Propagations of the Surface Fatigue Cracks in Various Kinds of Notched Specimens*

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In this report, the crack propagations in several kinds of notched round bar specimens of SK 5 and SUS 50 steels are investigated. The main results are summarized as follows:
1. It is possible to estimate the S-N curves for any notched specimen by using the idea of \( C_0 \), which has been often adopted to predict the endurance limit for crack initiation.
2. If the same crack length is obtained for the imposition of the same number of cycles, the crack propagation rates coincide with each other irrespective of the notch root radius and the materials.
3. In opposition to the results in (2), the effect of specimen shape appears on the crack propagation in the radial direction.
4. The index "m" for the formula, \( dL/dN=A K^m \) or \( dL/dN=A K_1^m \), is considered to be related with the gradient of S-N curve and varies from 4 to 5 depending on the stress amplitude.

1. Introduction

Up to this time, many studies about the fatigue crack propagation behaviours have been performed and also both theoretical and experimental crack propagation laws have been proposed. It is considered, however, that those laws are imperfect on account of some obscure experimental constants involved and they can not always estimate completely the experimental results which are proposed by the other investigators.

Therefore, the authors feel the necessity for re-examination of such laws containing the uncertain matters and for establishment of a more generalized crack propagation law. Especially, according to the microscopical observation of the crack propagation behaviours, it is recognized that the crack propagation behaviour is different in different localities. The following fact is considered typical: The fatigue crack propagation is controlled by the shear stress when the crack tip is situated near the crack initiation point (in the case of surface crack initiation), while it is controlled by the nominal stress when the crack tip is situated sufficiently apart from the crack initiation point.

The above matter has been, however, hardly considered in the studies until now and in many cases the crack propagation behaviours have been investigated under the simultaneous existences of different crack propagation mechanisms. Moreover, since the past theoretical crack propagation laws have been examined, unfortunately, by using those experimental results, it is doubtful whether any suitable investigation of the crack propagation law has been truly performed or not.

For instance, the reversal tension fatigue test of a plate specimen is considered to be a typical example of the above mentioned experiments. Namely, if the cyclic stress is imposed on a plate specimen which contains a central circular hole as shown in Fig.1(a), cracks initiate at both sides of the circular hole and propagate in both right and left directions (Fig.1(b)) indicates the cracked

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surface at AA section). Referring to Fig. 1(b), it is expected that the cracked parts which are situated in the regions near both surfaces (B3 and C1 sides) of plate propagate based on the mechanism of surface crack propagation. On the other hand, the cracked part which is situated in the region sufficiently apart from both sides of plate or from the central hole propagates based on the mechanism of inner crack propagation.

As mentioned above, however, the discussions in almost all the studies until now have been performed by using the average crack propagation rates obtained under the mixed condition of the above two crack propagation mechanisms (usually, the crack length which is measured on the surface is adopted for the investigation). Therefore, the past experiments are considered to be insufficient and unsuitable for the exact investigation of crack propagation law and consequently it is necessary to investigate the above two mechanisms independently.

Then, the authors planned to make clear the propagation mechanisms for the surface crack (the crack is situated in the region near the surface) and the inner one (the crack is situated sufficiently apart from the surface) individually and propose a new crack propagation law. As a preliminary to a series of experiments, the surface crack propagations in several kinds of notched specimens are investigated in this report and several matters become clear.

2. Experimental procedure

A canti-lever type rotating bending fatigue testing machine is used for this experiment. SK 5 steel is mainly used for the testing material and SUS 50 steel is also used a little. Since the grain sizes of these materials are too small to confirm the grain boundary by a microscope of 400 magnifications, the fatigue cracks are supposed to behave as if they propagated in homogeneous materials. Therefore, this experiment is considered to be suitable for the comparison between the experimental crack propagation and the theoretical law which is derived by using the continuum mechanics.

Several kind of notched round bar specimens are used and a specimen whose notch root radius is 30mm is considered to be a smooth one. The chemical composition and the mechanical properties are shown in Tables 1 and 2. Three or four kinds of notched specimens are used for SUS 50 and SK 5 steels, respectively. Moreover, the dimensions and the stress concentration factors for these specimens are shown in Fig. 2.

