Researches on the Positioning Device by Automatic Balancing

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In the case of applying a lever which swings about the horizontal axis by proportional control of an electric servomechanism to a manipulator's arm etc., it is difficult to position the manipulator's arm correctly according to the unbalance due to the grasping and releasing an object.

In this paper, a balancing device using a movable counterweight is proposed for the purpose of positioning the arm correctly, and its characteristics and stability are studied. As a result of analyses, it is confirmed that such a balancing device is available for the manipulator's arm and automatic balancing device.

1. Introduction

In order to balance a lever which swings about the horizontal axis, a balancing method using a counterweight and other new balancing methods have been suggested. Constant, such balancing methods are available, but it is already recognized that these methods cannot balance automatically when the unbalance changes. For example, in applying to a manipulator's arm, the unbalance of the arm changes when an object is grasped or released.

In this paper, we propose a balancing device using a movable counterweight for the purpose of investigating the automatic response to the unbalance change, and examining its applicability to a manipulator's arm.

Nomenclature

\( C_1 \): name and damping coefficient of a damper attached to a movable counterweight
\( C_2 \): name and damping coefficient of a damper attached to a arm axis
\( D \): damping coefficient of a servomotor (converted to a pinion axis)
\( F \): dissipation function
\( I \): moment of inertia of an arm (includes moments of inertia of an object, a movable counterweight, etc.)
\( I_{a} \): moment of inertia of an arm in itself
\( I_{b} \): moment of inertia of a movable counterweight
\( I_{c} \): moment of inertia of an object
\( I_{r} \): moment of inertia of a rotor of the servomotor (converted to a pinion axis)
\( K_e \): gain constant of a servomech-anism (converted to a pinion axis)

\( L \): Lagrange's function
\( l \): length, measured from an arm axis to the center of gravity of an object
\( l_{a} \): length, measured from an arm axis to the center of gravity of an arm
\( r \): radius of a pinion pitch circle
\( T \): kinetic energy
\( T_0 \): torque, exerted on a pinion axis
\( U \): potential energy
\( W \): weight of an object
\( W_{c} \): dead load of an arm
\( W_{r} \): weight of a movable counterweight

\( O-XYZ \): fixed Cartesian coordinate system

\( x \): displacement of the center of gravity of a movable counterweight from an arm axis
\( \theta \): angular displacement of an arm from horizon (output angle)
\( \theta_i \): input angle
\( \psi \): objective angle of an arm
\( \psi \): converted angular displacement of a movable counterweight from its balancing position
\( \psi_{c} \): converted angular displacement of balancing position of a movable counterweight

2. Balancing device using movable counterweight

In a balancing device using a movable

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Balancing device using movable counterweight}
\end{figure}
counterweight, as shown in Fig.1, a servomotor included in this mechanism is driven by the angle error of the arm, and a movable counterweight is controlled according to an unbalance change, therefore, the arm may be positioned.

2.1 Characteristics of mechanism

Though, in Fig.1, the axis of the arm coincides with the one of a pinion, both can move independently of each other.

Moreover, the center of gravity of a movable counterweight moves along the center line of the arm and a rack which transmits the driving force from the pinion to a movable counterweight is assembled parallel to the center line of the arm. Therefore, when torque $T_1$ is exerted on the pinion axis, driving force $T_1/r$ acts on the movable counterweight and torque $T_1$ acts on the arm axis. Then, considering that the torque $T_1$ is an external force and $x, \phi$ are generalized coordinates, the equation of motion is given from Lagrange's equation as follows:

$$\begin{align*}
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial F}{\partial \dot{x}} &= T_1, \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} + \frac{\partial F}{\partial \dot{\phi}} &= 0,
\end{align*}$$

where $L=T-U$.

