Multi-Product Production Scheduling in Automated Continuous Processes

By Fumio KODAMA

In the multi-product production of coating and heat treatment, various kinds of jigs and fixtures are used exclusively for each product. As the daily fluctuation of delivery requirements is large, either the number of jigs and fixtures maintained or the amount of inventories, has to be kept large. Therefore, in the scheduling, the trade-off between them has to be taken into account. This type of the multi-product scheduling is called here the multi-product production scheduling in the automated continuous process. The paper formulates this type of the scheduling problem mathematically and presents a mixed integer programming model. An example involving 8 products and 9 days is numerically solved. The optimal solution and its dynamic characteristics are discussed.

1. Introduction

At the stage of rapid economic growth, the "mass production" has been the main type of production. As the needs and value of the users have been pluralistic, the importance of multi-product production has been recognized.

In the researches about the production, the main subject has been the mass production, as we can find typical examples in the studies of Line Balancing. On the contrary, the multi-product production has not been well studied. The multi-product production has been so far discussed in two aspects; in the field of Group Technology, attention has been focused on the production technology such as production design, tool design and processing design. The other aspect was the trial to cope with multi-product production through the improvement of scheduling.

The multi-product production scheduling has been so far studied in the field called Multi-Product Multi-Period Production Scheduling. It is defined as a planning problem faced by a machine shop required to produce many different items by batch production so as to meet a rigid delivery schedule. The difficulties are summarized; (i) the presence of "setup costs" characterized by indivisibilities, (ii) the necessity to determine the lot-size, taking the future delivery cycle into account. The examples of the successful use of linear programming in overcoming these difficulties are Maneke Model and K.S.S. Model.

Although the Multi-Product Multi-Period Production Scheduling deals with the planning problem in batch production, as can be seen in the definition, the machine industry has many types of multi-product production, where the setup costs are negligible but only the exclusive use of jigs or fixtures is allowed for each product. Coating and heat treatment are typical examples. In these types of production, the materials are hung on the conveyor by means of jigs and fixtures and passed through the automated continuous processing equipment. Here, we call it the multi-product production in the automated continuous process.

In the multi-product production in the automated continuous process, only the exclusive use of jigs or fixtures is allowed for each product and the daily fluctuation of delivery requirements is large, therefore, many jigs or fixtures have to be maintained. On the other hand, in order to keep the number of jigs or fixtures to be maintained small, large amount of inventory is necessary at each stage of production. Therefore, it is necessary to think of a scheduling problem which minimizes the running cost composed of the maintenance cost of jigs or fixtures and the inventory cost.

2. Description of the production system

The production system in the automated continuous process is described as shown in Fig. 1. The materials arrive at the material stockyard.
The materials are gathered in some units and hung on the conveyor line by means of jigs* and passed through the automated continuous processing equipment. The materials which are processed by the equipment are stored at the parts stockyard as the parts which are to be assembled. After the parts are passed through the assembly line, they become products and are stored at the warehouses which are used for each product, and they are delivered whenever the delivery requirements come in.

As we see in Fig. 1, we can absorb the daily fluctuations of delivery requirements for the product, either by maintaining many jigs or by keeping the inventory at high level. And when the daily fluctuation of delivery requirements is given, a lessening of the number of jigs maintained contradicts with a reducing of the amount of inventory. That is, if many jigs are maintained, the amount of inventory can be kept at a high level. And if the amount of inventory is kept at a high level, the number of jigs maintained can be lessened.

Therefore, it is necessary to have a scheduling which minimizes the running cost composed of the maintenance cost of jigs and the inventory cost.

In general, the flow of jigs in the automated continuous processing equipment is described as shown in Fig. 2. A jig can pass through the equipment along the conveyor line several times within a day. Therefore, the time required for a jig to go around the conveyor line is called a round and it is appropriate to set the round as the minimum unit of scheduling. Therefore the scheduling can be defined as the combination of jigs hung on the conveyor line for each round of each day.

