A Study on the Flows of Dilute Polymer Solutions*
(4th Report, On the Velocity Measurement by a Pitot Tube Method)

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Flow visualization, measurements of velocity fields etc. are utilized to determine flow behaviours of dilute polymer solutions. However, except in the case such as using a doppler anemometer, it is expected that the influence of insertion of probes such as a pitot tube on the measurement is comparatively large. In this report, a problem of determination of a flow velocity by a pitot tube method is discussed analytically and experimentally.

Velocity measurements in a turbulent pipe flow of dilute polymer solutions show that the flow rate determined from apparent velocities, which are obtained with a customary method assuming fully developed stresses, is less than the true value (of the order of 10%), but the resultant velocity profile corrected by the analysis of the flow field around a pitot tube is in fairly good agreement with that by other methods and the disagreement of the flow rate is improved. These results show the necessity of the pitot tube coefficient caused from viscoelastic behaviour of dilute polymer solutions.

1. Introduction

Some kinds of dilute polymer solutions show drag reduction phenomena such as in turbulent pipe flows, which is known as Toms' effect. Such drag reduction phenomena are receiving attention from a viewpoint of theoretical research and application to industry. With development of chemical engineering and industry, viscoelastic fluids including dilute polymer solutions are exploited extensively and application of them is extended in a wide range. However, up to the present the interior structure of the flow of dilute polymer solutions has not been completely investigated, and so in order to study it use is made of the method of flow visualization, velocity measurement and measurement of velocity fluctuation. Except in the case of exploiting the method of flow visualization or of measuring the velocity by a laser doppler velocimeter, effects of inserting probes such as a hot wire and a pitot tube into flows are expected qualitatively by Metzner et al. (4) and Savins (9) to be rather great compared with that for Newtonian fluid. In spite of these circumstances, the measured pitot tube pressure (difference) is sometimes assumed to have the same relationship as that between stresses developed in the flow without inserting a probe. From such a viewpoint it becomes also necessary to get the information about the stress state in addition to the pitot tube pressure in order to determine the velocity. Although interpretations of the measured quantities with probes are similar to that noted above, effects of insertion of a pitot tube seem not to have been discussed in detail except the case given by Hurd (2) on the effect of viscosity, and up to the present analytical and experimental treatment of the effects of viscoelasticity on the measured quantities has not been established.

So in this report, the problem on the velocity measurement by a pitot tube method in dilute polymer solutions is treated analytically and experimentally.

Nomenclature

a : outer diameter of a pitot tube/2
B_αγ : first kind of Rivlin-Ericksen acceleration tensor
B_αν : second kind of Rivlin-Ericksen acceleration tensor
D : pipe diameter
ν : metric tensor
p : pressure
Re : Reynolds number =VD/ν
s : index which indicates elastic behaviour of a solution
\tau_{ij} etc. : physical component of deviatoric tensor
u : x-component of velocity
U_v : uniform velocity, local velocity
u_* = \nu/\nu* : friction velocity \sqrt{\tau/\nu}
\nu_* : apparent velocity/\nu*
v : y-component of velocity
V : average velocity

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velocity vector
w : z-component of velocity
x : orthogonal coordinate
y : orthogonal coordinate
y_0 : distance from the wall
x' : y=y/v
z : orthogonal coordinate
\lambda : coefficient which indicates elastic behaviour of the solution
\mu : coefficient of viscosity
\nu : kinematic viscosity
\rho : density
\tau : shear stress at the wall
\nu^\prime : deviatoric stress tensor
\j : covariant derivative operator
\Delta : Laplacian operator

2. Points at issue on the velocity measurement with a pitot tube

2-1 In general as a principle for a total pressure tube it is considered that the opening point of a pitot tube is a stagnation point of the flow and that the same expression (1) that is derived from Bernoulli's theorem for a perfect fluid may hold:

\[ p_{\text{m}} = \frac{1}{2} \rho U^2. \quad (p_{\text{m}}: \text{measured pressure}, \quad p_s: \text{static pressure}) \quad (1) \]

