536.33 : 535.23

Radiative Heat Transfer in Cylinder-Row Systems

(Part 1, Gray Cylinder Rows and Black

Cylinders of Semi-Infinite Rows)*

By Hidetoshi MASUDA**

The system of cylinder-rows is favored in heat-exchangers or radiators, but estimation of its radiative transfer is often unavailable due to insufficiency of data provided in few reports. The purpose of this study is to identify the characteristics of radiative transfer in such cylinder-row systems. Analysis is made on two-row gray circular cylinders arranged on square-centers to obtain local radiative fluxes. Then the influences on radiative transfer are found of emissivity and the center-to-center distance of cylinders. Approximate solutions are additionally evaluated to compare with the numerical solutions. The same procedures are employed in one-row cylinders to see the influence of the number of rows. In black, semi-infinite-row cylinders (arranged infinitely from the first row on one side), variations in radiative heat rate are evaluated in several rows from the initial, in the range where the heat transfer is significant.

1. Introduction

In heat exchangers designed to attain an effective heat transfer performance with size and weight reduced as much as possible, the heat-transfer surfaces have been devised into various shapes. In practical use of these, even in the atmosphere of the earth, however, the heat transfer will be affected significantly by radiative transfer, depending possibly on the shapes and temperatures of these surfaces, and the surrounding conditions. This eventually needs exact analysis of radiative transfer on such surfaces. Studies in radiative transfer between surfaces have been reported on various shapes, but they are generally confined to the cases of geometrically simple system, leaving out the radiative transfer in practical cases mostly difficult to estimate.

Tube or cylinder rows, presumed to constitute favorable surfaces for heat transfer, are preferably used in tube-bundle-type heat exchangers or pin-fin-type radiators⁽¹⁾; the latter are appreciated highly efficient⁽²⁾. The heat-transfer surfaces are usually composed of cylinders (or tubes) arranged at certain intervals in rows. Radiative heat transfer in such a system is done between the cylinders, their fixed base surfaces and the external environment. Concerning these, however, very few investigations have been per-

formed, probably because of complexity in numerical analysis involved. Included in a handful of reports on radiative transfer between cylinders are the study of Hottel⁽³⁾ dealing with two rows of black tubes as the problem within the furnace, and others of Sparrow and Eckert⁽⁴⁾ with some relation to the problem, referring to the heat transfer between the surfaces of fin-and-tube-type radiators. In another study, Sikka and Iqbal⁽⁵⁾ analyzed temperature distributions on a single fin of pin-fin surfaces, taking into account radiation interaction with the cylinder base.

The purpose of this study is to identify the characteristics of radiative heat transfer from the cylinder-row system to the external environment. On a system of two-row gray cylinders, infinitely long, arranged on square-centers, analysis is made to obtain the local radiative heat fluxes and the mean fluxes, identifying the effect on them of both emissivity and the center-to-center distance of cylinders. The results of the analysis are applicable as well to the radiative transfer in the system in which the cylinder rows are thermal receivers⁽⁶⁾. Comparative evaluations are made between approximate solutions based on the simplified theory⁽⁷⁾ and numerical solutions in these two-row cylinders and also in one-row cylinders to estimate the influence of the number of rows. In black cylinders of semi-infinite rows as a basic configuration of a large number of cylinder rows favorably used in practical designs, the radiative heat fluxes from the cylinders are evaluated in a significant range of several rows from the initial. Changes in the radiative transfer rate from a cylinder in each of the rows in the system are also assessed.

* Received 28th March, 1973.

** Assistant Professor, the Institute of High Speed Mechanics, Tohoku University, Sendai.

2. Nomenclature

- $B, \bar{B}$: radiosities
 C : center-to-center distance
 F : angle factor
 F_{i-iR} : angle factor of all cylinders in i th row as viewed from cylinder 1
 $q, \bar{q}$: net heat fluxes
 Q : heat transfer rate per unit length and unit time
 R : cylinder radius
 T : temperature, degrees absolute
 $\beta, \bar{\beta}$: dimensionless radiosities, $B/\epsilon\sigma T_w^4$, $\bar{B}/\epsilon\sigma T_w^4$
 γ : dimensionless distance, C/R
 ϵ : emissivity
 θ : angular position coordinate on cylinder
 σ : Stefan-Boltzmann's constant
 ϕ : incident angle in Fig. 2
 ϕ', ϕ'' : angles to both edges of cylinder in Fig. 2 ($\phi' > \phi''$)
 Subscripts
 1, 2 : first and second row
 11, 20, ... : cylinder 11 and 20, ...
 w : cylinder surface

