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PLASTIC DEFORMATION OF MILD STEEL UNDER COMBINED LOAD OF AXIAL FORCE AND TORSION WITH STRAIN TRAJECTORIES OF CONSTANT CURVATURE*

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A plastic deformation whose strain trajectory has constant curvature is investigated on mild steel by applying the combined load of axial force and torsion to thin-walled tubular specimen. The experiment is performed by using a full-automatic complex loading testing machine.

Experimental result shows that the relation between the magnitude of stress vector and the arc length of plastic strain trajectory depends mainly on the curvature, and the curve showing such a relation appears higher than the hardening curve for pure torsion. On the other hand, cyclic loading makes the curve lower than the hardening curve for pure torsion owing to the effect of cyclic softening.

This may be an experimental evidence that the conventional flow theory is not satisfactory for complex loading. However, the flow theory may be found as a sufficient approximation to plastic deformation under complex loading with strain trajectory of constant radius of curvature larger than 2 percent.

1. INTRODUCTION

The coaxiality of stress tensor σ_{ij} and plastic strain increment tensor $d\epsilon_{ij}^p$ or the so-called plastic potential has been assumed in the conventional plastic flow theory. However, the assumption of coaxiality of the stress and plastic strain increment tensors restricts a priori the generality of the relation between these tensors⁽¹⁾⁻⁽⁶⁾. Moreover, as the plastic deformation is one of the irreversible processes and thus the deformation state should depend on the preceding strain history besides the stress state at an instant considered, the concept of potential cannot be applied in its original meaning. Thus the plastic potential may be applied in the above mentioned non-holonomic relations only as an approximation in which the history of preceding processes is considered as a parameter, not as a variable. Though a number of experiments have been performed to investigate variations of plastic potential due to deformation history on an assumption of its existence, it cannot be said that a general rule for extensive configurations of plastic deformation may be formulated precisely according to such a conception.

On the other hand, as the stress and strain as well as their derived amount for expressing plastic deformation may be

regarded as symmetric tensors of the rank two in the three dimensional spaces, the general rule of plastic deformation should be formulated as tensor equations according to the general character of these spaces from extensive experimental observations. Ilyushin⁽⁷⁾ is one of the investigators who proposed such a conception, and he obtained a stress-strain relation of plastic deformation under complex loading in a form of vector equation, which depends on an arc length and geometry of strain trajectory in a five dimensional vector space of strain deviator. However, as part of generality in the original tensor space had been lost in his vector equation because of the use of vector space⁽⁷⁾, Novozhilov⁽⁸⁾ pointed out that and proposed his tensor equation which could express the plastic stress-strain relation under complex loading rationally⁽⁹⁾. After that, Ilyushin also has extended his vector equation to a tensorial one⁽¹⁰⁾. According to these equations, the conventional plastic flow theory, which has been regarded as a sufficiently general one for complex loading, is found to be an approximation only in the simplest case of strain trajectory with small curvature⁽¹¹⁾. However, the above-mentioned tensorial theories still remain within a rational principle of formulating the experimental results, and the experiments performed by Lensky⁽⁴⁾ are rather qualitative ones for verifying the propriety of these theories.

The above-mentioned historical background is related closely with the development of analytic technique. Before the dissemination of numerical calculation by electronic computer, the main purpose of formulation was to find linear relations which might be managed easily by analytic method neglecting the secondary effects

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appearing in real plastic deformations. Moreover, the experimental apparatus was used not always to realize accurately an assigned configuration of deformation state or its history. However, the recent development of numerical calculation together with electronic computer has removed the difficulty to obtain the solution even in a detailed deformation analysis according to the formulations reproducing precisely the real deformation phenomena. On the other hand, the recent development of automatic testing machine enable us to perform precise experiments of plastic deformation under complex loading with sufficient accuracy so that the secondary effects in plastic deformation may be formulated precisely.