Next, $T$, $U$, and $F$ are given as follows:

$$\begin{align*}
T &= \frac{1}{2} J_1 + \frac{W}{g} \ddot{x} \ddot{\phi} + \frac{1}{2} W_r x^2 \\
U &= (W_1 + W_2 - W_3 x) \sin \theta \\
F &= \frac{1}{2} C_1 x^2 + \frac{1}{2} C_2 \phi^2,
\end{align*}$$

From Equations (1) and (2), we obtain

$$\begin{align*}
\frac{\partial L}{\partial \dot{x}} &= \frac{\partial L}{\partial x} + \frac{\partial F}{\partial \dot{x}} = 0, \\
J_1 + \frac{W}{g} x^2 \ddot{\phi} + 2 \frac{W_r x \phi}{g} + C_1 \phi &= 0, \\
+ (W_1 + W_2 - W_3 x) \cos \theta - T_1 &= 0,
\end{align*}$$

Then we substitute displacement of the center of gravity of the movable counterweight from the arm axis as follows:

$$x = x' \phi,$$

Where $\phi$ satisfies the following equation

$$W_r x' \phi - W - W_3 x = 0.$$  

In Eq. (5), the unbalance $W_3 x$ due to the dead load of the arm is constant, but the unbalance $W_3 x$ due to the object changes as it is grasped or released. But, as its change is approximately stepwise, $\phi$ may be considered constant, then the displacement of the movable counterweight can be expressed by $\phi$ only.

In this mechanism in which the pinion and the rack form a differential gear, torque $T_1$ is expressed as follows:

$$T_1 = K_1 (\theta - \phi) - D_1 (\dot{\theta} + \dot{\phi}) - J_1 (\ddot{\theta} + \ddot{\phi}).$$

From equations (3) and (6) the equation of motion is rewritten as

$$\begin{align*}
\frac{W}{g} + \frac{W_r}{g} x' \phi &= 0, \\
J_1 (\theta - \phi) + D_1 (\dot{\theta} + \dot{\phi}) + J_1 (\ddot{\theta} + \ddot{\phi}) &= 0.
\end{align*}$$

In the case of the arm being positioned, provided the mechanism is stable, the following relation should be satisfied.

$$\tan \theta = \frac{W_r}{K_1} \sin \theta = \theta,$$

Namely, from equations (8) and (9) in the case that the arm is positioned at an arbitrary angle $\theta$, the movable counterweight displaces $\tan \theta$ from its balancing position. When input angle $\theta$ is given as $\theta = 0$ in Eq. (9), the arm is positioned in a well-balanced condition, and the relation between $\theta$ and $\phi$ is not influenced by the object. Therefore, assuming a frictionless condition, the arm is required to be positioned at angle $\theta$, by giving input angle $\theta$ as

$$\theta = \theta + K_1 \sin \theta.$$  

Then, the angle $\theta$ coincides with $\theta$, and the positioning error ($\theta - \theta$) vanishes. These characteristics are available for the manipulator's arm or automatic balancing device. But, in order to apply this mechanism, dynamic characteristics and stability should be studied. Referring to Eq. (7), let us neglect the nonlinear terms $(W_1/g) \phi^2 x^2$ and $2W_2/g \phi x$, and approximate the moment of inertia of the arm as

$$J = J_1 + \frac{W_r}{g} \phi^2.$$  

Moreover, put

$$\sin \phi = \theta, \cos \phi = \phi.$$  

Then Eq. (7) becomes linear equations with respect to new variables $\theta$ and $\phi$,

$$\begin{align*}
R \dot{\phi} + D \phi - K \phi = -W_r \theta, \\
\left\{J_1 + \frac{W_r}{g} \phi^2 \right\} \ddot{\phi} + (D_1 + C_1 \phi) \phi = 0, \\
(J + \phi)^2 + (D_1 + C_1 \phi) - K \phi = 0, \\
\phi \theta + \phi \theta = 0.
\end{align*}$$

where $\theta'$ is the weight of the movable counterweight and includes the weight of the rack. In order to examine a response for handling operation, at first we study the step-response of this mechanism.

