reasons are considered as the cause of the phenomenon that the electric current makes the flow more turbulent: (1) The electric current reduces the turbulence suppression effect of the field; (2) the electric current produces an unbalanced electromagnetic force and the secondary flow is induced due to the unbalanced force. As the electric current itself flowing in mercury produces the secondary magnetic field (in the x-direction), it seems,

Fig. 11

Fig. 12

Table 2

M	4.4	7.7	9.2
IA			
0	850	1500	1700
+5	900	950	850
-5	850	900	1400
+10	900	800	800
-10	900	900	1200
+15	850	800	—
-15	850	850	—
+20	900	700	—
-20	700	800	—

Fig. 13

Fig. 14

Fig. 15

however, to suppress the turbulence in fluid (see Appendix) and therefore it is supposed that the main cause of this phenomenon is due to the second reason (2).

The electric current in mercury is composed of two parts: one due to the induced electromotive force and the other due to the electric field caused by the voltage difference between the two electrodes. At the entrance and exit regions of the pump, that is, near the ends of the electrodes, electric field is not uniform, causing the current density to vary in magnitude and direction in the entrance and exit regions and the electric current lines become as shown in Fig.16. Therefore, even if a uniform magnetic field is applied, the magnitude and direction of the electromagnetic force near the ends are naturally different from those in the field of an infinite length. In addition, the magnetic flux is not uniform near the ends of the magnetic poles and a leakage of the magnetic field occurs. Near the ends of the pump, the magnetic field in the y-direction is induced by the electric current, causing an increase of the magnetic field at one end and a decrease of the magnetic field at the other end (Armature Reaction Effect). The nonuniform and the leakage magnetic field near the ends interfere with the electric current with nonuniform distribution as shown in Fig.16 or the leakage electric current and in consequence, a more complex distribution of the electromagnetic forces will occur near the ends of the pump. Consequently, it is supposed that the secondary flow is induced in fluid near the entrance of the magnetic field and it flows into the pump; this promotes the transition to a turbulent flow. If the region where the magnetic and the electric field are applied is very wide (in the x-direction), the laminar flow will be naturally obtained downstream as mentioned by Murgatroyd, since the induced secondary flow at the entrance is gradually suppressed due to the viscosity and the magnetic effects as it advances downstream.

5. Conclusions

We conducted an experiment on mercury flows in a kind of conduction electromagnetic pump under the magnetic field applied transversely over a relatively short

Fig. 16

section. In consequence, the following conclusions were obtained:

- (1) A magnetic field generally suppresses the pulsations in an electrically conducting fluid and delays the transition from laminar to turbulent flow. On the contrary, the electric and the magnetic field of short and finite length, however, promote the transition to turbulent flow and decrease the critical Reynolds number.
- (2) The critical Reynolds number decreases with an increase of the absolute value of the electric current flowing in mercury in spite of its direction and seems to approach the value for the zero-field. From this fact, it is considered that the flow laminarized by the magnetic field is relatively unstable in the region of low Reynolds numbers.
- (3) The end effect is found to be the cause of the phenomenon that the transition to turbulent flow is promoted. To put it concretely, the secondary flow which is produced by complex distribution of the electromagnetic forces induced near the ends seems to promote the transition to turbulent flow.
- (4) Murgatroyd's criterion for the transition $M/R > 1/225$ can not apply to the flow in the conduction electromagnetic pump as it stands. Accordingly, it will be necessary to investigate further on various cases and simultaneously to devise the electrodes, and the magnetic poles and the shape of the channel in which the nonuniformity of the electric and magnetic field and the leakage of the electric current and the magnetic flux near the ends are reduced.

As the present experimental results are connected with the region of relatively low Reynolds numbers ($Re < 4000$), that is, the region of the turbulence suppression effect being predominant, an investigation of the region of high Reynolds numbers, that is, the region of the Hartmann effect prevailing will be necessary.

Appendix

We attempt to discuss analytically, in relation to the subjects stated in the text, the influence of the electric current flowing in liquid metals on the stability of flow under the condition that a uniform magnetic field is applied transversely over a wide range in the flow-direction. Fluid is assumed to be incompressible and to be electromagnetically homogeneous and isotropic.