In this interpretation two points at issue will remain; the first is whether the assumption that the opening point of a pitot tube is a stagnation point is appropriate or not and the second is whether the measured value is the same as the stagnation point even in the case when the former holds. In general the so-called measured pressure is equal to the mean pressure at the cross section of a pitot tube far from its opening point. In case of a perfect fluid, since the flow penetrates into the pitot tube, it is not generally appropriate to consider the opening point as a stagnation point. Thus Eq. (1) holds for a long pitot tube. In case of a viscous fluid, flow velocity in a pitot tube is relatively small and effects of viscosity cannot be neglected. In this case it is rather difficult to estimate the state of flow inside and around the pitot tube, and moreover experiments in general show almost the same results from the latter standpoint that the measured pressure is equal to the stagnation pressure (or the mean value around the stagnation point, i.e. the opening point of the pitot tube). So here the following discussion is limited to this latter standpoint.

2-2 Case of infinitesimal thickness of wall of a pitot tube and slow flow of a main stream
Let the main stream be uniform and x-axis be parallel to the flow direction, x- and y-axes be perpendicular to the z-axis. Then the equation of motion in z-direction becomes

\[ \rho \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) = \frac{\partial P}{\partial x} + \frac{\partial T_x}{\partial y} + \frac{\partial T_y}{\partial z} \]

The assumption that a flow boundary consists of the wall (of the pitot tube) and of an imaginary wall at rest ahead of the opening end of the pitot tube is adopted, i.e. the probe is considered to be of the form without any hole. Integrating Eq. (2) with respect to z from -\infty to z (at the wall at rest), we obtain

\[ p - p_s = \frac{1}{2} \rho U^2 \int_{-\infty}^{z} \left( \frac{\partial P}{\partial x} + \frac{\partial T_x}{\partial y} + \frac{\partial T_y}{\partial z} \right) dy \int_{-\infty}^{z} \left( \frac{\partial P}{\partial x} + \frac{\partial T_x}{\partial y} + \frac{\partial T_y}{\partial z} \right) dz \quad (3) \]

To determine the so-called measured pressure \( p_0 \) (which is defined as a mean value of the total stress on the wall at rest), it is necessary to specify the shape of the wall at rest and velocity distributions. If the flow field is approximated as one around a sphere of a radius \( \kappa \) and if a denotes a radius of the pitot tube (\( \rho \approx \kappa \)), then velocities become (as Stokes' approximation)

\[ u = \frac{U}{\sqrt{\rho}} \left( \frac{r}{\rho} - 1 \right) \]
\[ v = \frac{U}{\sqrt{\rho}} \left( \frac{r}{\rho} - 1 \right) \]
\[ w = \frac{U}{\sqrt{\rho}} \left( \frac{r}{\rho} - 1 \right) \frac{1}{2} \left( 4 + \frac{1}{\rho} + \frac{1}{\rho^2} \right) \]

Hence \( p_0 \) becomes

\[ p_0 = \frac{1}{2} \rho U^2 \int_{-\infty}^{z} \left( \frac{\partial P}{\partial x} + \frac{\partial T_x}{\partial y} + \frac{\partial T_y}{\partial z} \right) dy \int_{-\infty}^{z} \left( \frac{\partial P}{\partial x} + \frac{\partial T_x}{\partial y} + \frac{\partial T_y}{\partial z} \right) dz \quad (4) \]

where \( \rho = 3 \rho \left( 1 - \sqrt{\rho - r} \right) \)

\[ r = \rho \kappa \]

The term including \( r \) in Eq. (4) denotes the contribution from viscosity. The difference between Eq. (1) and Eq. (4) is approximately 10% when \( r = 0.1 \), if the term including \( r \) is omitted. On the other hand in the case that the thickness of the wall of the pitot tube is finite, \( r = \rho \kappa \) (the inner radius of the pitot tube) \( \rho \kappa \) and \( r = \rho \kappa \) (the outer radius) and so when \( 0.65 < r < 0.8 \), the difference is approximately 5%.