![Diagram of Flow of Jig in the Equipment](image)

**Fig. 2 Flow of the jig in the equipment**

Generally speaking, we do not necessarily use a jig for each material which is to be processed. In the coating, for example, it is standard practice that several materials are hung on a jig. Therefore, it is convenient that the quantity of materials passed through the equipment is measured, with the quantity of materials which can be hung on a jig as a unit. Then, the quantity of materials passed through the equipment is equal to the number of jigs needed for each round.

Moreover, the size of jigs is same for each product, although only their exclusive use for each product is allowed.

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* In this paper, "jigs" is used to mean "jigs or fixtures".

3. System identification

3.1 Notations

Although there exists several assembly lines as shown in Fig. 1, the system identification is done as to the case where there is only one assembly line, for simplicity (Fig. 3).

![Diagram of System Identification](image)

**Fig. 3 System Identification**

The notations used for the system identification are as follows:

- $r_{i,j}$: arrival quantity of $i$-th material on $j$-th day ($i=1, \ldots, n; j=1, \ldots, m$)
- $a_{i,j}$: holding quantity of $i$-th material at the materials stockyard on $j$-th day ($i=1, \ldots, n; j=1, \ldots, m$)
- $r_{i,j}$: initial inventory at the materials stockyard ($i=1, \ldots, n$)
- $a_{i,j}$: the quantity of $i$-th material passed through the automated continuous processing equipment on $q$-th round of $j$-th day ($i=1, \ldots, n; j=1, \ldots, m; q=1, \ldots, q$)
- $a_{i,j}$: holding quantity of $i$-th part at the parts stockyard on $j$-th day ($i=1, \ldots, n; j=1, \ldots, m$)
- $r_{i,j}$: initial inventory at the parts stockyard ($i=1, \ldots, n$)
- $a_{i,j}$: quantity of $h$-th product passed through the assembly line on $j$-th day ($h=1, \ldots, h; j=1, \ldots, m$)
- $a_{i,j}$: holding quantity of $h$-th product at the warehouse on $j$-th day ($h=1, \ldots, h; j=1, \ldots, m$)
- $r_{i,j}$: initial inventory at the warehouse ($h=1, \ldots, h$)
- $d_{i,j}$: delivery quantity of $h$-th product on $j$-th day ($h=1, \ldots, h; j=1, \ldots, m$)
- $v_{1}$: the volume of an $i$-th material which occupies the materials stockyard ($i=1, \ldots, n$)
- $v_{1}$: the volumetric capacity of the materials stockyard
- $G$: the number of jigs which can be hung on the conveyor line per round
- $G$: the number of jigs which can be hung on the conveyor line after the round (the last round)
- $v_{2}$: the volume of an $i$-th part which occupies the parts stockyard ($i=1, \ldots, n$)
- $v_{2}$: the volumetric capacity of the parts stockyard
- $r_{i,h}$: the quantity of $i$-th parts which are needed for a $h$-th product assembled ($i=1, \ldots, n; h=1, \ldots, h$)
- $a_{h}$: the speed of passing of $h$-th product on the assembly line
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\( (h=1,\ldots,H) \) (1/hour),

\( A_k \): the net working hour per day of the assembly line (hour),

\( S_{h, k} \): the capacity of the warehouse for \( h \)-th product \((h=1,\ldots,H)\),

\( s_{h,k} \): the safety inventory of \( h \)-th product \((h=1,\ldots,H)\),

\( e_{k} \): the cost of maintaining a jig used for \( i \)-th part during the planning period \((i=1,\ldots,n)\),

\( c^{(1)}_{i} \): the holding cost of an \( i \)-th material per day at the materials stockyard. (yen/day) \((i=1,\ldots,n)\),

\( c^{(2)}_{i} \): the holding cost of an \( i \)-th part per day at the parts stockyard (yen/day) \((i=1,\ldots,n)\),

\( g^{*} \): the labour cost of making a jig hang on the conveyor line,

\( g^{*} \): the labour cost of making a jig off the conveyor line.

