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On the Experimental Plane Elastoplastic Stress Analysis
by a Method Combining Birefringent Coating Method
and Conducting Sheet Method. (Reinforcing Effect
and Assumption for the Strain Normal to the Plane.)*

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From among many problems contained in this experimental method by which the stress analysis can be performed using only the indirect informations from the surface layer, the above two were picked up and the case of "a tension of a strip with semicircular notches" was investigated. As the specimen, an aluminium alloy strip regarded as isotropic solid that obeys von Mises' yield criterion and the flow theory was used and as the coating material, epoxypolysulfide copolymer was used. Mainly, the following conclusions were obtained: (1) The necessity of taking the reinforcing effect into consideration. (2) The appropriateness of the assumption $\nabla^2 \epsilon_3 = 0$ in a state of the restricted plastic deformation. (3) The unsuitableness of applying the deformation theory to the problems of this kind.

1. Introduction

As one of the experimental solutions for the plane elastoplastic stress problems, there is a method combining birefringent coating method and conducting sheet method.⁽¹⁾ Each method has an advantage required in the experimental solution for the problems of this kind by which it is possible to obtain, though indirectly, the continuous informations over a wide region.

But, there are many problems awaiting solution. Namely, for the former the thickness and the reinforcing effect due to the thickness and the rigidity of coating, and for the latter the limit of applicability of the assumption⁽²⁾ that the principal strain normal to the plane may satisfy the plane harmonic equation all over the plastic as well as the elastic region. For thin coating in a state of the plane elastic stress the reinforcing effect must be examined rather than the thickness effect. And for the specimen in a state of the elastic deformation, the correction factor⁽³⁾ for reinforcing effect by Zandman et al.⁽³⁾ can be applied and the principal strain normal to the plane satisfies the plane harmonic equation according to the theory of elastic body obviously, but, in a state of the plastic deformation, the above theories can not be applied with the deterioration of rigidity and the non plane stress condition caused

by the nonuniform deformation in the direction of thickness.

Therefore, in this paper, we examine the reinforcing effect and the limit of applicability of the assumption for the principal strain normal to the plane in a state of the restricted plastic deformation and intend to enhance the reliability of the experimental stress analysis by this method and to grasp the mechanical behavior of solid properly.

As the specimen, an aluminium alloy strip regarded as homogeneous (in the elastic deformation) and isotropic solid that obeys von Mises' yield criterion and Prandtl-Reuss' stress-strain relation based on the flow theory was used and as the coating material, epoxypolysulfide copolymer.

In order to facilitate the examination of the results, a case of "a tension of a strip with semicircular notches" which had been solved by the relaxation method⁽²⁾ numerically and by the same method^{(1),(4)} experimentally already was treated. Besides, the applicability of the deformation theory to the problems of this kind was discussed.

Notations

- x, y, z: cartesian coordinates in the directions of width, axis, & thickness
- t, 2w: thickness & width of net section
- E, G, ν : Young's modulus, modulus of rigidity & Poisson's ratio
- C: correction factor for reinforcing effect
- α : photoelastic sensitivity
- β' : principal strain difference sensitivity = $\alpha E / (1 + \nu)$
- N(dN): fringe order (increment)

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- P: tensile load
 $\sigma(d\sigma)$: stress(increment)
 $\bar{\sigma}, \sigma_s$: equivalent stress & yield stress in uni-axial tension
 σ : nominal stress across net section
 $\epsilon(d\epsilon)$: strain(increment)
 $d\epsilon^P$: equivalent plastic strain increment
 $(\epsilon_1 - \epsilon_2)_n$: nominal principal strain difference across gross section
 H' : strain hardening coefficient = $d\bar{\sigma}/d\epsilon^P$
 K : stress concentration factor = $\sigma_{\max}/\sigma_n$
 K_ϵ^σ : principal strain difference concentration factor = $(\epsilon_1 - \epsilon_2)_{\max}/(\epsilon_1 - \epsilon_2)_n$
 L : constraint factor
 ∇^2 : Laplacian in plane = $\partial^2/\partial x^2 + \partial^2/\partial y^2$
 Subscripts
 a, c: pertaining to specimen & coating in a composite
 o: pertaining to specimen having no coating
 1, 2, 3: pertaining to values in the principal directions
 e, p: pertaining to elastic & plastic components of strain
 i: step number = 0, 1, 2, ...