2-3 Case of fast flow of a main stream
The so-called pitot tube pressure \( p_0 \) becomes

\[ p_0 = \frac{1}{2} \rho U^2 \left( \frac{1}{2} + \frac{1}{\rho} \right) \quad (5) \]

From the same standpoint as in 2-2, where a potential flow of so-called pitot-tube flow is assumed and \( \rho, r \) denote a radius of curvature at a stagnation point and an inner radius of the pitot tube at the opening end respectively. According to Eq. (5), when \( \rho = 0.5 \), the value \( p_0 \) shows 21% smaller value (based on dynamic pressure) than that in Eq. (1) and it shows 8% and 4% smaller values when \( r = 0.3 \) and 0.2 respectively. Therefore if actual coefficients of pitot tubes are taken into consideration, from the standpoint of the
mean pressure it is only inferred that r/\kappa \\
<0.2. Hence from a viewpoint of utility, in the case that a radius of flow curvature is relatively large compared with the radius of the pitot tube, it is nearly reasonable to consider that the pitot tube pressure is nearly equal to that at the stagnation point, (i.e. opposite sign but with the same value of the normal stress there in general), and it may not be necessary to adopt the mean value there.

3. On the velocity measurement by the pitot tube method in a dilute polymer solution

According to the previous results, the following analysis is based on the assumption that the measured pressure is nearly equal to the normal stress (but opposite sign) at the stagnation point. The flow of a dilute polymer solution is regarded to be locally laminar and it is assumed that the velocity gradient in the main stream does not strongly affect the flow field around a pitot tube. That is, the flow field without inserting a pitot tube is regarded to be locally uniform and the flow field around a pitot tube is regarded to be axisymmetric.

Let the axis of the pitot tube be z-axis, and x- and y-axes be perpendicular to it (Fig.1). Equations of motions in x-, y- and z-directions are

\[
\begin{align*}
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \\
= -\frac{\partial p}{\partial x} + \frac{1}{2} \rho \frac{\partial (u^2 + v^2)}{\partial x} + \frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{xz}}{\partial z} + \frac{\partial T_{yz}}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial \tau_{yz}}{\partial y} \quad \cdots \cdots \cdots (6)
\end{align*}
\]

\[
\begin{align*}
\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \\
= -\frac{\partial p}{\partial y} + \frac{1}{2} \rho \frac{\partial (u^2 + v^2)}{\partial y} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} + \frac{\partial T_{yz}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial \tau_{yz}}{\partial x} \quad \cdots \cdots \cdots (7)
\end{align*}
\]

\[
\begin{align*}
\rho \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \\
= -\frac{\partial p}{\partial z} + \frac{1}{2} \rho \frac{\partial (u^2 + v^2)}{\partial z} + \frac{\partial T_{xy}}{\partial z} + \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} - \frac{\partial \tau_{xy}}{\partial z} - \frac{\partial \tau_{xz}}{\partial x} - \frac{\partial \tau_{yz}}{\partial y} \quad \cdots \cdots \cdots (8)
\end{align*}
\]

respectively. As in the preceding section, the region of flow is assumed to be divided into two parts by an imaginary stationary wall located at the opening end of the pitot tube. Since u and v = 0 on the z-axis according to the assumption, integrating Eq.(8) with respect to z from \( -\infty \) to \( z \) (stagnation point) gives

\[
-\frac{1}{2} \rho U^2 = -(s-T_m)z + \int_{-\infty}^{z} (\frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{xz}}{\partial z}) dz
\]

Hence

\[
\begin{align*}
U^2 = \frac{2}{\rho} \int_{-\infty}^{z} (\frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{xz}}{\partial z}) dz + \frac{z-T_m}{s}
\end{align*}
\]

\[
\begin{align*}
p = p_0 + T_m \frac{z}{s} - \frac{z-T_m}{s}
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dx} \frac{\partial T_{xy}}{\partial x} + \frac{d}{dy} \frac{\partial T_{xz}}{\partial z} + \frac{d}{dz} \frac{\partial T_{yz}}{\partial y} = \frac{d}{dx} \frac{\tau_{xy}}{\partial x} + \frac{d}{dy} \frac{\tau_{xz}}{\partial z} + \frac{d}{dz} \frac{\tau_{yz}}{\partial y}
\end{align*}
\]

where

\[
p = p_0 + T_m \frac{z}{s} - \frac{z-T_m}{s}
\]