3. Heat transfer from two-row gray cylinders

3.1 Basic equation

Supposed is a system of two-row cylinders, R in radius, infinite in length, lined infinitely and arranged on square-

centers with center-to-center distance C as shown in Fig. 1. Analysis is made for radiative transfer from the system to the external environment (to the imaginary surfaces shown by dotted lines in Fig. 1). In the analysis, the following assumptions are provided:

(i) Each cylinder surface is a gray diffuse emitter and reflector with the same emissivity ϵ .

(ii) The temperature T_w is uniform on each surface and the same in all cylinders.

(iii) The environment (imaginary surfaces) is black with zero temperature, and the intervening medium is transparent for radiation.

Under these assumptions, local radiative flux on cylinders is symmetrically distributed about the central axis XX' (Fig. 1). Also in each cylinder, its distribution is considered to be symmetrical about the center-line of the circle perpendicular to XX' (e.g., YY'). Then the radiative transfer in the system can be determined by analyzing only the flux on the semicircumference of an arbitrary cylinder.

A cylinder in the first row is denoted by 1 (cylinder 1) and other cylinders by 11; 20, ..., 2k, ..., 2'1, ..., 2'k', ..., as shown in Fig. 1. In each cylinder, an angle between its center-line perpendicular to XX' and a radius to a point on the circumference is designated by θ , with the direction as shown in Fig. 2

Fig. 1 Configuration of cylinder-row system

Fig. 2 Angular coordinate system

(suffix denoting the corresponding cylinder). An angle between the normal at a point A of angular position (θ_1) on the circumference of cylinder 1 and a straight line from A to a point of angular position (θ) on the circumference of another cylinder is designated by ϕ , positive in the clockwise direction (Fig. 2). Two angles from the normal at A to the lines to both edges of the visible arc of the related cylinder are designated by ϕ' and ϕ''

($\phi' > \phi''$).

When radiosities on the cylinders in the first and the second row are denoted by B_1 and B_2 respectively, and the corresponding dimensionless radiosities by β_1 and β_2 , the dimensionless radiosity $\beta_1(\theta_1)$ for the elementary area at A, inclusive of all possible irradiation from the cylinders in the first and the second row, can be expressed by the following equation, within $0 \leq \theta \leq \pi$:

$$\beta_1(\theta_1) \equiv \frac{B_1(\theta_1)}{\epsilon \sigma T_w^4} = 1 + \frac{(1 - \epsilon)}{2} \left\{ \int_{\phi_{11}''}^{\phi_{11}'} \beta_1(\theta_{11}) \cos \phi \, d\phi + \sum_{k=n_1}^{n_2} \int_{\phi_{2k}''}^{\phi_{2k}'} \beta_2(\theta_{2k}) \cos \phi \, d\phi + \sum_{k'=1}^{n'} \int_{\phi_{2'k'}''}^{\phi_{2'k}'} \beta_2(\theta_{2'k'}) \cos \phi \, d\phi \right\} \quad (1)$$

where θ_{11} , θ_{2k} and $\theta_{2'k'}$ are given by

$$\theta_{11}(\theta_1, \phi) = \frac{3}{2} \pi - \theta_1 - \phi - \cos^{-1} \{ (1 + \gamma^2 - 2\gamma \sin \theta_1)^{1/2} \sin \alpha_1 \} \quad (2.a)$$

$$\alpha_1 = -\pi + \theta_1 + \phi + \tan^{-1} \frac{\gamma - \sin \theta_1}{\cos \theta_1} \quad (2.b)$$

$$\theta_{2k}(\theta_1, \phi) = \frac{3}{2} \pi - \theta_1 - \phi - \cos^{-1} \{ [1 + \gamma^2(1 + k^2) - 2\gamma(k \sin \theta_1 + \cos \theta_1)]^{1/2} \sin \alpha_2 \} \quad (3.a)$$

$$\alpha_2 = \theta_1 + \phi - \tan^{-1} \frac{k\gamma - \sin \theta_1}{\gamma - \cos \theta_1} \quad (3.b)$$

$$\theta_{2'k'}(\theta_1, \phi) = \frac{\pi}{2} + \theta_1 + \phi + \cos^{-1} \{ [1 + \gamma^2(1 + k'^2) + 2\gamma(k' \sin \theta_1 - \cos \theta_1)]^{1/2} \sin \alpha_2' \} \quad (4.a)$$

$$\alpha_2' = \theta_1 + \phi + \tan^{-1} \frac{k'\gamma + \sin \theta_1}{\gamma - \cos \theta_1} \quad (4.b)$$