After machining, the notch root of specimen is polished and annealed about 1.5 hrs at 650°C in vacuum. Then, electric polishing is also done before testing. The surface crack is measured with a microscope of 400 magnifications and in the case of SK 5 steel specimen the crack depth is also measured by the oxidization method.

3. Experimental results and considerations

First, the S-N curves for this experiment are obtained and shown in Fig. 3(a)(b). According to this figure, it is recognized that the fatigue limit for the smooth specimen of SK 5 steel (Fig. (a)) is about 8 kg/mm² higher than that of SUS 50 steel.

Table 1 Chemical composition (%)

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<th></th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
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<tr>
<td>SK 5</td>
<td>0.37</td>
<td>0.32</td>
<td>0.39</td>
<td>0.010</td>
<td>0.009</td>
<td></td>
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<tr>
<td>SUS 50</td>
<td>0.13</td>
<td>0.30</td>
<td>0.31</td>
<td>0.025</td>
<td>0.014</td>
<td>12.31</td>
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Table 2 Mechanical properties

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<tr>
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<th>Yield point kg/mm²</th>
<th>Tensile strength kg/mm²</th>
<th>Elongation %</th>
<th>Contraction %</th>
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<tr>
<td>SK 5</td>
<td>34.0</td>
<td>55.2</td>
<td>16.5</td>
<td>50.6</td>
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<tr>
<td>SUS 50</td>
<td>34.4</td>
<td>59.2</td>
<td>31.2</td>
<td>70.1</td>
</tr>
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</table>

Fig. 2 Dimensions of test specimens

Fig. 3 S-N curves for final fracture of SK 5 and SUS 50 steel specimens
(Fig.(b)), while no difference appears between the results for SK 5 and SUS 50 steels of 0.3mm notch root radius. Next, the crack length at the notch root is measured referring to these S-N curves. The surface crack propagation curves for SK 5 steel smooth specimen and 0.15mm notch root radius one are shown in Figs. 4 and 5, while those for SUS 50 steel are shown in Figs. 6 and 7. These results are typical examples for dull-and sharp notched specimens, respectively. Although only about four crack propagation curves are shown for each stress amplitude of SK 5 steel, more cracks are really observed and the numbers are indicated in the parentheses of these figures. Namely the crack propagation curves in these figures show the average ones. On the other hand, only several crack propagation curves which have been obtained earlier are plotted for the case of SUS 50 steel.

These cracks mainly initiate from the vicinity of inclusions and slip lines are hardly observed on the surface. It is also observed that the cracks are combined with each other with an increase of their length and the crack length before combination only is measured in this report. Therefore, several differences appear in the crack propagation behaviours between the dull-and the sharp notched specimens. For example, surface cracks longer than 3.0mm can be measured in smooth specimens of SK 5 steel, while ones shorter than about 1.0mm can only be measured in specimens of 0.15mm notch root radius. Therefore, the comparison between these results is only performed at the common measurable crack length.

In addition, it is recognized from the results in Figs. 5 and 7 that the crack propagation rates in sharp-notched specimens decrease just before the crack combinations. This phenomenon is caused by the discrepancy of the propagation courses of two cracks which are expected to combine and the crack propagation rate decreases when the crack propagates in slant direction as shown by the dashed curve in Fig. 8.

Then the crack propagation rates are
derived from the results in Figs. 4 ~ 7. Although the crack propagation curves are remarkably scattered for each stress amplitude, the crack propagation patterns do not considerably differ from each other. This fact is explained by using a smooth specimen of SK 5 steel. Namely, each crack propagation curve in Fig.4 is replotted by shifting them by a suitable amount along the abscissa to get the best fit of these curves. Since most experimental points can be replotted on each smooth curve as shown in Fig.9, the scatter of the crack propagations is considered to be very small. Therefore, this fact means that the scatter in the crack propagation is smaller than

Fig. 8 Scheme of the combination process of the surface crack

Fig.9 Relation between the surface crack length and the number of cycles for SK 5 steel specimen of $\rho = 30$mm (the crack propagation curves shifted parallel to the abscissa)

Fig.10 Relation between the surface crack length and the crack propagation rate for SK 5 steel

Fig.11 Relation between the surface crack length and the crack propagation rate for SUS 50 steel
that in the crack initiation (pay attention to the fact that the number of cycles for this figure does not a real one, because the experimental results are shifted by a certain amount).