In Eq.(9), \( p_0 \), \( p_m \) and \( T_m \) denote the so-called pitot tube pressure, the pressure at \( z=\infty \) and the deviatoric stress at \( z=\infty \) respectively. The integral term of the right-hand side of Eq.(9) represents the character of insertion of a pitot tube and it depends on the dimension of a pitot tube and rheological behavior of the solution. Let's estimate this character approximately. As a constitutive equation:

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} = \frac{1}{2} \frac{\partial \tau_{xy}}{\partial x} + \frac{1}{2} \frac{\partial \tau_{xz}}{\partial z} + \frac{1}{2} \frac{\partial \tau_{yz}}{\partial y}
\]

where

\[
\begin{align*}
-B_{01}^{(1)} & = \frac{1}{2} \frac{\partial \tau_{xy}}{\partial x} + \frac{1}{2} \frac{\partial \tau_{xz}}{\partial z} + \frac{1}{2} \frac{\partial \tau_{yz}}{\partial y} \\
-B_{01}^{(1)} & = \frac{1}{2} \frac{\partial \tau_{xy}}{\partial x} + \frac{1}{2} \frac{\partial \tau_{xz}}{\partial z} + \frac{1}{2} \frac{\partial \tau_{yz}}{\partial y}
\end{align*}
\]

and in orthogonal Cartesian coordinates \((x, y, z) \), \( T_m = \rho \eta \) etc. The index \( s \) in Eq.(10) lies in the range 0 < \( s < \infty \). In a dilute polymer solution over an appropriate range of shear rates it is considered that \( s \approx 2 \).

5-1 Slow flow

5-1.1 Case \( s=2 \): The terms representing acceleration in Eqs.(6)-(8) are omitted as Stokes' approximation and Oseen's approximation is applied to each component of \( \dot{\lambda} \dot{\omega} \) which represents viscoelastic behavior of the solution. That is, we substitute \( u, v, w \) for \( u, v, w \) respectively and neglect the terms such as product of \( u, v, w \) etc. Then equations of motion become

\[
\begin{align*}
0 & = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) \\
0 & = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right) \\
0 & = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left( \mu \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial w}{\partial z} \right)
\end{align*}
\]

And the equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

From these equations we obtain \( \delta p = 0 \). Let \( \rho \) be a harmonic function of degree \( n \), and if we assume

\[
\begin{align*}
\epsilon = \sum \left( -\frac{1}{\mu} \frac{\partial \phi}{\partial x} \right) \\
\nu = \sum \nu_0, \quad \kappa = \sum \kappa_0, \quad \omega = \sum \omega_0
\end{align*}
\]

then

\[
\begin{align*}
\frac{\partial \phi}{\partial x} = \mu \frac{\partial u}{\partial x}, \quad \frac{\partial \phi}{\partial y} = \mu \frac{\partial v}{\partial y}, \quad \frac{\partial \phi}{\partial z} = \mu \frac{\partial w}{\partial z}
\end{align*}
\]

Hence general solutions become
In the case that the imaginary stationary wall is a sphere of a radius $\kappa$, non-zero functions among $P_n$, $\phi_n$ and $\chi_n$ are

\begin{align*}
\rho_{,1} = A \nu^2, \quad \phi_{,1} = B \nu^2
\end{align*}

and according to the boundary conditions ($r = \kappa$, $w = 0$, $w = U_0$) the following are obtained:

\begin{align*}
A = -1.5 \nu U_0, \quad B = -0.25 \nu U_0^2
\end{align*}

With this velocity distribution, Eq.(9) becomes

\begin{align*}
\frac{\rho_{,1} - \rho_{,2} + T_{\nu,2}}{\rho U_0^2/2} = 1 + \frac{3\nu^2}{4} \frac{\kappa}{R^2} \text{..............(12)}
\end{align*}

where $R = \kappa/\nu$.