In the braces in the right-hand member of Eq. (1), the first term accounts for the irradiation from cylinder 11 to cylinder 1, and the second and the third term for the sum of irradiation to cylinder 1 from the cylinders that line up rightward from cylinder 20, and for that from the cylinders lining up leftward from cylinder 2'1. So, the second term drops out if the point A is not irradiated by the cylinders leftward from cylinder 2'1, and the second and the third term drop out if the point A is not at all irradiated by the cylinders in the second row. In Eq. (1), n_1 , n_2 and n' specify the range of cylinders in the second row that can irradiate A. These can be determined geometrically; particularly when A is at $\theta_1 = 0$, they are assigned to:

$$n_1 = 0, \quad n_2 = n' = \infty \quad (5)$$

The upper and the lower limits of integral of each integral term in Eq. (1), ϕ' and ϕ'' , are angles from the normal at A to the tangents to any cylinder when the cylinder is entirely visible from A (e.g., cylinder 20 in Fig. 2); when the sight from A to any cylinder is partly intercepted by some other cylinder, either ϕ' or ϕ'' is replaced by an angle to the tangent to the intercepting cylinder (e.g., cylinder 2k in Fig. 2). When a part of a cylinder is out of the range of sight ($-\pi/2 \leq \phi \leq \pi/2$), the relation becomes either $\phi' = \pi/2$ or $\phi'' = -\pi/2$ (e.g., cylinders 11 and 2'k' in Fig. 2). When the sight to any cylinder is not intercepted by any other cylinder, ϕ' and ϕ'' can be expressed as follows:

ϕ_{11}' and ϕ_{11}'' are

$$\begin{aligned} 0 \leq \theta_1 \leq \tan^{-1} \frac{2}{\sqrt{\gamma^2 - 4}} & \quad ; \quad \phi_{11}' = \frac{\pi}{2}, \quad \phi_{11}'' = \phi_{11}^- \\ \tan^{-1} \frac{2}{\sqrt{\gamma^2 - 4}} \leq \theta_1 \leq \pi - \tan^{-1} \frac{2}{\sqrt{\gamma^2 - 4}} & \quad ; \quad \phi_{11}' = \phi_{11}^+, \quad \phi_{11}'' = \phi_{11}^- \\ \pi - \tan^{-1} \frac{2}{\sqrt{\gamma^2 - 4}} \leq \theta_1 \leq \pi & \quad ; \quad \phi_{11}' = \phi_{11}^+, \quad \phi_{11}'' = -\frac{\pi}{2} \end{aligned} \quad (6)$$

where

$$\begin{aligned}\phi_{11}^+ &= \frac{\pi}{2} - \theta_1 + \tan^{-1} \frac{\cos \theta_1}{\gamma - \sin \theta_1} + \tan^{-1} \frac{1}{(\gamma^2 - 2\gamma \sin \theta_1)^{1/2}} \\ \phi_{11}^- &= \frac{\pi}{2} - \theta_1 + \tan^{-1} \frac{\cos \theta_1}{\gamma - \sin \theta_1} - \tan^{-1} \frac{1}{(\gamma^2 - 2\gamma \sin \theta_1)^{1/2}}\end{aligned}\quad (7)$$

ϕ_{2k}' and ϕ_{2k}'' are

$$\begin{aligned}0 \leq \theta_1 \leq \frac{\pi}{2} - \tan^{-1} \frac{1}{k} + \tan^{-1} \left\{ \frac{\gamma^2}{4} (1 + k^2) - 1 \right\}^{1/2} &; \quad \phi_{2k}' = \phi_{2k}^+, \quad \phi_{2k}'' = \phi_{2k}^- \\ \frac{\pi}{2} - \tan^{-1} \frac{1}{k} + \tan^{-1} \left\{ \frac{\gamma^2}{4} (1 + k^2) - 1 \right\}^{1/2} \leq \theta_1 \leq \pi - \tan^{-1} \frac{1}{k} &; \quad \phi_{2k}' = \phi_{2k}^+, \quad \phi_{2k}'' = -\frac{\pi}{2}\end{aligned}\quad (8)$$