After reploting the results as shown in Fig.9 and drawing the average crack propagation curves, the relations between the crack propagation rate and the crack length are obtained as shown in Figs.10 and 11.

According to the results in Fig.10, it is recognized that a linear relation exists between dL/dN and 1 on logarithmic scale for a dull-notched specimen (Fig.10(a)), while such relation does not exist for a sharp-notched specimen (Fig.10(b)). As mentioned previously, this difference is considered to be caused by the crack combination phenomenon which is often observed after the cracks in the sharp-notched specimens grow to about 0.5 ~ 1.0mm. Although the same behavior as that caused by the crack combination is, by the way, observed under the stress amplitude below the fatigue limit (σf = 8kg/mm² in Fig.10(d) and 10kg/mm² in Fig.11(c)), this phenomenon is considered to come from the special character of a non-propagating crack rather than that of the crack combination.

Next, the relation among the crack propagations in several kinds of notched specimens are investigated for further examination of the crack propagation. First, the S-N curves for SK 5 and SUS 50 steels at l=0.2mm are obtained from the above results and shown in Figs.12 and 13, respectively. Since the ordinate in Fig.12(a) for both diagrams is the nominal stress, σN, the S-N curves for sharper notched specimens are located on the lower side of the diagram. On the other hand, since the ordinate in Fig.12(b) for both diagrams is the maximum stress, σmax (α is the stress concentration factor), the S-N curves for sharper notched specimens are located on the upper side of the diagram. This phenomenon means that a crack in the sharp notched specimen propagates more slowly than one in the dull notched specimen in spite of a higher maximum stress amplitude in the sharper notched specimen (here, the stresses are expected to be kept in the elastic region).

Judging from these results, it is supposed that the S-N curves for several kinds of notched specimens can be summarized on a certain master curve by multiplying a suitable coefficient which takes the place of stress concentration factor α. Then, for this purpose the authors adopt the so-called idea of μ, which was proposed by T. Ishiawashi for the explanation of the relation among the fatigue limits for the crack initiation of different notched specimens, and perform some investigations. This rearranging method was also adopted by the authors to estimate the S-N curves for crack initiations and better results were obtained.

![Fig.12: S-N curves of SK 5 steel for l=0.2mm](image)

![Fig.13: S-N curves of SUS 50 steel for l=0.2mm](image)

![Fig.14: Bending stress distributions in several kinds of notched specimens](image)
Fig. 14 shows the stress distribution near the notch root in several kinds of specimens used in this report. According to this figure, the most suitable values of $\varepsilon_r$ to rearrange the S-N curves for SK 5 and SUS 50 steels are 0.05 and 0.03 mm, respectively. Also, the rearranged results are shown in Figs. 12(c) and 13(c), in which the ordinate is $\alpha'\sigma_{rad}$ ($\alpha'$ is the value of the stress concentration at the distance of $\varepsilon_r$ from the notch root surface to the inner direction (cf. Fig. 14)). Judging from these results, it is clear that the S-N curves for a specific crack length can be rearranged on a certain curve by using a suitable value of $\varepsilon_r$. Namely, this fact means that the S-N curve for a certain notched specimen can be estimated from that for the other one. In addition to the results in Figs. 12 and 13, where the crack length is fixed at 0.2 mm, the same tendency is observed for the other crack length, for example, as in the case of 1=1.0 mm. Although the magnitude of $\varepsilon_r$ is supposed to be affected by the plastic zone size at the vicinity of notch root and consequently to vary with the crack length, no such phenomenon is observed in this experiment.