3-1.2 Case $s < 2$: It is very difficult to handle this case reasonably, and so approximate estimation for Eq.(9) with the same velocity distributions as in (3-1.1) gives

\begin{align*}
\frac{\rho_{,1} - \rho_{,2} + T_{\nu,2}}{\rho U_0^2/2} = 1 + \frac{3\nu^2}{4} \frac{\kappa}{R^2} \frac{\Gamma(s+1)}{\Gamma(s+1)} \frac{\nu}{\rho U_0^2/2} \text{R}^s \text{.........(13)}
\end{align*}

where $\Gamma(\cdot)$ denotes a Gamma function, and $R = \kappa/\nu$.

The second term of the right-hand side of Eqs.(12) and (13) denotes contribution from viscosity and the third term denotes contribution from viscoelastic behaviour. As a result in the case of a slow flow of a dilute polymer solution, it is found that the effect of viscosity on the character of pitot tube coefficient is greater than that of viscoelasticity when $1 < s < 2.5$.

3-2 Fast flow

Let $\kappa$ be a radius of curvature (at the stagnation point) of an imaginary stationary wall ahead of the opening end of the pitot tube. Since in a dilute polymer solution for Reynolds number $R$ ($eU_0\kappa/\nu > 100$):

\begin{align*}
\frac{\nu}{\rho U_0^2/2} = \frac{\nu}{\rho U_0^2/2} \text{R}^s \text{..............(11)}
\end{align*}

it is appropriate to consider an external flow around the pitot tube to be a potential flow. Also in the vicinity of the stationary wall the flow will be one of boundary layer type. So the dependence of the character of pitot tube coefficient in Eq.(9) can be divided into two parts, i.e. the first element is due to a potential flow and the second is due to a boundary-layer flow. In the flow with relatively weak viscoelasticity such as in a dilute polymer solution, from the analogy of the results for the Newtonian fluid flow the contribution of a potential flow is expected to be of order $\sqrt{R}$-times that of a boundary-layer flow. As a potential flow the following two cases are adopted: a flow around a sphere with a radius $\kappa$, and a so-called pitot-tube flow with a radius $\kappa$ of curvature at the stagnation point. Stream functions $\psi$ are

\begin{align*}
\phi = 0.5U_0^2(1-x^2)^{1/2}, \quad r = (a^2+y^2)^{1/2}
\end{align*}

for the former:

\begin{align*}
\frac{\nu}{\rho U_0^2/2} = \frac{\nu}{\rho U_0^2/2} \text{R}^s \text{..............(11)}
\end{align*}

As velocity distributions in the boundary-layer, the solutions (5) obtained by Prüssing for a flow around an axisymmetric body are exploited. These velocity distributions give, as the estimate for Eq.(9),

in the former case:

\begin{align*}
\frac{\rho_{,1} - \rho_{,2} + T_{\nu,2}}{\rho U_0^2/2} = 1 + \frac{6}{R(1 + \sqrt{3}/R)} \frac{22.5}{R \sqrt{R}} \frac{1}{R^2} \frac{\nu^2}{R^2} \frac{1}{\rho U_0^2/2} \text{R}^s \text{..............(14)}
\end{align*}

in the latter case:

\begin{align*}
\frac{\rho_{,1} - \rho_{,2} + T_{\nu,2}}{\rho U_0^2/2} = 1 + \frac{16}{3R(1 + \sqrt{3}/R)} \frac{21.5}{R \sqrt{R}} \frac{36.0}{R^2} \frac{\nu^2}{R^2} \frac{1}{\rho U_0^2/2} \text{R}^s \text{..............(15)}
\end{align*}

where the functions $\lambda(\cdot)$, $\kappa(\cdot)$, $\kappa(\cdot)$ are functions (Fig.15) given by Prüssing. In the case when $\nu > 100$, not so great differences between Eqs.(14) and (15) can appear. Here we rewrite Eqs.(14) and (15) as

\begin{align*}
\frac{\rho_{,1} - \rho_{,2} + T_{\nu,2}}{\rho U_0^2/2} = 1 + \frac{\nu}{\rho U_0^2/2} \text{R}^s \text{..............(16)}
\end{align*}
The curves $\varepsilon_1$ vs. $R$ and $\varepsilon_2$ vs. $R$ are shown in Fig.2; Figs.3 and 4 respectively. Since

\[
\lim_{R \to \infty} \varepsilon_1 = \text{const.}, \quad \lim_{R \to \infty} \varepsilon_2 = \text{const.}
\]

according to Eqs. (14) and (15), it is found that the effect of viscoelasticity on the character of pitot tube coefficient is greater than that of viscosity when $1 < s < 2$ as in dilute polymer solutions.