where

$$\begin{aligned}\phi_{2k}^+ &= -\theta_1 + \tan^{-1} \frac{k\gamma - \sin \theta_1}{\gamma - \cos \theta_1} + \tan^{-1} \frac{1}{\{\gamma^2(1 + k^2) - 2\gamma(k \sin \theta_1 + \cos \theta_1)\}^{1/2}} \\ \phi_{2k}^- &= -\theta_1 + \tan^{-1} \frac{k\gamma - \sin \theta_1}{\gamma - \cos \theta_1} - \tan^{-1} \frac{1}{\{\gamma^2(1 + k^2) - 2\gamma(k \sin \theta_1 + \cos \theta_1)\}^{1/2}}\end{aligned}\quad (9)$$

$\phi_{2k'}^+$ and $\phi_{2k'}''$ are

$$\begin{aligned}0 \leq \theta_1 \leq -\frac{\pi}{2} + \tan^{-1} \frac{1}{k'} + \tan^{-1} \left\{ \frac{\gamma^2}{4} (1 + k'^2) - 1 \right\}^{1/2} &; \quad \phi_{2k'}^+ = \phi_{2k'}^+, \quad \phi_{2k'}'' = \phi_{2k'}^- \\ -\frac{\pi}{2} + \tan^{-1} \frac{1}{k'} + \tan^{-1} \left\{ \frac{\gamma^2}{4} (1 + k'^2) - 1 \right\}^{1/2} \leq \theta_1 \leq \tan^{-1} \frac{1}{k'} &; \quad \phi_{2k'}^+ = \phi_{2k'}^+, \quad \phi_{2k'}'' = -\frac{\pi}{2}\end{aligned}\quad (10)$$

where

$$\begin{aligned}\phi_{2k'}^+ &= -\theta_1 - \tan^{-1} \frac{k'\gamma + \sin \theta_1}{\gamma - \cos \theta_1} + \tan^{-1} \frac{1}{\{\gamma^2(1 + k'^2) + 2\gamma(k' \sin \theta_1 - \cos \theta_1)\}^{1/2}} \\ \phi_{2k'}^- &= -\theta_1 - \tan^{-1} \frac{k'\gamma + \sin \theta_1}{\gamma - \cos \theta_1} - \tan^{-1} \frac{1}{\{\gamma^2(1 + k'^2) + 2\gamma(k' \sin \theta_1 - \cos \theta_1)\}^{1/2}}\end{aligned}\quad (11)$$

If the point A is not irradiated from any of the cylinders in the second row, Eq. (1) becomes:

$$\begin{aligned}\beta_1(\theta_1) &= 1 + \frac{(1 - \epsilon)}{2} \\ &\times \int_{\phi_{11}''}^{\phi_{11}'} \beta_1(\theta_{11}) \cos \phi \, d\phi\end{aligned}\quad (12)$$

Equation (1) is an integral equation on β_1 and β_2 . The dimensionless radio-sities β on a cylinder surface are distributed symmetrically about a central axis (e.g., YY' in Fig. 1); on the other hand, the relation between β_1 and β_2 is satisfied for symmetrical positions on cylinders in the first and the second row about the axis XX' by the following equation:

$$\beta_1 = \beta_2 \quad (13)$$

With these relations, Eq. (1) can be solved by the iterative method⁽⁷⁾. In the actual process of solution, however, n_2 and n' are considerably large for a small value of θ_1 . Then some integral terms in Eq. (1) are presumed to take an extremely small value, and for any of them approximation may be desirable. In the braces of the right-hand terms of Eq. (1), when k in the second and k' in the third term are considerably large, the corresponding integral terms will have very small values with $(\phi_{2k}' - \phi_{2k}'')/\pi \ll 1$ and $(\phi_{2k'}^+ - \phi_{2k'}^-)/\pi \ll 1$, respectively. If cylinders corresponding to these integral terms, viewed from A, partly overlap each other, the sum of terms with indexes k and k' larger than arbitrary k_0 and k'_0 can be approximated by

$$\sum_{k=k_0}^{n_2} \int_{\phi_{2k}''}^{\phi_{2k}'} \beta_2(\theta_{2k}) \cos \phi \, d\phi = \frac{\beta_2(\theta_{2k_0}') + \beta_2(\theta_{2k_0}'')}{2} \int_{\phi_{2k_0}''}^{\phi_{11}'} \cos \phi \, d\phi \quad (14)$$

$$\sum_{k'=k'_0}^{n'} \int_{\phi_{2k'}''}^{\phi_{2k'}'} \beta_2(\theta_{2k'}) \cos \phi \, d\phi = \frac{\beta_2(\theta_{2k'_0}') + \beta_2(\theta_{2k'_0}'')}{2} \int_{-\pi/2}^{\phi_{2k'_0}'} \cos \phi \, d\phi \quad (15)$$