3.2 State equation

The state of the system can be described by the following three difference equations:

The difference equation of the materials at the stockyard is

\[
s_{i,j}^{(1)} = s_{i,j}^{(1)} - c_{i,j}^{(1)} \sum_{q=1}^{\bar{q}} x_{i,j,q} - \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} \sum_{q=1}^{\bar{q}} x_{i,j,q}, \quad (i=1,\ldots,n; j=1,\ldots,m)
\]

The solution of the equation is

\[
s_{i,j}^{(1)} = s_{i,j}^{(1)} - c_{i,j}^{(1)} \sum_{q=1}^{\bar{q}} x_{i,j,q} - \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} \sum_{q=1}^{\bar{q}} x_{i,j,q} \quad (1)
\]

The difference equation of the parts at the stockyard is

\[
s_{i,j}^{(2)} = s_{i,j}^{(2)} - c_{i,j}^{(2)} \sum_{q=1}^{\bar{q}} x_{i,j,q} - \sum_{q=1}^{\bar{q}} c_{i,j}^{(2)} \sum_{q=1}^{\bar{q}} x_{i,j,q}, \quad (i=1,\ldots,n; j=1,\ldots,m)
\]

The solution of the equation is

\[
s_{i,j}^{(2)} = s_{i,j}^{(2)} - c_{i,j}^{(2)} \sum_{q=1}^{\bar{q}} x_{i,j,q} - \sum_{q=1}^{\bar{q}} c_{i,j}^{(2)} \sum_{q=1}^{\bar{q}} x_{i,j,q} \quad (2)
\]

The difference equation of the products at the warehouse is

\[
s_{h,j}^{(3)} = s_{h,j}^{(3)} + c_{h,j}^{(3)} \sum_{q=1}^{\bar{q}} x_{h,j,q} - \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \sum_{q=1}^{\bar{q}} x_{h,j,q}, \quad (h=1,\ldots,H; j=1,\ldots,m)
\]

The solution of the equation is

\[
s_{h,j}^{(3)} = s_{h,j}^{(3)} + c_{h,j}^{(3)} \sum_{q=1}^{\bar{q}} x_{h,j,q} - \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \sum_{q=1}^{\bar{q}} x_{h,j,q} \quad (3)
\]

4. Optimization problem

4.1 Running cost

The objective function is the running cost which has to be paid during the planning period. The running cost consists of the following three cost items; (1) the maintenance cost of jigs, (2) the holding cost, (3) the labour cost of changing the jigs combination from round to round.

As for the maintenance cost of jigs, the number of jigs used for \( i \)-th part which have to be maintained on \( j \)-th day is \( m_{i,j} \), and the number of jigs which have to be maintained during the planning period is \( \max_{i,j} m_{i,j} \). Therefore, the maintenance cost of jigs paid during the planning period is \( \sum_{i,j} m_{i,j} \max_{i,j} m_{i,j} \) .... (4)

The holding cost consists of those at the materials stockyard, parts stockyard and warehouse. Therefore, the holding cost paid during the planning period is

\[
\sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) + \sum_{h,j} h_{h,j} \left( \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \right)
\]

Substituting each term in Eq. (5) by Eq. (1) - Eq. (3),

\[
\sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) + \sum_{h,j} h_{h,j} \left( \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \right) = \sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) + \sum_{h,j} h_{h,j} \left( \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \right)
\]

Substituting each summation in the above equation by

\[
\sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) = \sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right)
\]

where, \( \sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) \) is substituted in the above equation by

\[
\sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) = \sum_{i,j} l_{i,j} \left( \sum_{q=1}^{\bar{q}} c_{i,j}^{(1)} + c_{i,j}^{(2)} \right) + \sum_{h,j} h_{h,j} \left( \sum_{q=1}^{\bar{q}} c_{h,j}^{(3)} \right)
\]