4. Experimental results and discussion

Since it is very hard to realize a uniform flow of a dilute polymer solution, measurements of velocity are executed in a turbulent region of a pipe flow. The pipes used are made of acrylic with 1.615 cm and 0.862 cm inner diameter, and the pitot tubes used are 0.08 cm, 0.07 cm, and 0.06 cm outer diameter. The solutions used are 5.30 ppm aqueous solution of PEO 18 (polyethylene oxide) and 2.30 ppm aqueous solution of SEPARAN AP-30 (poly-acrylamide). These solutions are expected to show Newtonian viscosity. The experimental apparatus is shown in Fig.5 schematically. We measure pressure difference between the so-called pitot tube pressure and the so-called static pressure which is transmitted through a hole (0.5 mm) on the wall. In incompressible fluids, pressure does not generally coincide with the mean normal stress with a negative sign in three mutually perpendicular directions. (In Newtonian fluids, it does necessarily.) This character depends on the rheological behaviour of the solution. According to Eq.(10) which represents rheological behaviour of dilute polymer solutions rather well, it is found that in a simple shear flow no deviatoric stress appears in the direction perpendicular to the shear surface. Also in general it is considered that a simple shear flow field is approximately constructed in the vicinity of a stationary wall. Since it is found experimentally that the value of total stress $\tau_w$ does not vary in a radial position, in the case that the flow is almost parallel to the stationary wall and that a velocity component perpendicular to the wall is relatively small, such as in a pipe flow, the measured pressure through a hole on the wall may be regarded as a static pressure. So the pressure difference measured is considered to be nearly $(p_0 - p_s)$. The effect of the velocity fluctuation in a turbulent flow on measurement may not be so different from that in Newtonian fluid, and so consideration for it is omitted in the following discussion.

4-1 Checking whether the pitot tube coefficient is unity

A pitot tube coefficient is defined as the value appearing in the left-hand side of Eq.(16). Since in water $\tau_{nw} = 0$ and $\lambda = 0$, the pitot tube coefficient depends only on viscosity. Dimensionless velocity distribution for water obtained from the assumption $\varepsilon_1 = 0$ is shown in Fig.6. The flow rate calculated with these dimensionless velocity distributions is a little greater than that measured by weighing it, but the differences remain within $\pm 3\%$. In the case of polymer solutions, apparent dimensionless velocity distributions obtained from the assumptions $\varepsilon_1 = 0$ and $\tau_{nw} = 0$ are shown in Figs.7 and 8. In these figures, apparent velocities in the buffer layer are less than those in Newtonian fluid and in the sublayer the...
relation $\omega = \omega'$ (which may hold in a simple shear flow and is confirmed by measurement with laser etc.) does not hold. These show that at least the term $T_{e-m}$ cannot be omitted. Figure 9 shows an example of a velocity distribution corrected with $T_{e-m} = 24 \frac{\partial \bar{u}}{\partial y}$, $(\sigma = 3, \lambda = 2 \times 10^5 \text{g/cm})$. This correction does not produce the relation $\omega = \omega'$ in the sublayer and although the flow rate computed from apparent velocity distribution is about 10% less than the true flow rate, this correction improves the difference between flow rates only by an order of 1%. From this it is predicted that due to viscoelastic behaviour of the solution the character of a pitot tube coefficient, i.e. the right-hand side of Eq.(16), has a greater influence on determination of velocities than the effect of stress $T_{e-m}$ developed in an undisturbed flow. That is, it is unnatural to assume that the pitot tube coefficient is unity.

4-2 Pitot tube coefficient and discussion

The ratios of the difference between an apparent mean velocity (obtained from an apparent flow rate) and a true mean velocity to the true mean velocity are shown in Table 1-(a),(b). These show that due to viscoelastic behaviour of the solution the apparent mean velocities are less than true values. In order to determine the velocity according to Eq.(16), it is necessary to know the characteristic values of the solution, i.e. $\lambda$ and $s$.