2. Basic theories

2.1 Stress-strain relations (5)

The stress-strain relations for the isotropic material in a state of the plane stress can be expressed as follows. In the elastic region, by Hooke's law

$$\begin{aligned} \frac{\sigma_1}{2G} &= \frac{\nu}{1-2\nu}(\epsilon_1 + \epsilon_2 + \epsilon_3) \\ \frac{\sigma_2}{2G} &= \frac{\nu}{1-2\nu}(\epsilon_1 + \epsilon_2 + \epsilon_3) \\ \epsilon_3 &= \frac{\nu}{1-\nu}(\epsilon_1 + \epsilon_2) \end{aligned} \quad (1)$$

In the plastic region, by the inverse form of Prandtl-Reuss's stress-strain relation together with von Mises' yield criterion

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_s^2 \quad (2)$$

$$\frac{d\sigma_1}{2G} = \frac{\nu}{1-2\nu}(d\epsilon_1 + d\epsilon_2 + d\epsilon_3) - \frac{(2\sigma_1 - \sigma_2)d\epsilon_1 + (2\sigma_2 - \sigma_1)d\epsilon_2 - (\sigma_1 + \sigma_2)d\epsilon_3}{6\bar{\sigma}^2(H'/3G+1)}(2\sigma_1 - \sigma_2)$$

$$\frac{d\sigma_2}{2G} = \frac{\nu}{1-2\nu}(d\epsilon_1 + d\epsilon_2 + d\epsilon_3) - \frac{(2\sigma_1 - \sigma_2)d\epsilon_1 + (2\sigma_2 - \sigma_1)d\epsilon_2 - (\sigma_1 + \sigma_2)d\epsilon_3}{6\bar{\sigma}^2(H'/3G+1)}(2\sigma_2 - \sigma_1) \quad (3)$$

$$\frac{d\sigma_1 + d\sigma_2}{2G} = \frac{1+\nu}{1-2\nu}(d\epsilon_1 + d\epsilon_2 + d\epsilon_3) \quad (d\epsilon_1^P + d\epsilon_2^P + d\epsilon_3^P) = 0, \quad d\sigma \geq 0$$

In right hand side of above equations, for the principal stresses the values at previous step are used, but, for H' concerning the loading path from the known $\bar{\sigma}_{i-1}$ at previous step to the unknown $\bar{\sigma}_i$ at new step, the value at previous step must not be used because the relation of equivalent stress-equivalent plastic strain is not followed rightly. That is, $\int d\epsilon^P = A\bar{\sigma}^B$ and H' in difference expression is

$$H'_i = \Delta\bar{\sigma}_i / \Delta(\int d\epsilon^P)_i = (\bar{\sigma}_i - \bar{\sigma}_{i-1}) / \{A(\bar{\sigma}_i^B - \bar{\sigma}_{i-1}^B)\} \quad (3)'$$

For the above reason, the stress analysis needs the repetitive calculation for $\bar{\sigma}_i$ until it converges. In order to precipitate the convergence, the value in the first approximation may be obtained by extrapolation (Newton-Gregory method).

And for $d\sigma < 0$, $H' \rightarrow \infty$.

In the large plastic strain field, the nonuniform deformation in the direction of thickness and the plastic anisotropy must be considered, but these matters may be neglected here because a state of the restricted plastic deformation is treated.