Basic equations are as follows:

Equation of continuity,
 $\nabla \cdot \mathbf{V} = 0$ .....(1)

Momentum equation,

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla p + \eta \nabla^2 \mathbf{V} + \mathbf{J} \times \mu \mathbf{H}$$
 .....(2)

Maxwell's equation,
 $\nabla \cdot \mathbf{H} = 0$ .....(3)

$\nabla \times \mathbf{H} = \mathbf{J}$ .....(4)

$\epsilon \nabla \cdot \mathbf{E} = \rho$ .....(5)

$\nabla \times \mathbf{E} = -\mu (\partial \mathbf{H} / \partial t)$ .....(6)

Ohm's law for a moving fluid;

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mu \mathbf{H}) \quad (7)$$

where $\mathbf{V}$ and p are respectively the velocity and pressure of the fluid; $\mathbf{H}$, $\mathbf{J}$, $\mathbf{E}$ and $\mathbf{e}$ are magnetic field, current density, electric field and charge density; ρ , η , μ , ϵ and σ are the density, viscosity, permeability, dielectric constant and electrical conductivity of the fluid. In arriving at the above set of equations, we have assumed that the displacement current $\epsilon \partial \mathbf{E} / \partial t$ and the convection current $\mathbf{eV}$ can be neglected. These assumptions are, however, applicable for liquid metals in good approximation.

Now let us consider a steady laminar flow between parallel planes under a uniform magnetic field applied transversely (see Fig. 5). The following expression are obtained:

$$\begin{aligned} \mathbf{V} &= [V(y), 0, 0], \quad \mathbf{H} = [\beta(y)H_0, H_0, 0] \\ \mathbf{J} &= [0, 0, J(y)], \quad \mathbf{E} = (0, 0, E) \end{aligned} \quad (8)$$

where

$$\begin{aligned} V &= V_0 \frac{M(\cosh M - \cosh My/a)}{(M \cosh M - \sinh M)} \\ \beta(y) &= \left[\frac{\eta \sqrt{\sigma \eta}}{\rho a} \frac{M \tanh M}{(\tanh M - M)} \right. \\ &\quad \times \left. \left(\frac{y}{a} - \frac{\sinh My/a}{\sinh M} \right) R - I_m y \right] \frac{1}{H_0} \\ I_m &= I/4ac \text{ (mean current density)} \\ V_0 &= \text{mean velocity} \\ H_0 &= \text{uniform magnetic field applied transversely} \end{aligned} \quad (9)$$

The pressure and each component of the velocity and magnetic field vectors can be split into time-mean and fluctuation components as below:

$$\begin{aligned} \mathbf{v}' &= (v'_x, v'_y, v'_z) \\ \mathbf{H} &= (\beta H_0 + h_x, H_0 + h_y, h_z) \\ p &= (p + p') \end{aligned} \quad (10)$$

Here we consider a two-dimensional small disturbance and assume the disturbance to be generally written as

$$g(x, y, t) = g(y) e^{ik(x - \alpha t)} \quad (11)$$

Using Eq. (11) reduces Eq. (1) to

$$ikv_x + \frac{\partial v_y}{\partial y} = 0 \quad (12)$$

We assume that the disturbance is small and that the squares of the small quantities in all the equations can be neglected. Eliminating the pressure p from the components of x and y of Eq. (2), we obtain

$$\begin{aligned} v_y'' - 2k^2 v_y'' + k^2 v_y - (v_y'' - k^2 v_y)(V - \alpha) \\ \times \frac{ik}{V} + V'' v_y \frac{ik}{V} = -\frac{\mu H_0}{\eta} (h_y'' - k^2 h_y) \\ - \frac{\mu H_0 \beta}{\eta} ik(h_y'' - k^2 h_y) - \frac{\mu H_0 \beta'}{\eta} ik h_y \end{aligned} \quad (13)$$

where primes denote differentiation with respect to y . We can make the above equation nondimensional by taking

$$\begin{aligned} \phi = \frac{v_y}{V_0}, \quad \varphi = \frac{h_y}{H_0}, \quad \bar{V} = \frac{V}{V_0} \\ \bar{\alpha} = \frac{\alpha}{V_0}, \quad \bar{k} = ak, \quad \bar{y} = \frac{y}{a} \end{aligned} \quad (14)$$