Transform Eq. (13) by the method of Laplace transformation

$$\theta = 0, \quad \phi = 0, \quad \phi = 0, \quad \phi = 0,$$

and put the root of characteristic function $S_1$, then $\theta$ is given from Heaviside expansion equation as follows:

$$\theta = \theta_1 \left\{K \theta + \sum \frac{f(S)}{S - S_0} g(S) \right\}.$$  

Therefore, the response of $\theta$ may be in proportion to the magnitude of the step. Next, we examine a transitional response for a momentary grasping or releasing of the object.

Put the initial conditions as

$$\begin{align*}
\theta = 0, \quad \phi = 0, \quad \phi = 0, \quad \phi = 0, \\
\theta = 0, \quad \phi = 0, \quad \phi = 0, \quad \phi = 0,
\end{align*}$$

and $\theta_1 = 0$, then $\theta$ is given by

$$\theta = \frac{W_r \theta_1}{g} \sum \frac{f(S)}{S - S_0} g(S) \theta_1.$$
where coefficient $W_r\psi_\omega$ should satisfy the following equation

$$W_r\psi_\omega = W_l$$

(18)

Therefore $\theta$ may be in proportion to the magnitude of the unbalance change. If this mechanism is stable, the following equation should be satisfied:

$$\lim_{t \to \infty} \frac{f(S_0)}{g(S_0)} = 0$$

(19)

From equations (17) and (19), the arm angle $\theta$ should approach the initial position, i.e., $\theta = 0$.

2.2 Stability of mechanism

As the result of the analyses in section 2.1, the characteristics of mechanism become clear. But, in order to apply this mechanism to the manipulator's arm or automatic balancing device, it should be stabilized. Then we study the stability of this mechanism by the method of the Hurwitz stability criterion. From Eq.(13), the characteristic equation is given by

$$a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0$$

where

$$a_4 = J_1 + \frac{W_r}{g}r^2J_1$$
$$a_3 = J_1 C_l + \frac{W_r}{g}r^2(D_1 + C_l)$$
$$a_2 = 2WR_1J_1 + \frac{W_r}{g}r^2K_l$$
$$a_1 = 2WR_1D_1 + C_l r^2 K_l$$
$$a_0 = WR_1(K_l - W_r)$$

The device in Fig.1 has three dampers; a damping effect of the servomotor, a damper $C_l$ for the movable counterweight and a damper $C_r$ for the arm axis.

In order to compare the effects of $C_l$ and $C_r$, put the damping effect of $C_l$ as $C_l r^2$.

At first, under the condition of $D = 40$ kg mm $s$, $C_l r^2 = C_l = 0$ kg mm $s$, stability of mechanism is obtained as follows:

$$\frac{W_r}{K_l} < 1$$

(21)

$$0 < \frac{W_r}{g}r^2$$

(22)

where Eq.(22) is a very severe condition to be satisfied.

The experimental apparatus is shown in Fig.2 and each constant value is given as follows:

$$W_{fr} = 91.8 \text{ kg mm}, \quad r = 22.5 \text{ mm}$$
$$W_r = 0.870 \text{ kg}, \quad W_r = 0.965 \text{ kg}$$
$$K_l = 220 \text{ kg mm/rad}, \quad D_1 = 37 \text{ kg mm s}$$
$$J_1 = 4.28 \text{ kg mm}^2, \quad J_1 = 4.46 \text{ kg mm}^2$$
$$C_l r^2 = C_l = 0 \text{ kg mm}^2$$

Apart from Eq.(21), since Eq.(22) is not satisfied, the mechanism is considered unstable.

As the result of the experiment of step-response, in Fig.3(a), the mechanism is confirmed to be unstable. But Fig.3(b) and (c) show the possibility of stabilizing of the mechanism by attaching $C_l r^2$ or $C_r$. In these figures, $\theta$ shows the arm angle from horizon and $\phi$ shows the angular displacement of the movable counterweight from its balancing position. Then, we analyze stability of this mechanism theoretically by attaching not only $D_1$ but also either $C_l r^2$ or $C_r$. The stability limit diagram is given as shown in Fig.4. The upper side of this curve shows the stable region, and the lower side shows the unstable region, and the effect of stabilization of $C_l r^2$ is greater than that of $C_r$.