2.2 Basic theories in measurements

By the fundamental concept in birefringent coating method $\epsilon_{1a} = \epsilon_{1c}$ and $\epsilon_{2a} = \epsilon_{2c}$, the following relations are obtained

$$(\epsilon_1 - \epsilon_2)_a = N/(2\beta't_c), \quad d(\epsilon_1 - \epsilon_2)_a = dN/(2\beta't_c) \quad (4)$$

$$\text{And by the assumption for } \epsilon_3 \quad \nabla^2 \epsilon_{3a} = 0 \quad (5)$$

with the conditions for free boundaries in a state of the uni-axial stress, in the elastic region

$$\epsilon_{3a} = -\nu_a \epsilon_{1a} = -\{\nu_a/(1+\nu_a)\}\{N/(2\beta't_c)\}$$

and in the plastic region

$$\begin{aligned} \epsilon_{3a} &= -(\nu_a \epsilon_{1a}^e + 0.5 \epsilon_{1a}^p) = -\nu'_a \epsilon_{1a} \\ &= -\{\nu'_a/(1+\nu'_a)\}\{N/(2\beta't_c)\} \end{aligned} \quad (5)'$$

where ν'_a is dependent upon ϵ_{1a} or N .

Accordingly, first, for the elastic or the plastic region, each principal strain or principal strain increment can be separated by the 3rd Eq. of (1) and the 1st Eq. of (4) or by the 3rd Eq. of (3) and the 2nd Eq. of (4), and, next, each principal stress or principal stress increment is obtained from the 1st and 2nd Eqs. of (1) or from the 1st and 2nd Eqs.

of (3). In this procedure for the plastic region the repetitive calculation concerned with Eq.(3)' must be performed obviously.

The elastic-plastic boundary can easily be obtained by introducing the 1st and 2nd Eqs. of (1) expressed by N and ϵ_{3a} into the criterion of Eq.(2) without the stress analysis for all over the field,

$$(N/a)^2 + (\epsilon_{3a}/b)^2 = 1 \quad \text{where} \quad (6)$$

$$a = \sigma_s / \{\sqrt{3}G_a/(2\beta't_c)\}; \quad b = -\sigma_s / \{G_a(1+\nu_a)/\nu_a\}.$$

On the free boundary $N = \sigma \beta' t / G_a$ (6)
This procedure (a-h) is shown in Fig.1.

2.3 Reinforcing effect

This effect due to the coating appears in the principal strain difference (the fringe order) and for the specimen in a state of the elastic deformation the wellknown correction factor C by Zandman et al. (3) introduced from the equilibrium equations

$$\begin{aligned} t_a dy \sigma_{10} &= t_a dy \sigma_{1a} + t_c dy \sigma_{1c} \\ t_a dx \sigma_{20} &= t_a dx \sigma_{2a} + t_c dx \sigma_{2c} \end{aligned} \quad (7)$$

may be adopted as follows:

$$\begin{aligned} (\epsilon_1 - \epsilon_2)_0 / (\epsilon_1 - \epsilon_2)_a &= C \\ C &= \{1 + (t_c/t_a)(G_c/G_a)\}. \end{aligned} \quad (8)$$

But, for the specimen in a state of the plastic deformation the value C can not be expressed in a simple form like Eq.(8) because of complexity in the stress-strain relations Eq.(3). Now, considering the incremental forms of Eqs.(7), the correction factor C corresponding to Eq.(8) can be obtained as follows*:

$$\begin{aligned} d(\epsilon_1 - \epsilon_2)_{0,i} / d(\epsilon_1 - \epsilon_2)_{a,i} &= C' \\ C' &= \left\{1 - \frac{G_a}{H'_{a,i}} \frac{(2-S_a)T_{1a} + (2S_a-1)T_{2a} - (1+S_a)T_{3a}}{(1-S_a) + \frac{t_c}{t_a} \frac{G_c}{G_a}}\right\} / \left\{1 - \frac{G_o}{H'_{o,i}} \frac{(2-S_o)T_{1o} + (2S_o-1)T_{2o} - (1+S_o)T_{3o}}{(1-S_o)}\right\} \end{aligned}$$