The labour cost of changing the jig combination consists of that of making jigs hang on the conveyor line and that of taking jigs off the conveyor line. The number of jigs which should be made during the planning period is \( \sum_{i,j} x_{i,j} \), where \( x_{i,j} \) is defined by

\[
x_{i,j} = \begin{cases} x_{i,j} & \text{if } x_{i,j} \geq y_{i,j} \\ 0 & \text{if } x_{i,j} < y_{i,j} \end{cases}
\]

And, on the first round, all the jigs needed for the round should be made to hang on the conveyor line. Therefore, the labour cost of making jigs hang on the conveyor line to be paid during the planning period is

\[
g^{*} \sum_{i,j} \left( \sum_{q=1}^{\bar{q}} x_{i,j} q^{q-1} \right) = \sum_{i,j} \left( \sum_{q=1}^{\bar{q}} x_{i,j} q^{q-1} \right)
\]

Similarly, the number of jigs which should be taken off the conveyor line is

\[
\sum_{i,j} \left( \sum_{q=1}^{\bar{q}} x_{i,j} q^{q-1} \right)
\]

And after the \( q \)-th round (the last round), \( q = 1 \) is over, all the jigs hanging on the conveyor line should be taken off. Therefore, the labour cost of taking
jigs off the conveyor line to be paid during the planning period is

\[
G = \sum_{q=1}^{Q} \sum_{i,j} a_{ij} \left( x_{i,j,q-1} - x_{i,j,q} \right) + \sum_{i,j} \bar{x}_{i,j} \ldots (8)
\]

4.2 Supply condition

The products delivered on j-th day have to arrive at the warehouse by j-1-th day. Therefore,

\[ a_{ij} \geq d_{ij}, (h=1, \ldots , H; \ j=1, \ldots , m). \]

Substituting Eq. (3), we get the supply condition of the products as follows;

\[ \frac{V_{ij}}{d_{ij}} \geq \sum_{j=1}^{i} b_{ij} \ldots (9) \]

The parts which are to be assembled on j-th day, are supplied either from those at the parts stockyard on j-th day, or from those which have been passed through the equipment by q-1 round of j-th day. Therefore, we get

\[ s_{i,j-1} \geq q_{i,j} \sum_{i=1}^{n} b_{ij} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij}, \ (i=1, \ldots , n; \ j=1, \ldots , m). \]

Substituting \( s_{i,j-1} \) by Eq. (2), we get the following supply condition of parts;

\[ \frac{s_{i,j-1}}{q_{i,j}} \geq \sum_{j=1}^{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \ldots (10) \]

The materials which are passed through the equipment on j-th day have to arrive at the parts stockyard by j-1-th day. Therefore, we get

\[ s_{i,j-1} \geq q_{i,j} \sum_{i=1}^{n} b_{ij} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij}, \ (i=1, \ldots , n; \ j=1, \ldots , m). \]

Substituting \( s_{i,j-1} \) by Eq. (1), we get the following supply condition of materials;

\[ \frac{s_{i,j-1}}{q_{i,j}} \geq \sum_{j=1}^{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \ldots (11) \]

4.3 Processing condition

The processing capacity of the equipment can be described by the upper limit of the number of jigs which can be hung on the conveyor line per round. Therefore, we get

\[ \sum_{j=1}^{Q} x_{i,j,q} \leq G, \ (i=1, \ldots , m; \ q=1, \ldots , Q-1), \ldots (12) \]

\[ \sum_{j=1}^{Q} \bar{x}_{i,j} \leq G, \ (i=1, \ldots , m). \ldots (13) \]

As for the quality of processing by means of the automated continuous processing equipment, the following situation arises. That is, in the case of processing different items on the same round by making jigs for different parts hang on the conveyor line, the parts of two different jigs which are adjacent with each other on the conveyor line are apt to have low quality. For example, in Fig. 4, where three different jigs A, B, C are hung on the conveyor line, the parts of the cross-hatched jigs are apt to have low quality.