The experimental results are different from the theoretical results and the reasons are considered chiefly as:

1. the influence of the nonlinear terms which were neglected or approximated when the equation of motion was linearized, and

2. the influence of the friction.

Since it is very difficult to analyze the stability of a mechanism theoretically with due consideration of these influences, let us simulate by use of an analog-computer considering the friction. Putting the friction torque about the pinion axis as $T_f$ and that about the arm axis as $T_r$, the equation of motion is given by

$$J_1 \ddot{\theta} + D_1 \dot{\theta} - K_l (\theta - \theta_0) - W_r \phi$$

$$+ \left( J_1 + \frac{W_r}{g}r^2 \right) \ddot{\phi} + (D_1 + C_l r^2) \dot{\phi} \pm T_f = 0$$

$$\left( J_1 + J_1 \right) \ddot{\theta} + (D_1 + C_l) \dot{\theta} - K_l (\theta - \theta_0)$$

$$+ J_1 \ddot{\phi} + D_1 \phi - W_r \phi \pm T_f = 0$$

(23)

where the sign of $T_f$ is the same as that of $\phi$, and the sign of $T_r$ is the same as $\theta$. From Eq.(23), a setup diagram of the analog-computer is given as shown in Fig.5. At first, we simulate in respect to Fig.3(a) by neglecting the friction. The result of simulation is, as shown in Fig.6, more unstable than the experimental results. Then we measured static friction torque $T_m$ about the pinion axis and $T_m$ about the arm axis, both of them being obtained as follows:

$$T_m = 3.5 \text{ kg mm}, \quad T_m = 1.4 \text{ kg mm}$$

(24)

When the mechanism is in operation, the dynamical friction torque should be taken into consideration instead of the static friction torque. But, since it is very difficult to measure the dynamical friction torque, we substitute a-times of static friction torque for $T_f$, $T_r$, and simulate as $a = 0.6, 0.7, 0.8$. As the result of the simulation, as shown in Fig.7, it becomes clear that the dynamic characteristics of the mechanism change on a large scale only by a small fluctuation of the friction.
torque, i.e., the mechanism is stable when $\alpha = 0.8$ and the simulation result agrees very well with the experimental one when $\alpha = 0.6$. The reason why the period of the simulation is shorter than that of the experimental result may be that the increase of the moment of inertia of the arm by the displacement of the movable counterweight is neglected. Therefore, the difference between the theoretical and experimental results, in Fig. 4, may be caused by the influence of the friction and nonlinear terms which are neglected or approximated to linear terms. Since the friction exists without fail, we can stabilize the mechanism by estimating the maximum moment of inertia of the arm with consideration of the movable range of the movable counterweight and deciding the damping coefficient theoretically.

3. Positioning device by automatic balancing

In the case of applying the balancing device using a movable counterweight to a manipulator's arm, it is necessary to set a movable range of the movable counterweight at optimal position of the arm. Although the movable range may be set at an arbitrary position in case of using movable counterweight, the moment of inertia of the arm increases, and dynamic characteristics become worse. Then we represent a method of changing the unbalance by use of a tension coil spring without increasing the moment of inertia of the arm, and apply it to the balancing device using a movable counterweight.

3.1 Balancing device using tension coil spring

Methods of balancing the arm which swings about the horizontal axis have been suggested. One of them is a balancing device using a compression coil spring, which needs a rigid bar for a connector, and its dynamic characteristics is worse because of the increase in the moment of inertia of the arm. The other device using a close-wound coil spring having the desired initial tensile force cannot be applied to the arm because of difficulty to make such a spring. Then we suggest a balancing device in Fig. 8 which contains a flexible link, for example.