where,

$$\begin{aligned} S_{a(o)} &= (\sigma_2/\sigma_1)_{a(o),i-1}, \\ T_{1a(o)} &= \{d\epsilon_1/d(\epsilon_1 - \epsilon_2)\}_{a(o),i}, \\ T_{2a(o)} &= \{d\epsilon_2/d(\epsilon_1 - \epsilon_2)\}_{a(o),i}, \\ T_{3a(o)} &= \{d\epsilon_3/d(\epsilon_1 - \epsilon_2)\}_{a(o),i}, \\ f d(\epsilon_1 - \epsilon_2)_0 / f d(\epsilon_1 - \epsilon_2)_a &= C. \end{aligned} \quad (9)$$

In the state of the uni-axial tension, as $v \neq 0$, $d\sigma_2 \neq 0$, and $d\epsilon_2 = d\epsilon_3$ may be assumed, Eq.(9) becomes as follows

$$\begin{aligned} C' &= [1/\{3(1/3 + G_a/H'_{a,i}) + (t_c/t_a)(G_c/G_a)\}] / [1/\{3(1/3 + G_o/H'_{o,i})\}] \\ f d(\epsilon_1 - \epsilon_2)_0 / f d(\epsilon_1 - \epsilon_2)_a &= C. \end{aligned} \quad (9')$$

The values in the numerator in right hand side of the 2nd Eq. of (9) are known from the results in the case of the reinforcing effect not being taken into consideration, but the values at new step in the denominator $d\epsilon_{10,i}$, $d\epsilon_{20,i}$, $d\epsilon_{30,i}$, and $H'_{o,i}$ (or T_{1o} , T_{2o} , T_{3o} , and $H'_{o,i}$) are unknown. Among these, only $d\epsilon_{30,i}$ can be obtained by solving Eq.(5) with the boundary condi-

* There is another method of solving simultaneous equations by introducing the 1st and 2nd Eqs. of (3) into the incremental forms of Eqs.(7). In this method stress analysis needs the repetitive calculation only for $\bar{\sigma}_i$ concerning $H'_{o,i}$.

Fig.1 Method of determination of elastic-plastic boundary.

tion corresponding to the principal strain difference corrected by Eq.(9) anew. Therefore, the stress analysis of this case needs the repetitive calculation for the approximate values of $d\epsilon_{10,i}$, $d\epsilon_{20,i}$, and $\bar{\sigma}_i$ concerning $H'_{o,i}$ as well as $\bar{\sigma}_i$ for Eq.(3) mentioned before.

3. Experiments

3.1 Various properties of materials used

In Table 1 the chemical composition, the mechanical properties, etc. of the specimen and coating material are shown,

and in Fig.2 the stress-strain difference relation, etc. in the uni-axial tension. For the measurement of strain, wire strain gauges of 6 mm gauge length were used considering the cross sensitivity and for the measurement of β' , Babinet's compensator. The proportional limit was taken as the yield point of specimen and the equivalent stress-equivalent plastic strain relation was obtained in the form noted down in the

right hand side of Table 1. Furthermore, the homogeneity and the isotropy in small strain field $(\epsilon_1 - \epsilon_2) \leq 10 \times 10^{-3}$ were checked.

Fig.2 Mechanical properties in uni-axial tension.

3.2 Reinforcing effect in uni-axial tension

Fig.3 represents the experimental result and the numerical result by Eq.(9)'. In this calculation $d(\epsilon_1 - \epsilon_2)_{a,i} = 186 \times 10^{-6}$ corresponding to the fringe order of 0.1.

Fig.3 Reinforcing effect in uni-axial tension.

3.3 Tension test of strip with semi-circular notches

3.3.1 Preliminary test

In order to obtain the reference data for a state of the elastic deformation in the main test (the 0th step), a plane photoelastic test using an epoxy resin plate was made.