However, since it is very hard to measure the characteristic values over relatively low shear rates such as shear rate at the wall $(\leq 1500 / \text{s})$ or mean velocity $V$ (a radius of a pitot tube) $(\leq 2000 / \text{s})$ in this experiment, these values are inferred from those over the range of higher shear.

![Fig.7](image_url)

**Fig.7** Apparent dimensionless velocity distributions for aqueous solution of SEPARAN

![Fig.8](image_url)

**Fig.8** Apparent dimensionless velocity distributions for aqueous solution of PEO 15

### Table 1 (a)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Water</th>
<th>Aqueous solution of PEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration ppm</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Stabilizer</td>
<td>Sundex C</td>
<td></td>
</tr>
<tr>
<td>Reynolds number Re</td>
<td>4960</td>
<td>7050</td>
</tr>
<tr>
<td>Pipe diameter D cm</td>
<td>1.615</td>
<td>0.862</td>
</tr>
<tr>
<td>Pitot tube diameter cm</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>$(\bar{V} - V') / V$</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>$\sigma$ dyn/cm²</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>$\bar{V} - V'$ / V</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma$ dyn/cm²</td>
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<td>-5.2</td>
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<tr>
<td>$\sigma$</td>
<td>300</td>
<td>150</td>
</tr>
<tr>
<td>$\bar{V} - V'$ / V</td>
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<td>1.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-18.2</td>
<td>-2.1</td>
</tr>
</tbody>
</table>

### Table 1 (b)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Water</th>
<th>Aqueous solution of Separan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration ppm</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Stabilizer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reynolds number Re</td>
<td>4200</td>
<td>4550</td>
</tr>
<tr>
<td>Pipe diameter D cm</td>
<td>0.862</td>
<td>3.165</td>
</tr>
<tr>
<td>Pitot tube diameter cm</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>$(\bar{V} - V') / V$</td>
<td>5.8</td>
<td>-7.3</td>
</tr>
<tr>
<td>$\sigma$ dyn/cm²</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$(\bar{V} - V') / V$</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma$ dyn/cm²</td>
<td>-1.0</td>
<td>-0.7</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>$(\bar{V} - V') / V$</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-2.1</td>
<td>-5.6</td>
</tr>
</tbody>
</table>

$\bar{V}$: Mean velocity obtained from apparent velocity distribution

$V$: Mean velocity

$V_0$: Mean velocity obtained from corrected velocity distribution
rates. In spite of this the values of $s$ are not so clearly determined, and so correction according to Eq. (16) is based on some suitable values of $s$. In these computation it is assumed that $2\kappa$ = the outer diameter of the pitot tube and it is noted that $n = 2.5$ for $e_1$ and $c_2$, the average of the values according to Eq. (14) and (15) is adopted. The resultant dimensionless velocity distributions are shown in Figs. 10, 14.

These figures show the following: (1) Underestimation for the velocity near the wall is improved. (2) Figures 12 and 14, in which a clear turbulent core region exists, exhibit the relation $u' = 2.5 \ln y' + \text{const.}$ in turbulent region, which is reported by Rollin et al. (8) (with optical measurement). (3) It is inferred that the relation $u' = A_0 \ln y' - B_0$ (1) in the appropriate range may hold (different values for $A_0$, $B_0$ are reported, but $A_0 = 10$ and $B_0 = 13$ according to Ref. (1)). Furthermore, the comparison between the average velocities calculated with these corrected
velocity distributions and those measured is shown also in Table 1. This shows that agreement of the flow rate corrected is better than that based on apparent velocity distribution.

From the above, it is found that better results of the velocity measurement are obtained if Eq.(16) is utilized as the pitot tube coefficient.

5. Conclusions

It is found that in the case of measuring the velocities of a dilute polymer solution by a pitot tube method, viscoelastic behaviour of the solution has a great influence on the character of the pitot tube.

These effects might be estimated approximately with Eq.(16) in this report. However because of the complexity of the flow behaviour, further discussions will be necessary, e.g. by utilizing a laser doppler velocimeter.

References

(5) Frössling, N., cited from "Boundary-Layer Theory (H. Schlichting, McGraw-Hill)"