According to the above fact, in which the crack propagations in different notched specimens can be estimated from each other by using the idea of $\varepsilon_r$, the common mechanism is supposed to be applicable to the crack propagations in these specimens (also including unnotched specimen). Therefore, this matter is re-examined minutely from

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Fig. 15 Relation between the number of cycles and the crack propagation rate for $l=0.2$ mm

Fig. 16 Relation between the number of cycles and the crack propagation rate for $l=1.0$ mm

Fig. 17 Crack propagation in the inner direction of SK 5 steel
the viewpoint of crack propagation rate. The results are shown in Figs.15 and 16, in which the abscissa is the number of cycles necessary to make a certain amount of crack length for each stress amplitude (here, the crack length is chosen as 0.2 or 1.0mm) and the ordinate is the crack propagation rate.

From these figures, it is clear that the results for different notched specimens are located within a certain band. In other words, if the numbers of cycles necessary to make a certain surface crack length are the same, namely if the fatigue lives for a specific surface crack length coincide with each other, the same crack propagation rate is estimated irrespective of notch shape. Moreover, since the results for SK 5 and SUS 50 steels are located in the same band, this relation is also considered to hold irrespective of material properties. In addition, the experimental results are plotted on a straight line inclined about 45° in logarithmic scale, namely d1/dN varies in inverse proportion with N.

Referring to the above results, in which the surface crack propagation behaviour of different notched specimens and materials are rearranged by a simple method as shown in Figs.12, 13, 15 and 16, more minute consideration is performed through observation of the behaviour of the surface crack in the inner direction by the oxidation method.

According to the oxidation method, it is recognized that the crack propagation patterns in the inner direction of both sharp and dull notched specimens differ from each other as shown in Fig.17(a)(b). Namely, in the case of a sharp notched specimen the contour of the crack front becomes like a circular arc in the initial condition but soon changes into a concentric circle about the axis of the specimen (Fig.(a)), while in the case of a dull notched specimen the shape of circular arc is maintained over a remarkably deep crack region (Fig.(b)).

Above fact becomes more clear from the results in Fig.18 in which the relations between the crack length in the surface direction and the depth in the inner direction are indicated about different notched SK 5 steel specimens. Consequently, for the same length of the surface crack, the smaller the notch root radius, the deeper the crack depth. This means that the crack propagation behaviour in the inner direction is influenced by the notch root radius. Therefore, at first sight this fact seems to be contradictory to the results in Figs. 15 and 16. Such apprehension, however, vanishes by considering the fact that the crack propagation is only controlled by the condition in the extremely small region near the crack tip as indicated by $E_0$. Because, the stress condition near the tip of a crack which propagates in the inner direction changes with the stress distribution(cf. Fig.14) depending on the notch root radius (in this case the average stress condition in the region of $E_0$ is supposed to differ from each other with the difference of the notch root radius), while the stress condition at the tip of the crack which propagates along the surface is considered to be hardly affected by the shape of notch root (in this case the average stress condition in the region of $E_1$ is supposed to be constant irrespective of notch shape).

According to these results, the effect of the stress gradient on the crack propagation does not seem to appear either. Since the physical meaning of $E_0$ is, however, not clear yet, an exact conclusion is reserved for another occasion.

Next, the investigation of the crack propagation laws is performed using these surface crack propagation data. The laws which have been proposed can be summarized as follow: $d1/dN = CK^{\frac{m}{2}}$, $d1/dN = C' \cdot 1^m$. $k$ is the stress intensity factor and $C$, $m$ and $n$ are experimental constants. Supposing $l$ is a constant, the index "m" in these formulas possesses the same meaning. Then first, the investigation of the index "m" is performed. Since the gradient of the curve for $\log d1/dN = \log \sigma$ at specific crack length coincides with the value of the index "m" the relations between $\log d1/dN$ and $\log \sigma$ for $l=0.2$ and 1.0mm are obtained for reference and shown in Figs. 19-21.