\[ \begin{array}{c}
A \\
B \\
C
\end{array} \]

Fig.4 Explanation about processing quality

From the viewpoints of the above situation and of management, it is necessary that at least \( x \) jigs used for the same item be hung adjacent on the conveyor line. Therefore, the processing condition is described as follows;

\[ x_{i,j,q} \geq u \text{ or } x_{i,j,q} = 0, \ldots (14) \]

\[ (i=1, \ldots , n; \ j=1, \ldots , m; \ q=1, \ldots , Q). \]

The capacity of the assembly line is described by the net working hours per day. Therefore, we get

\[ \sum_{i} y_{ij} (u_{ij}/q_{ij}) \leq A, \ (i=1, \ldots , m). \ldots (15) \]

4.4 Condition about stockyards

The volume of the materials occupying the materials stockyard should be smaller than its volumetric capacity. Then, we get

\[ \sum_{i} a_{ij} \leq V, \ (j=1, \ldots , m). \]

Substituting \( s_{i,j-1} \) by Eq. (1), we get the following condition;

\[ \sum_{i} a_{ij} \leq \sum_{j=1}^{i} q_{i,j} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \leq a_{ij} \leq \sum_{i} a_{ij} \frac{V}{V}, \]

\[ \sum_{i=1}^{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \ldots (16) \]

Similarly, we get the following condition about the parts stockyard;

\[ \sum_{i} s_{i,j-1} (a_{i,j-1} \leq 1) \leq \sum_{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \leq \sum_{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \leq \sum_{i} b_{ij} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \ldots (17) \]

As for the warehouse, we get

\[ s_{i,j-1} \leq s_{i,j-1}, \ (i=1, \ldots , m; \ j=1, \ldots , m). \]

Substituting \( a_{ij} \) by Eq. (3), we get the following condition about the warehouse;

\[ s_{i,j-1} \leq s_{i,j-1} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \leq \sum_{i} s_{i,j-1} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{h=1}^{H} a_{ij} \ldots (18) \]

5. Linear Programming formulation

In order to formulate the problem by means of linear programming, the objective function and constraint conditions which are nonlinear have to be linearized.

Among the constraint conditions, Eq. (14) is nonlinear. By introducing integer variables \( n_{1,j} \), \( n_{2,j} \), \( n_{3,j} \), this condition can be replaced by a set of the following linear conditions. Let \( M \) be a sufficiently large number, then we get

\[ 0 \leq n_{1,j} \leq n_{1,j} + M_{1,j} \ldots (14.a) \]

\[ n_{1,j} \leq n_{1,j} + M_{1,j} \ldots (14.b) \]

\[ n_{1,j} + n_{2,j} = 1 \ldots (14.c) \]

When \( n_{2,j} = 1 \) and \( n_{1,j} = 0 \), we get \( n_{2,j} = 0 \ldots (14.b) \)

When \( n_{2,j} = 0 \) and \( n_{1,j} = 1 \), we get \( u_{i,j} = M-1 \ldots (14.c) \)

As for the objective function, the maintenance cost of jigs and the labour cost of changing the jig combination have nonlinearity. The maintenance cost of jigs can be linearized by introducing the auxiliary variables \( \lambda_{i,j} \) and \( u_{i,j} \), and \( \lambda_{i,j} \). Let

\[ \lambda_{i,j} = \max (s_{i,j}, w_{i,j} \max (\lambda_{i,j})), \text{ then the maintenance cost can be described by } \lambda_{i,j} \]

\[ \text{And the constraint conditions should be the same.} \]
be added by linearization are as follows;

\[ x_{i,j,q} \leq z_{i,j} \quad (i=1, \ldots, n; j=1, \ldots, m; q=1, \ldots, q_{0}) \]  

\[ x_{i,j} \geq z_{i,j} \quad (i=1, \ldots, n; j=1, \ldots, m) \]  