3.3.2 Main test (measurements of N & ϵ_3)

The specimen was such a thin plate (thickness:3mm, width of gross section: 200mm, length:600mm and radii of semicircular notches:50mmR, and 10mm×10mm mesh on its surface) that the state of it might be regarded as the plane stress during gradually increasing loading. As for the loading, a step by step process was adopted. From the limit of the elastic deformation at the bottoms of the notches till the beginning of plastic flow, for each 100kg of loading, the monochromatic and the coloured pictures of the fringe pattern in dark field by normal incidence were taken. The fringe distribution at each step was finally decided, considering the continuous variances in the directions of x and y at each one step and for the

change of loading at each point.

The principal strain normal to the plane was solved by means of the apparatus shown in Fig.4. The conducting sheet was cut four times as large as real specimen for the first quadrant for the sake of symmetry and connected at the boundary to the cords by silver paint.

C.S.:Conducting sheet,
V.D.:Voltage divider,
P:Probe,
D.V.:Digital voltmeter
(sensitivity 1/1000V).

Fig.4 Schematic diagram of apparatus for conducting sheet method.

Now, the stress analysis by step by step process based on the flow theory can be performed in two ways. Namely, each step is settled by equal load increment or by equal maximum principal strain difference increment. The latter method that keeps the maximum value of strain increment at each step constant and makes the extrapolation in the repetitive calculation easy and, as the result, enhances the accuracy in calculation is preferable to the former. Therefore, the latter called "equal maximum principal strain increment method" was used and the state of a true limit of elastic deformation at the bottoms of the notches was named the 0th step and the each successive step was taken by $dN = 0.1$ at the same points. But in the case of the reinforcing effect being taken into consideration this restriction is not made of course.

4. Results

Hereafter, case A means a case of the reinforcing effect not being taken into consideration and case B does one of the reinforcing effect being taken into consideration.

4.1 Elastic-plastic boundary

Table 1 Chemical compositions, mechanical properties, etc.

	Compositions %			t mm	E kg/mm ²	ν	σ_s kg/mm ²	The others
Specimen (A2P1-1/2H)	Al 97.25	Mg 2.50	Cr 0.25	3.00	7120	0.344	16.0	$f d \epsilon^P = 5.43 \times 10^{-29} \times \bar{\sigma}^{19.147}$ ($\bar{\sigma}$: kg/mm ²)
Coating material (Epoxy resin)	Epoxy prepolymer : 77.0 Polysulfide : 15.0 Diethylene triamine : 8.0			1.84	300(0.469)	—	—	$\alpha = 0.714 \text{ mm/kg}$ ($\lambda = 5461 \text{ \AA}$) $1/(2t_c \beta') = 1860 \times 10^{-6}$

Fig.5 Evolution of elastic-plastic boundary. (a) Case A (Tresca's yield criterion). (b) Case A (Mises' yield criterion). (c) Case B (Mises' yield criterion).

Fig.5 represents the evolution of the elastic-plastic boundary for an increasing σ/σ_s . (a) shows $(\epsilon_1 - \epsilon_2)_a = \sigma/(2G)$ (corresponding to Tresca's yield criterion and $N = 1.624$) in case A. (b) represents the manner of the evolution in case A wherein two fronts of the plastic regions developed from the bottoms of the notches join at (20,50) in $\sigma/\sigma_s = 1.23$ with other fronts developed from the plastic core appearing at (0,70) in $\sigma/\sigma_s = 1.21$. (c) case B wherein two fronts developed from the bottoms of the notches to the center of plate join at the center (0,0) in $\sigma/\sigma_s = 1.13$. The constraint factor $L = 1.13$ is, to some degree, higher than the lower and the upper limit of 1.097-1.113 by Hill and Ford et al. (8), but, considering the behavior after the restricted plastic deformation, this seems to be right. As the manner of the evolution in (b) is very similar to the equi-equivalent stress lines in the elastic deformation, the reinforcing ef-

fect is obviously conceivable.