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\includegraphics[width=\columnwidth]{fig4}
\caption{Stability limit diagram}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig5}
\caption{Simulation diagram of analog-computer}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig6}
\caption{Simulation of Fig. 5(a), neglecting friction}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig3}
\caption{Experimental results of step-response}
\end{figure}
a steel band, and a tension coil spring. The stiffness \( k \) of the spring is given by

\[
k = \frac{OB \cdot OD}{W_d}
\]

(25)

and the spring is connected to a flexible link in series and the flexible link is wrapped around the fixed pin D. Fixing the spring at the position C so that the spring force \( P \) satisfies the following equation

\[
P = k \cdot DB
\]

(26)

the arm may be balanced at any position. It has been confirmed experimentally that the arm can be well-balanced with this apparatus, as shown in Fig.9. As the result of the experiment of step-response by electric servomechanism, the dynamic characteristics of the new balancing device using a spring are demonstrated to be better than those of the conventional balancing method using a counterweight as shown in Fig.10 (a).

Therefore the arm is balanced without increasing the moment of inertia by using a light material for the flexible link and the dynamic characteristics do not become worse in spite of a spring being used.

Considering the relation

\[
 DB^2 = OB^2 + OD^2 - 2OB \cdot OD \cdot \sin \theta = (W_d - k \cdot OB \cdot OD) \sin \theta
\]

(27)

in Fig.8, potential energy \( U \) is obtained by

\[
 U = (W_d - k \cdot OB \cdot OD) \sin \theta
\]

(28)

If the stiffness \( k \) satisfies Eq. (25), \( U \) is always zero regardless of the arm angle \( \theta \), and the arm may be balanced at any angle. But, if \( k \) is given by

\[
k = \frac{(W_d - W_d_0)}{OB \cdot OD}
\]

(29)

\( U \) takes the form

\[
 U = (W_d - (W_d - W_d_0) \sin \theta
\]

(30)

This means that the unbalance of the arm is equivalent to \( W_d - (W_d - W_d_0) \), and this device may be used as an unbalance transformer without increasing the moment of inertia.

3.2 Experimental results

By use of the experimental apparatus as shown in Fig.11, considering the action of handling, i.e., (1) an object is grasped, (2) step-input is switched on, (3) an object is released, or (4) step-input is switched off, we examined the dynamic response. Fig.12(a) shows the response for balancing the arm by fixing the movable counterweight, and the positioning error arises from an object being grasped or released. Fig.12(b) shows the response for changing the unbalance of the arm by counterweight, and Fig.12(c) shows that for changing the unbalance by tension coil spring.

\[
 K_s = 45 \text{ kg mm/rad}, \quad D_s = 4.4 \text{ kg mm s}, \quad f_s = 4.4 \text{ kg mm s}^2
\]

(a) In the case of using tension coil spring

\[
 J_s = 0.1 \text{ kg mm s}^2
\]

(b) In the case of using counterweight

Fig.10 Comparison of dynamic characteristics by step-response
spring. In both cases, there is a tendency to keep the initial angle under the unbalance change caused by handling an object. Since (c) is more stable than (b), it is found that the positioning device by automatic balancing is excellent.

Next, we examine the accuracy of positioning. In the case of the arm being balanced in itself by fixing the movable counterweight, the relation between the input angle $\Theta_i$ and the angle error ($\Theta_i - \Theta$) is given as shown in Fig.13. In this figure, $W1$ is the unbalance caused by handling an object, the thin solid line shows the experimental result when the unbalance $W1 = 0$ kg mm and the bold solid line shows the result when $W1 = 32.5$ kg mm. These curves mean that the angle error may be influenced by grasping or releasing an object. The broken line shows the theoretical result under the frictionless condition, and it is obtained by the following equation

$$\theta + \frac{W1}{K_r} \cos \theta = \Theta_i$$  \hspace{1cm} (31)

In order to position the arm correctly at an arbitrary angle $\Theta_i$, the arm should be controlled by detecting the unbalance change automatically, which is very difficult to control. On the other hand, the positioning device by automatic balancing has a constant characteristic, because the angle error is not influenced by the unbalance $W1$.