The labour cost of changing the jig combination can be linearized by introducing the nonnegative auxiliary variables

\[ t_{i,j,q} \]  

and \( \bar{t}_{i,j,q} \). Let \( t_{i,j,q} = x_{i,j,q} - z_{i,j,q} \), \( \bar{t}_{i,j,q} = z_{i,j,q} - 1 - x_{i,j,q} \), then, the labour cost can be described by

\[ \sum_{q=1}^{Q} \sum_{j=1}^{J} \gamma_{j} \bar{t}_{i,j,q} \]  

As for the constraint conditions which should be added by linearization, by the use of the relation \( x_{i,j,q} = x_{i,j,q} - x_{i,j,q-1} + (x_{i,j,q} - x_{i,j,q-1}) \), we get the following conditions:

\[ x_{i,j,q} - x_{i,j,q-1} \leq (x_{i,j,q} - x_{i,j,q-1}) \]  

\[ t_{i,j,q} \geq 0, \bar{t}_{i,j,q} \geq 0 \]  

Together with the above mentioned considerations and Eq. (4)–Eq. (8), the running cost to be paid during the planning period is described by

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{q=1}^{Q} \left[ \gamma_{j} \bar{t}_{i,j,q} \right] \]  

6. Numerical example

6.1 Input data

For the optimization problem as a numerical example, we adopted a mixed integer programming problem which minimizes Eq. (19) under the constraint conditions composed of Eqs. (9)–(13), Eqs. (14)-(a),(14)-(c), Eq. (15), Eq. (18), Eqs. (4.4a)-(4.4b) and Eqs. (7a)-(7b).

The input data concerning the delivery requirements and arrival of the materials are shown in Table 1. The data concerning the correspondence between part and product, \( b_{i,j} \), are shown in Table 2. As to the condition of processing quality, we adopted \( w=3 \), because the minimum unit describing the adjacent phenomenon is 3. Moreover, it is assumed that the first day and the 9-th day are holidays in the factory. Therefore, when \( j=1 \) and \( j=9 \), we get \( \bar{t}_{i,j} = 0, \bar{t}_{i,j} = 0, A_0 = 0 \).  

6.2 Search of optimal solution

The optimal solution was searched by I.B.M.'s, MPSX-MIP (Mathematical Programming System-Mixed Integer Programming). In the MPSX-MIP, the branch-and-bound method is used as the solution method for the mixed integer programming. This method is rather efficient compared with other methods, although there exists a limitation. Moreover, if we stop the computation before we get the true optimal solution, we can get a feasible solution and use it as the practical solution(10).

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</tbody>
</table>

Table 2 Correspondence between parts and products

But if we apply the method to a numerical example directly, it is expected that we need very long computation time. Therefore, the following procedure was taken. At first, we get the continuous optimal solution \( x_{i,j,q} \), and we set \( x_{i,j,q} = 0, x_{i,j,q} = 1 \), for \((i,j,q)\) such that \( x_{i,j,q} = 3 \) in the continuous solution, and \( x_{i,j,q} = 1, x_{i,j,q} = 0 \), for \((i,j,q)\) such that \( x_{i,j,q} = 0 \) in the continuous solution. And for \((i,j,q)\) such that \( 0 < x_{i,j,q} < 3 \) in the continuous solution, we let \( x_{i,j,q} = 1 \) and \( x_{i,j,q} = 2 \) remain as the integer variables and apply the branch-and-bound method.
to those \((i, j, \phi)\)'s.

And the computation was stopped when three feasible integer solutions were found. That is, the integer solution was improved three times. The improvement of the objective function is shown in Table 3.