In the present calculations, with the above situations, the first k and k' respectively satisfying the conditions of $(\phi_{2k}'' - \phi_{2k}')/\pi < 0.001$ and $(\phi_{2k'}'' - \phi_{2k'}')/\pi < 0.001$ were defined as k_0 and k'_0 .

Once dimensionless radiosity $\beta_1(\theta_1)$ has been found, net heat flux $q_1(\theta_1)$ can be obtained for $\epsilon \neq 1$ from

$$\frac{q_1(\theta_1)}{\epsilon \sigma T_w^4} = \frac{1}{1 - \epsilon} \{1 - \epsilon \beta_1(\theta_1)\} \quad (16)$$

and for $\epsilon = 1$, considering $\beta_1 = 1$, from

$$\frac{q_1(\theta_1)}{\sigma T_w^4} = 1 - (F_{d\theta_1-11} + \sum_{k=n_1}^{n_2} F_{d\theta_1-2k} + \sum_{k'=1}^{n'} F_{d\theta_1-2'k'}) \quad (17)$$

In Eq. (17), $F_{d\theta_1-11}$ expresses, for instance, the angle factor for an elemental area at θ_1 on cylinder 1 and the whole of cylinder 11. The radiative transfer rate from a unit length of a cylinder, Q_1 , and the mean flux on its circumference, q_m , are given by

$$Q_1 = 2R \int_0^\pi q_1(\theta_1) d\theta_1 \quad (18)$$

$$q_m = \frac{1}{\pi} \int_0^\pi q_1(\theta_1) d\theta_1 \quad (19)$$

3.2 Results and discussion

By using the iterative method (7), numerical solutions were obtained of Eq. (1). The step size of θ_1 was set as $\Delta\theta_1 = 0.01\pi$. The values of γ may practically be determined by taking account of various factors affecting the heat transfer, such as natural and forced convection, the number of rows, etc. In the present calculations, the values were selected within $\gamma = 3 \sim 7$, with primary concern for relatively large variations in local radiative heat flux on the cylinder, and with reference to the results of studies in convective heat transfer on pin-fin-type radiators (8)(9). Figure 3 illustrates some examples of the numerical solutions, showing variations in $\beta_1(\theta_1)$ on the cylinder circumference with ϵ and γ as parameters. The lower value of ϵ or γ leads to the greater variations in β_1 on the circumference, while with $\epsilon = 1$ or $\gamma =$

Fig. 3 Radiosity distribution along cylinder surface

∞ , the curves of β_1 conform to the abscissa in the figure.

The distributions of net heat fluxes with $\gamma = 3, 5$ and 7 are plotted with ϵ as a parameter in Figs. 4, 5 and 6, respectively. In each instance, the heat fluxes decrease when θ_1 decreases from π to 0 , and the trend is particularly remarkable for lower γ or higher ϵ . When γ remains constant, the dimensionless heat fluxes become larger with lower ϵ , reflecting the fact that a low value of emissivity means increased emergence to the environment due to a relative increase in reflected energy at other cylinder surfaces. At any value of γ , the distributions of net heat fluxes with fixed value of ϵ have a minimum, and occasionally even a maximum (e.g., $\gamma = 3$),