Therefore, under the frictionless condition, when the input angle $\Theta_i$ is given to satisfy Eq. (10), the arm may be positioned at $\Theta_i$ and the positioning error ($\Theta_i - \Theta$) may become zero.

Moreover, we examined a response of the positioning device by automatic balancing for the unbalance change. Fig.15 shows the response of releasing an object just at $t=0$ sec, when $W1 = 32.5$ kg mm. The initial angles of Fig. (a), (b) and (c) are 0.2, 0, -0.2 rad, respectively. In spite of the difference of the initial angle, this positioning device may respond to the unbalance change automatically, and may work.
to keep the initial position. Although $\varphi$ should approach the broken line in Fig.15 as time increases indefinitely, it produces an error due to the influence of the friction and the weight of the rack. As the result of above experiments, in the case of applying the balancing device using a movable counterweight to the manipulator’s arm etc., the movable range of movable counterweight is set at an arbitrary position of the arm by using a tension coil spring without increasing the moment of inertia of the arm, and then the dynamic characteristics of this mechanism do not become worse in spite of using a tension coil spring.

3.3 Instruction for mechanism design

Since the availability of the positioning device by automatic balancing has become clear, we can give an instruction for the mechanism design. Following constants in Fig.16 should be given by the designer:

- $W_{\text{max}}$: maximum weight of objects
- $l$: length, measured from the arm axis to the center of gravity of an object
- $-\theta_1 \leq \theta \leq \theta_2$: operative range of the arm
- $l_1$: length of the movable range of a movable counterweight
- $z_2$: position of the center of gravity of a movable counterweight under the condition of the arm not grasping an object and the arm being positioned horizontally
- $r$: radius of a pinion pitch circle

At first, designing the arm in itself, the unbalance $W_j$, caused by the dead load of the arm can be determined. Next, putting the weight $W_r$ of a movable counterweight as

$$W_r = \frac{W_{\text{max}}}{l_1 + r \tan (-\theta_1) - r \tan \theta_2} \quad (32)$$

the maximum moment of inertia of the arm can be estimated. In order to reduce the positioning error under the allowable value, the gain constant $K_r$ of a servomechanism should satisfy Eq.(21) and the following equation:

$$K_r \geq \frac{(T_r)_{\text{max}}}{\varepsilon} \quad (33)$$

where $(T_r)_{\text{max}}$ is the maximum static friction torque of the arm axis and $\varepsilon$ is the absolute allowable value of positioning. Therefore, when the input angle $\theta_1$ is given by Eq.(10), the positioning error may diminish under the allowable value.

In order to stabilize this mechanism, we should decide the damping coefficients theoretically. In order to set the movable range of the movable counterweight at an arbitrary position of the arm, we should determine the positions of the points B and D, decide the stiffness $k$ of the tension coil spring as

$$k = \frac{W_{\text{max}}}{OB} \quad (34)$$

and determine the position of the point C so that spring force $P$ satisfies Eq.(26), then the design may be accomplished.

4. Conclusions

The results obtained in this paper are summarized as follows:

1. In the case of applying a lever which swings about the horizontal axis and is proportionally controlled by an electric servomechanism to a manipulator's arm etc., the error angle of the arm is influenced by an unbalance change due to the grasping or releasing of an object. Then we suggested a balancing device using a movable counterweight, and as a result of this analysis, confirmed its availability for the manipulator’s arm.

2. In order to stabilize the balancing device using a movable counterweight, it is necessary to attach not only a damper to a servomotor but also the damper either to movable counterweight or to arm axis. Moreover, the damper attached to the movable counterweight is better than that to arm axis for the effect of stabilization.

3. In the case that the lever swings about the horizontal-axis and has an unbalance, it is balanced by a tension coil spring without increasing the moment of inertia. And this balancing mechanism can be used as an unbalance transformer.

4. In the case of item(1), it is necessary to set the movable range of movable counterweight at an arbitrary position of the arm. For this purpose, a tension coil spring is available.

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Reference