In the case of SK 5 steel, the stress amplitudes are expressed using the same

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* Since the crack propagation curves are scattered as shown in Figs.4-7, the average values are taken as the number of cycles.
factors as used in Fig. 12: nominal stress \( \sigma_0 \), maximum stress \( \sigma_{\text{max}} \), and the stress based on the idea of \( \sigma_{\text{eq}} \). According to the results in Figs. 12 and 19, the patterns of the S-N and the

d\(dN/d\alpha \) curves for \( \alpha = 0.2 \) mm are quite similar to each other (it becomes more clear if the curves in Fig. 12 are turned 90 degrees counterclockwise and compared with those in Fig. 19). Furthermore, this relation is confirmed by considering the existence of a linear relationship between \(dN/d\alpha \) and \(N\) on logarithmic scale (cf. Figs. 15 and 16). Therefore, it is recognized from these results that an intimate relation exists between the index "\( m \)" (gradient of the curve in Fig. 19) and the gradient of the S-N curve in Fig. 12. Since the S-N curves, generally, do not always show a linear relation on logarithmic scale, it is expected that the index "\( m \)" does, and it always become invariant for specific material, specimen shape, testing condition and so on. Actually, the index "\( m \)" varies continuously from about 4 to 5 with a change of stress amplitude in the case of Fig. 19.

Although it is dangerous to draw the above conclusion only from the tendency of generally obtained S-N curves which express only the final fracture phenomenon of the fatigue, an important idea in examination of the fatigue crack propagation law will be obtained in future by elaborate investigation of the S-N curves for a specific crack length.

Re-examining the past results including Paris's one(2) which were investigated by using the stress intensity factor, it is recognized that the experimental points fall on the same S style curves as those which are often observed for the generally obtained S-N curves. In other words, the fact that the index is not always invariant irrespective of crack propagation rate is confirmed by the other investigators' results. Therefore, after this the estimation method(9) for the crack propagation and the S-N curve which have been proposed by using the idea of constant index "\( m \)" must be reconsidered.

On the other hand, the index "\( n \)" expresses the crack propagating condition

* Since the number of cycles to final fracture is equivalent to the value at which an instantaneous transition from the fatigue crack propagation process to static fracture occurs and the crack length at the transition point changes with the stress, material properties and testing condition, the comparison of the S-N curves for the final fracture and the specific crack length is considered to be inappropriate. However, considering the acceleration of crack propagation just before the final fracture and the scatter in the numbers of cycles to final fracture, it is expected that the pattern of S-N curves for fracture and the specific crack length do not always differ remarkably from each other. Moreover, as the S-N curves for the latter case are considerably, the S-N curves for final fracture are conveniently used as the reference data.
at specific stress amplitude (in the first formula, \( n \) is included in \( K^n \)). Since the grain sizes of the materials in this experiment are too small to observe by a microscope of 400 magnifications, it is not necessary to consider the effect of the grain boundary on the crack propagation. The effects of the inclusions and the specimen dimensions, however, also exist even in the results of this experiment and it is difficult to conclude the exact meaning of index "\( n \)" only from the results in this report. But, from rough investigation of Figs.10 and 11, the index "\( n \)" becomes about unity for these specimens.

4. Conclusions

From the investigation of the fatigue crack propagation behaviours in the notched specimens of SK 5 and SUS 50 steels, in which the crack is considered to propagate without the effect of the grain boundary conditions, the following results are obtained.

(1) Adopting the idea of \( E_0 \) which was often used for the explanation of the relation among the fatigue limits for the crack initiation in different kinds of notched specimens, the relations among the S-N curves at a specific crack length in various kinds of notched specimens are estimated. In this report, the values of \( E_0 \) for SK 5 and SUS 50 steels are obtained as 0.05 and 0.05mm, respectively.

(2) Irrespective of the difference in notch root radius, the same surface crack propagation rate is obtained when the number of cycles which is necessary to create the same surface crack length is the same. Namely, the crack propagation along the surface is not affected by the difference in notch shape. Neither is it affected by the difference in material so far as the results of this experiment are concerned.

(3) Although the crack propagations along the surface have the same behaviour irrespective of notch root radius as shown in (2), the crack propagations in the inner direction depend upon the notch shape (stress distribution) and the ratio of the crack depth in the inner direction and the crack length along the surface becomes larger with an increase in notch root radius.

(4) The value of index "\( n \)" in \( d\lambda/dN=CK^n \) and \( d\lambda/dN=CG^m \) is considered to relate with the gradient of the S-N curves and varies with the stress amplitude or the crack propagation rate from about 4 to 5.

6. Acknowledgement

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References