Fig. 4 Local heat flux distribution on cylinder surface for $\gamma = 3.0$

Fig. 5 Local heat flux distribution on cylinder surface for $\gamma = 5.0$

Fig. 6 Local heat flux distribution on cylinder surface for $\gamma = 7.0$

in the range of $\theta_1 = 0 \sim \pi/2$. These changes are attributed to the angle factors as can be surmised from the curve with $\varepsilon = 1$ (black body). In other words, the curve with $\varepsilon = 1$, as can be easily understood from Eq. (17), shows, besides the heat fluxes, the angle factors at each position on the cylinder to the whole of the environment visible through the other cylinders. As in the present cases, angle factors do not vary monotonously when the opposing surfaces are not contiguous in arrangement. This influences all curves with $\varepsilon \neq 1$. Such changes become less significant with lower values of ε . In comparison with the characteristics of two-row cylinders, the distributions of net heat fluxes on a system of one-row cylinders are indicated by the dotted lines in Figs. 4, 5 and 6, representing numerical solutions, under the same assumptions, of Eq. (12) in the range within $0 \leq \theta_1 \leq \pi$ (symmetric about the center, $\theta = \pi/2$). The curves of one-row cylinders are more flat than those of two-row cylinders and their trend of falling near $\theta = \pi/2$ is less significant because of the radiation interaction only between the cylinders lined on both sides.

From Figs. 4, 5 and 6 it is clear that the local heat flux of two-row cylinders is large in the range of $\theta_1 = \pi/2 \sim \pi$ and small in the range of $\theta_1 = 0 \sim \pi/2$; the radiative transfer rates over the respective ranges differ considerably with the values of γ or ε . Figure 7 shows the ratios of radiative transfer rate from an

Fig. 7 Radiative transfer rate from outer semicircumference of cylinder

Fig. 8 Variation of mean heat flux with center-to-center distance

outer semicircumference of a cylinder ($\theta_1: \pi/2 \rightarrow \pi \rightarrow 3\pi/2$), Q_{out} , to that from the whole circumference, Q_1 , with emissivity in abscissa; the ratios of Q_{out}/Q_1 increase with the increase in ε or the decrease in γ , but they change nearly linearly.

The trend of mean heat flux q_m increasing with γ is shown with ε as a parameter in Fig. 8.

3.3 Approximation by simplified formula

Exact analysis of radiative transfer, as can be surmised in the above descriptions, is extremely complex in many cases. Then, in cases where angle factors are known between the finite surfaces concerned, frequently applied is the simplified theory(7) as an approximate solution to obtain radiative heat fluxes of a system, assuming that radiosity is uniform over each divided section of the system. This approximation technique is known to yield results occasionally satisfactory.

Approximate solutions by such simplified technique were obtained in the system of two-row cylinders and evaluated in comparison with numerical solutions based on exact analysis. Comparison was also made between approximate and numerical solutions in the system of one-row cylinders to assess the influence of the number of rows on radiative transfer. Actual calculations were made for the simplest form, in which the cylinder surface was not divided and the radiosity was supposed to be uniform over a single cylinder. In this case, the simplified formula on the radiosity $\bar{B}_1$, considering $\bar{B}_1 = \bar{B}_2$, can be expressed as

$$\bar{\beta}_1 \equiv \frac{\bar{B}_1}{\varepsilon \sigma T_w^4} = \frac{1}{1 - (1 - \varepsilon)(F_{1-1R} + F_{1-2R})} \quad (20)$$

F_{1-1R} and F_{1-2R} in the above equation represent the angle factor of cylinder 1 to all the cylinders in the first row and that in the second row in Fig. 1, respectively. These can be determined by application of the crossed-string method(10) (see Appendix). The net heat flux $\bar{q}_1$, by replacing $\beta_1(\theta_1)$ in Eq. (16) by $\bar{\beta}_1$, can be

Fig. 9 Approximate solutions by simplified formula

expressed as

$$\frac{\bar{q}_1}{\sigma T_w^4} = \frac{1 - F_{1-1R} - F_{1-2R}}{1 - (1 - \epsilon)(F_{1-1R} + F_{1-2R})} \quad (21)$$