4.2 Strain and stress distributions across net section

Comparing between Fig.6, case A and Fig.7, case B, the difference in quantity may be seen in the vicinities of the bottoms of notches. Furthermore, the stress distributions along the free boundary are shown in Fig.8. In above stress analysis, each process was the loading one.

4.3 K_ϵ and K_σ

The variance of K_ϵ at the bottoms of notches where the principal strain differences take the maximum values for an increasing σ/σ_s is shown in $x/w = 1.0$ of Fig.9 together with the variances at other points. The arrow indicates the beginning of the plastic deformation, and the distance between the straight lines does the restricted plastic deformation range. As the nominal value in the gross section is the value in the elastic deformation, the deviation from the line parallel to the horizontal axis means a deviation from the state of the elastic deformation all over the region. So, from Fig.9, it is understood that the influence of the local plastic region on the elastic regions may be small.

The points of the maximum stresses shift from the bottoms of the notches to the inner points. The variance of K_σ for an increasing σ/σ_s is shown in Fig.10, in which the fine line indicates the values at the bottoms of the notches. The elastic stress concentration factor was 1.64 , which is lower than 1.65 by Heywood (9) but higher than the numerical value (10) obtained up to the present.

4.4 Stress history

In the problems of this kind, it is interesting to pursue the stress history in connection with the numerical analysis. The stress histories at four points on the net section are represented in Fig.11, in which the curve of $\bar{\sigma}/\sigma_s = 1.00$ indicates von Mises' yield curve and the curve of $\bar{\sigma}/\sigma_s = 1.31$ that by the assumed yield stress decided by the point of intersec-

Fig.6 Strain and stress across net section for case A.

Fig.7 Strain and stress across net section for case B.

tion of two extended straight lines of the elastic and the plastic portion. Generally speaking, the region enclosed by these two curves is the one where the strain hardening characteristic changes suddenly. In a state of elastic deformation ($\bar{\sigma}/\sigma_s < 1$) the proportional loading relation is retained very nearly as it was mentioned in 4.3, but, in a state of the successive plastic deformation ($\bar{\sigma}/\sigma_s = 1.31$) it is not retained. Therefore, the application of the proportional loading relation to the problems of this kind may be limited only to a few incipient steps in a state of the plastic deformation, and the determination of yield stress must be performed carefully.

And it is interesting that the ratio of $(\sigma_2/\sigma_s)/(\sigma_1/\sigma_s)$ at $x/w=0$ in case B is almost the equal to 0.5 at the net section in the yield point of the plate with the same notches obtained by the stress trapezoid.

Fig. 8 Stress along free boundary.

Fig. 9 Principal strain difference concentration factor.

Fig. 10 Stress concentration factor. (Thin line indicates value at bottom of notches.)

5. Examinations

5.1 Accuracy of experiment and ϵ_3

The main test was performed up to the step $i=30$, but the treatments of results

Fig. 11 Stress history.

were brought to an end by $i=25$ because of the occurrence of innumerable slip lines in the vicinities of the bottoms of notches at the latter steps, which made the measurement of fringe pattern difficult. Usually, the estimation for the accuracy of experiment of these problems is done by comparison between the axial load calculated by the stress analysis and the one measured by the experiment. Accordingly, the error so-called contains all error elements concerning the fringe pattern reading and the assumptions (isotropy, plane stress state, stress-strain relation, and ϵ_3) used here.

Specially, the assumption for ϵ_3 ceases to hold with the appearance of a large strain field because the deformation becomes three-dimensional. For instance, as it is observed with the necking in the uni-axial tension, a state of the non plane stress such that the load carrying capacity in the middle layer is larger than in the surface layers may be supposed. As the birefringent coating method is a method that measures the strains on the surface, the axial load calculated may be expected to decrease naturally.