In the present paper, in such circumstances, a plastic deformation of a thin-walled tubular specimen of mild steel under combined loadings of axial force and torsion having strain trajectories with constant curvature is investigated as one of the fundamental experiments to establish a general plastic rule extensively applicable. The stress-strain relation may be determined rationally by an analogous method as in the previous paper⁽⁶⁾, in formulating the magnitude and direction of stress vector at each point on the strain trajectory as function of arc length and curvature of the trajectory. However, as our main purpose is to investigate the property of deformation of this kind, relations between variables concerning deformation are shown in the form of diagram. The experimental result of the plastic deformation having strain trajectory with constant curvature, which is a special example of general strain trajectories whose parameters vary with its arc length, may give a guide to formulation of the stress-strain relation considering the effect of deformation history.

Moreover, as cycles for each stress component may be repeated in a possible extent of cumulative plastic strain for a small radius of curvature, experimental results of this kind are compared with the corresponding ones of the experiment under alternating torsion. On the other hand, as the conventional flow theory may be a good approximation to express the deformation with strain trajectory of small curvature, a maximum curvature of strain trajectory for which the flow theory may be applicable is examined as an applicable range of the theory.

As shown in the previous paper⁽⁵⁾, relations between the magnitudes of stress and strain vectors for torsion and axial force do not always agree with each other, because of the effect of the third invariant of stress deviator which is not isotropic in the corresponding vector space. Thus a modification proposed in the previous paper⁽⁵⁾ is used to discuss only the effect of deformation history on eliminating the effect of the third invariant.

The experimental results are discussed on the plastic strain by removing

the elastic one which is in a reversible process.

2. PLANE OF PLASTIC STRAIN DEVIATOR

As combined loads of axial force and torsion are applied to a thin-walled tubular specimen, two dimensional vector space has only to be considered for each deviatoric tensor. Vector of strain deviator (called the strain vector hereafter) or that of stress deviator (called the stress vector hereafter) in each space is expressed as

$$\epsilon = \epsilon_1 \mathbf{e}_1 + (\gamma_1 / \sqrt{3}) \mathbf{e}_2 \quad \text{or} \quad \sigma = \sigma_1 \mathbf{e}_1 + \sqrt{3} \tau_1 \mathbf{e}_2$$

where ϵ_1 , γ_1 , σ_1 and τ_1 denote the axial and shear strains, the axial and shear stresses, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors in the directions of ϵ_1 - and $\gamma_1/\sqrt{3}$ -axis, respectively. Magnitude of stress vector $|\sigma| = \sqrt{\sigma_1^2 + 3\tau_1^2}$ is equal to the effective stress, and that of strain vector $|\epsilon| = \sqrt{\epsilon_1^2 + \gamma_1^2/3}$ is equal to the effective strain. An arc length of strain trajectory is described as $S = \int_0^t |d\epsilon/dt| dt$ and is reduced to the magnitude of strain vector when the strain trajectory is a straight line starting at the origin.

In what follows, since the deformation behaviour is discussed in some cases for an increasing amount of plastic strain under cyclic loading, a plastic strain vector and a plastic strain increment vector are introduced, describing them as

$$\epsilon^p = \epsilon_1^p \mathbf{e}_1 + (\gamma_1^p / \sqrt{3}) \mathbf{e}_2$$

and

$$d\epsilon^p = d\epsilon_1^p \mathbf{e}_1 + (d\gamma_1^p / \sqrt{3}) \mathbf{e}_2$$

where

$$\epsilon_1^p = \epsilon_1 - \sigma_1 / E,$$

$$\gamma_1^p / \sqrt{3} = \gamma_1 / \sqrt{3} - \sqrt{3} \tau_1 / 3G$$

and E : Young's modulus, G : modulus of rigidity, $d\epsilon_1^p$ and $d\gamma_1^p/\sqrt{3}$ denote their increments, respectively.

An arc length of plastic strain trajectory may be described as

$$S^p = \int_0^t \left| \frac{d\epsilon^p}{dt} \right| dt$$

The directions of the stress and plastic strain increment vectors are expressed as

$$\theta_\sigma = \tan^{-1}(\sqrt{3} \tau_1 / \sigma_1)$$

and

$$\theta_{\epsilon^p} = \tan^{-1}(d\gamma_1^p / \sqrt{3} d\epsilon_1^p)$$

The parameters introduced above are shown in Fig.1, on the plane of plastic strain deviator as well as the local plane of stress deviator.