<table>
<thead>
<tr>
<th>feasible integer solution</th>
<th>first solution</th>
<th>second solution</th>
<th>third solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>objective functional value</td>
<td>6815.35</td>
<td>6805.71</td>
<td>6796.86</td>
</tr>
</tbody>
</table>

Table 3 Improvement of objective functional value

6.3 Results

The computation results will be shown in this section. The optimal values are shown as variables attached with \(*\) (\(x_{i, j}^{(1)*}\), \(x_{i, j}^{(2)*}\), and \(x_{i, j}^{(3)*}\) are shown in Table 4). In this table, we can observe that the initial inventory of the warehouse is 2-5 times as much as the safety inventory level. On the other hand, the initial inventories at the materials stockyard and the parts stockyard are almost at zero level for almost all parts and materials. Moreover, it can be easily expected that the optimal initial inventories at any stage are far less than those of the traditional scheduling. Therefore, as for the initial inventory, it can be said that it is possible to start the production with far less level of the initial inventories of all stages than those of the traditional scheduling. In principle, we had better let the initial inventories at the materials stockyard and the parts stockyard be at zero level and the inevitable inventories concentrate on the warehouse.

The number of jigs passing through the equipment, \(x_{i, j}^{(1)*}\), is shown in Table 5. And the number on each day, \(x_{i, j}^{(1)*} \big|_{t=1}^{t=10} \), is shown in Table 6. The variables concerning the maintenance of jigs, \(y_{i, j}^{*}\), are shown in Table 7. In Table 6, we can get the following observations: As to 1st-6th parts, the production is concentrated at the middle of the planning period except 3rd part. As to 7th and 8th parts, there is no concentration of production. Therefore, in 1st-6th parts, the number of jigs needed on 5-th day is the number of jigs which have to be maintained during the planning period.

<table>
<thead>
<tr>
<th>(i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{i, j}^{(1)*})</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
</tr>
<tr>
<td>(y_{i, j}^{*})</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Table 5 Optimal values of the quantity of materials passed through the equipment

<table>
<thead>
<tr>
<th>(i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{i, j}^{(1)*})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(y_{i, j}^{*})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6 Daily quantity of materials passed through the equipment

<table>
<thead>
<tr>
<th>(i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{i, j}^{(1)*})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(y_{i, j}^{*})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7 Optimal values of number of jigs maintained

The number of products assembled, \(y_{i, j}^{*}\), is shown in Table 8. In this table, we observe that earlier peak of the production is on 3rd or 4-th day and the later peak of the production is on 7-th or 8-th day. Therefore, we can say that while the peak of the production in the equipment is at the middle of the planning period, the peak
of the production of the assembly line is in first quarter and in third quarter of the planning period.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
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</thead>
<tbody>
<tr>
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<td>4</td>
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<td>6</td>
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<td>8</td>
</tr>
</tbody>
</table>

Table 8: Optimal values of the quantity of products passed through the assembly line

As for the change of jigs combination \( t_{ij} \), we have to skip the details but the total number of jigs which should be hung or taken off on each day is shown in Table 9. In this table, we can observe that the number of jigs which should be hung is much larger than that which should be taken off, and that the number of jigs which should be taken off is large at the later half of the planning period.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
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<td>8</td>
<td>7</td>
<td>7</td>
<td>6</td>
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<td>6</td>
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<td>10</td>
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<td>4</td>
<td>5</td>
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<td>5</td>
<td>6</td>
<td>8</td>
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<td>10</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 9: Total number of jigs which should be hung or taken off on each day

7. Dynamic characteristics

The dynamic characteristics of the optimal values of the number of jigs passing through the equipment can be described depending on how the daily fluctuation of delivery requirements for the products can be smoothed at the stage of jigs passing through the equipment. Therefore it can be shown by plotting the daily delivery requirements of the product and the daily number of jigs passing through the equipment on the same graph. Some examples are shown in Fig.5.

In this figure, we can have the following observations: As to the pairs of 1st, 2nd, 3rd products and of the parts needed for these products, the daily fluctuation of delivery requirements is smoothed at the stage of jigs passing through the equipment. On the other hand, as to the pairs of 5th and 6th products and the parts needed for these products, the fluctuation is not smoothed at the stage of the equipment. We can point out the following two reasons: (1) The maintenance cost of jigs for 7th and 8th parts is cheaper than those for other parts. (2) The total delivery requirements for 5th and 6th products are small. But the number of adjacent jigs hung on the conveyor line has a lower limit, because of the condition of the processing quality. Therefore, the daily fluctuation of the number of jigs passing through the equipment is inevitable.