Fig. 12 Experimental error in axial load.

The error in the axial load at each step is represented in Fig. 12, in which both cases A and B show the same tendency. In case A the maximum absolute value considering the reading errors of dN and ϵ_3 and the repetitive calculation was estimated at about 10% and its sign had to be negative during all steps, and in case B the value was 15% considering the elements mentioned above in addition to the error in the calculation for the rein-

forcing effect, but, the maximum was 1.2% in positive sign in the former and 15.2% in the latter. In comparison between the former case where the assumption for ϵ_3 might nearly hold and the latter case, the same tendency was observed except at the latter steps, in which in the latter case the tendency of decrease was obvious. This tendency may be explained by the reason mentioned above. Therefore, in the experimental plane elastoplastic stress analysis, the assumption $\nabla^2 \epsilon_3 = 0$ may hold in a state of the restricted plastic deformation.

From the discussion mentioned above, the error in this experiment may be said to be in the allowable limit.

5.2 Reinforcing effect

When the stress analysis is performed by the flow theory, the reinforcing effect must also be considered for dN at each step. A fundamental examination of this problem was performed by the uni-axial tension test. From the results shown in Fig.3, the difficulties in both the experimental measurement and the numerical calculation occurred in the state of H' changing suddenly. But, from the figure, it seems that both results agree well. And the expansion to a two-dimensional problem is given by Eq.(9). In comparison between both cases for the strain and the stress components in Figs.6 and 7, though the differences in absolute values are reasonable, those tendencies of distributions are the same, but a remarkable difference is observed in the manner of the evolution of elastic-plastic boundary in Fig.5. This makes the reliability of the constraint factor having an important significance for the notch problems decrease. Fig.13 (1),(2),(4) represent some other results. Considering the manner of the evolution of the elastic-plastic boundary and the strain distributions, the results by the author correspond to the numerical results by Allen et al.⁽²⁾ which are seemingly appropriate, therefore the necessity of taking the reinforcing effect into consideration must be pointed out. As for the other results,^{(1),(4)} it is not clear whether those structures were treated as the composite as often seen or not.

6. Conclusions

The following matters are concluded from this experiment.

As for the birefringent coating method:(1) The assumption $\nabla^2 \epsilon_3 = 0$ holds nearly under the condition of the restricted plastic deformation. Therefore, the conducting sheet method is applicable to the problems of this kind and, as the result, the strain components can easily be separated. (2) The reinforcing effect must be considered in the relative comparison between the specimen and the coating material and the correction factor can be calculated by Eq.(9).

As for the stress analysis:(1) When the deformation theory is applied to the

Fig.13 Other results (evolution of elastic-plastic boundary).

Table 2 Other results (constraint factor, etc. in reference to Figs. 5 and 13)

	Method	Yield criterion	Material σ_s kg/mm ²	$2w \times t_a$ mm (t_c)	K_σ	L	The others
Allen- (2) Southwell	Relaxation	von Mises' (ideal body)	—	—	1.52	1.12	Fig.13(a)
Theocaris (1)	Coating-Conducting sheet	von Mises' (ideal body)	Alloy steel 84.7	203.2×3.2 (3.5)	1.52	1.15	Fig.13(b)
Dixon (4)	Coating	Tresca's	Aluminium alloy 30.8	6.4×1.2 (0.4)	1.75	1.10	Fig.13(c)
Author	Coating-Conducting sheet	von Mises'	Aluminium alloy 16.0	100.0×3.0 (1.8)	1.64	1.13	Fig. 5(c)

problems of this kind, the limit of application must be taken into consideration sufficiently. (2) The behavior of the material in the elastic region may be regarded as an elastic proportional loading one. (3) As the point of the maximum stress shifts from the bottom of notch to the inner point, the determination of the maximum stress must not be done at the bottom of notch every time. (4) In the analysis by step by step process, it is desirable that "equal maximum principal strain increment method" be used.

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