3. EXPERIMENTAL APPARATUS AND SPECIMEN⁽⁵⁾

Experiment is performed by using Autograph of IS-10-TS type, which is an automatic complex loading testing machine made by Shimazu Factory. The testing

machine enable us to give arbitrary stress or strain trajectory and to measure the response of a specimen very precisely and automatically.

The specimen is a thin-walled tube of mild steel S10C made of a bar of 43 mm diameter annealed according to JIS standard (cooled in furnace after kept at 880°C for an hour). Geometry and chemical composition of the specimen are shown in Fig.2 and Table 1. Central part of the specimen with gauge length 50 mm, outer diameter 21 ± 0.01 mm, inner diameter 19 ± 0.01 mm and thickness 1 mm, is finished carefully to obtain smooth surface. A measuring device for deformation is fitted to the small knife-edges as well as the V-shaped notches on them at both ends of gauge length. As in the previous paper⁽⁵⁾, grain sizes of these specimens are limited within $(2.5 \sim 3.5) \times 10^{-4}$ mm² of mean grain area. This assumes reproducibility of experimental results with sufficient accuracy.

Fig.1 Plane of plastic strain deviator

Table 1 Chemical composition of mild steel

Constituent	C	Si	Mn	S	P	Fe
Content (%)	0.10	0.24	0.44	0.009	0.018	99.193

Fig.2 Geometry of specimen
(dimension in mm)

4. EXPERIMENTAL PROCEDURE

4.1 STRAIN TRAJECTORY WITH CONSTANT CURVATURE

Strain trajectories with constant curvature in the plane of strain deviator ($\epsilon_1, \gamma/\sqrt{3}$) are considered as shown in Fig. 3 by combined loading of axial force and torque. An increasing rate of arc length of the trajectory is selected as $ds/dt = 3 \times 10^{-6} \text{ sec}^{-1}$. Experiment is conducted for an extent of plastic strain as for as $S^p = 6$ percent. Radius of curvature for each strain trajectory is shown in Table 2, where the symbol N or S denotes the experimental result whose strain trajectory starts with tension or torsion at the origin.

Though the experiments are performed with a fixed value of radius of curvature r in the plane of strain deviator, a radius of curvature r^p in the plane of plastic strain deviator is examined as well, because the experimental results are discussed using plastic strain. Figures 4 and 5 compare, for example, the configurations of the strain trajectory and the plastic strain trajectory for Experiment 1N and 0.09N. As found from Fig.4, the magnitude of r^p is almost the same as that of r for a fairly large value of radius of curvature, and thus the curvature of plastic strain trajectory may be said to be kept almost constant. On the other hand, for a very small radius of curvature, r^p decreases first with an increase of the arc length of plastic strain trajectory. Decrease of radius r^p is fairly remarkable first but becomes negligibly small afterwards, and a mean value of r^p has been determined excluding the first half-round, in which strain state is in a yield-range and unstable. Thus the first half-round of trajectory where the curvature r^p remarkably varies is not taken into account in

Fig.3 Strain trajectories with constant curvature in the plane of strain deviator

Table 2 Radius of curvature for each strain trajectory

Experiments	4n, 4S	2n, 2S	1N, 1S	0.47N, 0.47S	0.21N, 0.21S	0.09N
Radius of curvature (total strain) r %	4	2	1	0.5	0.25	0.14
Radius of curvature (plastic strain) r^p %	4	2	1	0.47	0.21	0.09

the discussion of deformation behaviour.

4.2 ALTERNATING TORSION WITH CONSTANT STRAIN AMPLITUDE

Experiment is conducted further under alternating torsion with constant strain amplitude $\gamma_s/\sqrt{3}$ with the previous strain rate $ds/dt = 3 \times 10^{-6} \text{ sec}^{-1}$. Strain amplitude a was selected as shown in Table 3, where the plastic strain amplitude a^p obtained by removing the elastic part is estimated corresponding to each value of a . The value of a^p decreases with strain cycles due to a hardening of the material, but as the decrease is sufficiently small, a mean value of a^p is estimated by averaging the value of a^p in every strain cycle.