The relations between γ and the heat flux obtained with this equation are shown in Fig. 9. Defects of this approximation technique lie in that it may overestimate heat flux when the surfaces of the system are relatively closed to the external environment, or the number of such closed surfaces is larger than that of open surfaces, as well as when the value of emissivity is relatively low⁽⁶⁾⁽⁷⁾. Figure 10 shows deviations of the approximate solutions from the numerical ones with changes in γ and ϵ . In the figure, Q_B in the ordinate indicates radiative transfer rates on the black cylinder-row system with the same configuration as on the gray one; the solid lines show the approximate solutions and the symbols $\oplus$, $\ominus$ and $\circ$ the numerical solutions. When γ and ϵ are both low, the deviations of the approximate from the numerical solutions are evidently great, for instance, about 12 % with $\gamma = 3$ and $\epsilon = 0.5$. Such deviations are attributed to the assumption of uniform radiosity on which calculations were based despite the fact, as obvious in reference to the variations in $\beta_1(\theta_1)$ in Fig. 3, that variations in radiosity are markedly great when γ and ϵ are both low. The deviation gradually decreases as ϵ approaches unity, showing a value less than 8 % with $\epsilon \geq 0.7$ in the range of present calculation ($\gamma = 3 \sim 7$). The symbol + and the dotted line in Fig. 10 indicate the numerical solutions and the approximate ones obtained with $\gamma = 3$ for one-row cylinders, respectively. A deviation of 1.3 % observed with $\epsilon = 0.5$ is distinctly smaller than the corresponding deviation in two-row cylinders, suggesting a larger error in approximate solutions with the increase in number of rows.

Fig. 10 Comparison of approximate and numerical solutions

4. Heat transfer in semi-infinite-row black cylinders

Hottel⁽³⁾ conducted investigations in two-row black cylinders in an attempt to identify the radiative transfer characteristics from the bank of tubes in a furnace

chamber or that from rod heaters in the electric furnace. The cylinder bundle in practical use for heat-transfer surfaces, however, usually has a considerable number of cylinder-rows. In addition, these cylinders are finite in length and individually have fixed-walls at their ends, involving more complicated and difficult analysis of radiative transfer.

In the present study, radiative transfer to the environment is discussed in the system of black cylinders infinite in length, lined side by side infinitely and square-center-arranged in semi-infinite rows on one side as a basic model. In such a system, the over-all heat transfer rate from the system is considered more significant than the local heat fluxes from individual cylinders. Provided here are the same assumptions, (ii) and (iii) as in section 3.1 for the case of two-row gray cylinders. The net local heat flux on a cylinder in the j th row can be obtained by the following equation similar to Eq. (17):

$$\frac{q_j(\theta_j)}{\sigma T_w^4} = 1 - \left(\sum_{i=1}^j F_{d\theta_j-iR} + F_{d\theta_j-BR} \right) \quad (22)$$

In this equation, $F_{d\theta_j-iR}$ represents the angle factor of an infinitesimal element at an angular position (θ_j) on a cylinder in the j th row to all the cylinders of the i th row, and $F_{d\theta_j-BR}$ that of the same element to all the cylinders of the $(j+1)$ th and further rows. $F_{d\theta_j-BR}$ is congruous with the angle factor of the same element to the infinite plate parallel to the j th row. The over-all heat transfer rate from a cylinder to the environment can be obtained by the integral of Eq. (22) over the surface. The mean heat flux on a cylinder in the j th row, $q_{b,j}$, can be expressed using the reciprocity relations of $F_{1-iR} = F_{i-1R}$ etc. and using

$$F_{j-BR} = \frac{1 - F_j - jR}{2} = \frac{1}{2} - \frac{1}{2\pi} \times (2 \sin^{-1} \frac{2}{\gamma} + \sqrt{\gamma^2 - 4} - \gamma) \quad (23)$$

as

$$\frac{q_{b,j}}{\sigma T_w^4} = \frac{1}{2} + \frac{1}{2\pi} (2 \sin^{-1} \frac{2}{\gamma} + \sqrt{\gamma^2 - 4} - \gamma) - \sum_{i=1}^j F_{1-iR} \quad (24)$$

In order to ascertain numerically the trend of the over-all heat transfer rate of a cylinder, declining progressively from the initial row in the system of semi-infinite rows, the calculations were made on cylinders in the significant range from the first to the fourth row. The angle factors F_{1-iR} necessary for calculation were obtained, as in the case of two-row gray cylinders, by using the crossed-string method (see Appendix). Figure 11 illustrates the over-all heat transfer,

Fig. 11 Radiative transfer rate from semi-infinite-row black cylinders

$Q_{b,j}/Q_{b(\infty)}$, from a cylinder of each row. $Q_{b,j}$ is the radiative transfer rate per unit length of a cylinder in the j th row, and $Q_{b(\infty)}$ the sum of such rates in the system of semi-infinite rows. $Q_{b(\infty)}$ corresponds to the radiative energy from a rectangular black surface with unit length and C in width. With the increase in γ , $Q_{b,j}/Q_{b(\infty)}$ decreases in the first row, but in the second and further rows temporarily rises and after giving a maximum generally follows a decreasing trend. The sum of radiative transfer rates from cylinders in the first through the j th row, $Q_b(j)$, is jointly shown in Fig. 11,

$$Q_b(j) = \sum_{i=1}^j Q_{b,i} \quad (25)$$

The figure indicates that the radiative heat dissipation from the system is done predominantly in the first several rows near the external environment. The curves in Fig. 12 show the trend of $Q_b(j)$ increasing with the number of rows, j . These curves implicitly suggest the trends of radiative transfer on cylinders in the fifth and further rows, for which calculations were not performed.