where a , V_0 and H_0 are respectively the characteristic length, velocity and magnetic field. Using Eq. (14), we find that Eq. (13) becomes

$$\begin{aligned} (\phi'' - 2\bar{k}^2 \phi'' + \bar{k}^2 \phi) - i\bar{k}R[(\bar{V} - \bar{\alpha})(\phi'' - \bar{k}^2 \phi) \\ - \bar{V}'' \phi] = -\frac{M^2}{R_m} [\varphi''' - \bar{k}^2 \varphi \\ + i\bar{k}\bar{\beta}(\varphi'' - \bar{k}^2 \varphi) + i\bar{k}\bar{\beta}' \varphi] \end{aligned} \quad (15)$$

where $R_m = \sigma \mu a V_0$ (magnetic Reynolds number)

The boundary conditions for the velocity field will contain the following features:

(1) from the condition of no-slip at the walls,

$$\phi(1) = \phi'(1) = 0 \quad (16)$$

(2) from the consideration of the symmetric disturbance,

$$\phi'(0) = \phi'''(0) = 0 \quad (16)$$

The following equation is induced from Maxwell's equation and Ohm's law:

$$\frac{\partial \mathbf{H}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{H} - (\mathbf{H} \cdot \nabla) \mathbf{V} = \frac{1}{\sigma \mu} \nabla^2 \mathbf{H} \quad (17)$$

Repeating the same operation as that performed in momentum-equation, we can write the y -component of Eq. (17) as below:

$$\varphi'' - \bar{k}^2 \varphi = -R_m [\phi' + i\bar{k}\bar{\beta}\phi - i\bar{k}(\bar{V} - \bar{\alpha})\varphi] \quad (18)$$

The electric current flowing in an electrically conducting fluid is composed of two parts: one due to the induced electromotive force and the other due to the electric field caused by the voltage difference between the two electrodes. However, the latter current predominates in the MHD devices such as the conduction electromagnetic pump and therefore the secondary magnetic field induced by the current depends mainly on the current due to the voltage difference between the electrodes. From the above consideration, $\beta(y)$ in Eq. (9) is approximated as follows:

$$\beta(y) \approx -\frac{I_m y}{H_0} \quad (19)$$

and then

$$\beta''(y) = 0 \quad (20)$$

is obtained.

In addition, the value of the magnetic Prandtl number is very small ($P = R_m/R = 1.5 \times 10^{-7}$, for mercury) and the magnetic Reynolds number R_m is far smaller than one, if $R < 10^5$. Accordingly, we assume in Eq. (18) as follows:

$$\bar{k}^2 \gg |R_m \bar{k}(\bar{V} - \bar{\alpha})| \quad (21)$$

Hereafter, we omit the mark "-" denoting the nondimensional quantities for simplicity and obtain the following equation as basic equation, substituting Eqs. (18), (21) into Eq. (15):

$$\begin{aligned} (\phi'' - 2\bar{k}^2 \phi'' + \bar{k}^2 \phi) - i\bar{k}R[(V - \alpha) \\ \times (\phi'' - \bar{k}^2 \phi) - V'' \phi] = M^2 [\phi''' \\ + 2i\bar{k}\bar{\beta}\phi' + (i\bar{k}\bar{\beta}' - \bar{k}^2 \bar{\beta}^2)\phi] \end{aligned} \quad (22)$$

$$\phi(1) = \phi'(1) = 0$$

$$\phi'(0) = \phi'''(0) = 0$$

We can obtain curves of neutral stability in the following manner: We reduce Eq. (22) to a finite difference equation, give the wave number k , Reynolds number R , mean current density I_m and Hartmann number M and calculate the corresponding eigenvalue α .

A part of the curves of neutral stability for $M=1$, $\bar{I}(=I_m/aH_0)=0$ and 10 is shown in Fig.17. We find that the critical Reynolds numbers are respectively 10300 for $\bar{I}=0$ ($R_c=10450$ obtained by Lock⁽⁴⁾) and 11000 for $\bar{I}=10$ and that the electric current flowing in liquid metals stabilize the flow though its effect is only a very little.

Fig. 17

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