The dynamic characteristics of the optimal values of the quantity of the products passing through the assembly line, \( Y_{hij} \), can be described depending on how their daily fluctuations are made corresponding to the daily fluctuations of delivery requirements. And it can be shown by plotting the quantity of products passing through the assembly line and the delivery requirements on the same graph. Some examples are shown in Fig.6.

In this figure, we can have the following observations: As to 1st, 2nd, 3rd, 6th products, there exists good correspondence between these two fluctuations. But, as to 2nd product, there exists almost no correspondence between these two fluctuations. We can point out the following reason; the holding cost of 1st, 3rd, 6th products at the warehouse is higher than those for other products, but the holding cost of 2nd product is extremely cheaper than those for other products. Therefore, in order to reduce the total holding cost at the warehouse, it is necessary to make the daily
fluctuations of the quantity of products assembled corresponding to those of the delivery requirements in the case of 1st, 3rd, 6-th products, but it is not necessary in the case of 2nd product.

The dynamic characteristics of the optimal values of holding quantities can be described depending on what amounts of the materials, parts and products are stored at what stage of the production. It can be shown by plotting the daily delivery requirements $d_{n,k}$, the daily arrival of the materials $r_{n,k}$, and the amounts of stocks at each stage, $s_{n,k}^{(1)}$, $s_{n,k}^{(2)}$, $s_{n,k}^{(3)}$ on the same graph. Some typical examples are shown in Fig.7.

![Fig.7 Dynamic characteristics of holding quantity at each stockyard](image)

In the figure, the case (A) is an example where $s_{n,k}^{(3)}$ is much larger than $s_{n,k}^{(2)}$ and $s_{n,k}^{(1)}$. The case (B) is an example where $s_{n,k}^{(3)}$ is relatively small compared with $s_{n,k}^{(2)}$ and $s_{n,k}^{(1)}$. Therefore, while $s_{n,k}^{(2)}$ and $s_{n,k}^{(1)}$ have large values compared with $s_{n,k}^{(3)}$ in (A), $s_{n,k}^{(3)}$ has large value compared with $s_{n,k}^{(2)}$ and $s_{n,k}^{(1)}$ in (B). And (C) is a case where the initial inventories of all stages are not at zero level and the mutual relation among $s_{n,k}^{(3)}$, $s_{n,k}^{(2)}$, and $s_{n,k}^{(1)}$ is a standard one. And (D) is a case where the total delivery requirements are small, and $s_{n,k}^{(3)}$ is about the safety inventory level.

8. Effectiveness of the optimization and concluding remarks

The effectiveness of the optimization was calculated in terms of the reduced percentages of the number of jigs maintained and of the amounts of the materials, parts and products at the stockyards, as compared with those of the traditional scheduling. The results are shown in Table 10.

<table>
<thead>
<tr>
<th>Initial inventory</th>
<th>Maintenance quantity</th>
<th>The change in the maintained</th>
</tr>
</thead>
<tbody>
<tr>
<td>124-130</td>
<td>124-130</td>
<td>0.2</td>
</tr>
<tr>
<td>124-130</td>
<td>124-130</td>
<td>0.2</td>
</tr>
<tr>
<td>124-130</td>
<td>124-130</td>
<td>0.2</td>
</tr>
<tr>
<td>124-130</td>
<td>124-130</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 10 Calculated effectiveness of optimization

The problems to be studied in the future are summarized as follows:
1. It takes very long to get the true optimal solution of a large scale mixed integer program. Therefore, it is necessary to develop a heuristic algorithm, based on the results of several case studies.
2. It is necessary to develop a method of splitting large problems into several small ones which can be solved with a reasonable amount of efforts.

References