Fig. 12 Variation of total heat rates from cylinders in first through j th row with numbers of rows

5. Conclusions

Radiative transfer was analyzed in

a system of two-row gray cylinders and a system of semi-infinite-row black cylinders, arranged on square-centers. Results are summarized as follows:

(1) The local radiative heat fluxes on the circumference of a cylinder in the two-row system change considerably with emissivity and the distance between cylinders, and its distribution is not so monotonous as in the system of one-row cylinders.

(2) The ratios of the radiative transfer rate from the outer semicircumference of a cylinder to that from the whole circumference become greater with the decrease in distance between cylinders, but the relations between the ratio and emissivity are almost linear.

(3) Approximate solutions of radiative transfer in the two-row system showed a deviation over 10 % from numerical solutions in the range of the present study. The comparison of approximate solutions with numerical ones in two-row and one-row systems suggests that the larger number of rows may give rise to greater discrepancy between them.

(4) In first several rows in which main radiative transfer is done in the system of semi-infinite rows, the radiative transfer rates from cylinders in each row were compared and identified with relation to the influence of the distance between cylinders. Also identified was the variation in the sum of radiative transfer rates with the number of rows in the system.

Acknowledgments

The author wishes to express his thanks to Professor Emeritus T. Tsubouchi, Tōhoku University, for his advices and encouragements extended in the study. He also extends thanks to Miss M. Kato, assistant official of the Institute of High Speed Mechanics, Tōhoku University, for her assistance throughout this study. Numerical computations in the study were made by using NEAC 2200-700 at Computer Center, Tōhoku University.

Appendix: Angle factors F_{1-iR}

The angle factors F_{1-iR} described in sections 3.3 and 4, were obtained by Hottel's crossed-string method⁽¹⁰⁾. F_{1-2R} , representing the angle factor of cylinder 1 to all the cylinders infinitely lined in the second row, can be obtained with this method; other angle factors to the cylinders in the third and any further row can be obtained with the sum of the factors individually determined as to the finite cylinders in the row concerned. F_{1-iR} can be given as

$$F_{1-iR} = \frac{1}{\pi} \left(2 \sin^{-1} \frac{2}{\gamma} + \sqrt{\gamma^2 - 4} - \gamma \right) \quad (26)$$

Figure 13 illustrates the angle factors, F_{1-iR} through F_{1-4R} , shown against γ .

Fig. 13 Angle factors F_{1-iR}

References

- (1) Tsubouchi, T., Heat Exchangers (in Japanese), (1968), Chap. 5, Asakura.
- (2) Aihara, T., Jour. Japan Soc. Mech. Engrs. (in Japanese), Vol. 70, No. 583 (1967-8), p. 1197.
- (3) Hottel, H.C., Trans. ASME, Vol. 53 (1931), p. 265.
- (4) Sparrow, E.M. and Eckert, E.R.G., Trans. ASME, Ser. C, Vol. 84, No. 1 (1962-2), p. 12.
- (5) Sikka, S. and Iqbal, M., AIAA J., Vol. 8, No. 1 (1970-1), p. 101.
- (6) Masuda, H., Rep. Inst. High Speed Mech., Tōhoku Univ., Japan, Vol. 27 (1973), p. 67.
- (7) Sparrow, E.M. and Gregg, J.L., Trans. ASME, Ser. C, Vol. 83, No. 4 (1961-11), p. 494.
- (8) Aihara, T. and Hatada, K., National Technical Rep. (in Japanese), Vol. 8, No. 3 (1962-7), p. 228.
- (9) Lukomsky, S.M. and Reabulchenko, G.V., Proc. 4th Int. Heat Transfer Conf., Vol. 1 (1970), HE 1.2.
- (10) Hottel, H.C. and Sarofim, A.F., Radiative Transfer, (1967), p. 31, McGraw-Hill.