5. EXPERIMENTAL RESULTS AND DISCUSSIONS

5.1 STRAIN TRAJECTORY WITH CONSTANT CURVATURE

5.1.1 MAGNITUDE OF STRESS VECTOR

Relations between the magnitude of stress vector $|\sigma|$ and arc length of plastic strain trajectory S^p for every value of r^p obtained from the experiment are described in Figs. 6(a) through (c), where the solid curves are obtained by connecting the corresponding symbols expressing

Fig.4 Plastic strain trajectory (1N)

Fig.5 Plastic strain trajectory (0.09N)

Table 3 Strain amplitude

Strain amplitude	a %	0.25	0.38	0.52	0.75	1.00
Plastic strain amplitude	a^p %	0.18	0.30	0.43	0.66	0.92

the experimental results, and the single and double dotted chain curves show the experimental results for pure torsion and uniaxial tension performed by using specimens of the same group. Though the magnitude of $|\sigma|$ increases with S^p for every case, the increase is not monotonic but rather wavy and the amount of increase is fairly different with respect to the magnitude of r^p .

In what follows, these trends will be discussed in detail. As the wavy increase of $|\sigma|$ is strongly related with the periodic variation of stress components, variation modes of $|\sigma|$, σ_s and $\sqrt{3}\tau_s$ are compared with each other with respect to S^p as shown in Figs. 7 through 10, for Experiments 2N, 0.47N, 0.47S and 0.09N. In these figures, the thick solid curves shows the relation between $|\sigma|$ and S^p , and the fine solid and dashed curves correspond to the variation of stress components σ_s and $\sqrt{3}\tau_s$ with an increase of S^p . The variations of stress components are shown with their

(a) $r^p = 4, 2\%$

(b) $r^p = 1, 0.47\%$

(c) $r^p = 0.21, 0.09\%$

Fig.6 Relation between $|\sigma|$ and S^p

absolute values for convenience of illustration as well as economy of space, and the symbol (+) or (-) distinguishes plus or minus side of these curves (plus side is taken on the tension for σ_z , or in the initial direction of torsion for $\sqrt{3}\tau_z$). The single and double dotted chain curves show the experimental results for pure torsion and uniaxial tension, respectively.

As mentioned in the previous paper⁽⁵⁾, an effect of the third invariant appears on the stress component σ_z . Thus, in order to obtain the magnitude of stress vector $|\sigma'|$ in an isotropic vector space as combined with the component $\sqrt{3}\tau_z$, the effect of third invariant on the component σ_z is eliminated by using a modification proposed in the previous paper⁽⁵⁾. The relation between $|\sigma'|$ thus obtained and S^p is described with a thick dashed curve in each figure.

As the strain trajectories for the experiments of N-group (Figs. 7, 8 and 10, where elastic strain does not appear because S^p is taken in the abscissa) start with tension, materials yield first in a state $\sigma_z \gg \sqrt{3}\tau_z$. Afterwards, each of the stress components varies as sine wave with an increasing amplitude with an increase of S^p . Since the component $\sqrt{3}\tau_z$ increases with a decrease of σ_z and vice versa, each of stress components is repeated with a phase difference 90° with an increase of S^p . The same trend appears also in the experiments of S-group, with only an interchange in sequence of stress components σ_z and $\sqrt{3}\tau_z$. As found from these figures, the $|\sigma'| - S^p$ curve appears as a wavy one whose hump or hollow corresponds with the crest of σ_z or $\sqrt{3}\tau_z$ -curve owing to the effect of third invariant, which consists in that the stress component σ_z appears higher than the component $\sqrt{3}\tau_z$ for the same plastic strain intensity. Actually, most of such a wavy trend has been removed on the dashed curve expressing the relation between $|\sigma'|$ and S^p .

In order to compare the experimental results in N- and S- groups, $|\sigma'| - S^p$ curves and $|\sigma'| - S^p$ curves for $r = 2$ and 0.47 percent of both groups are shown in Figs. 11 and 12. In these figures, the fine and thick solid curves correspond to the $|\sigma'| - S^p$ and $|\sigma'| - S^p$ curves in N-group, and the fine and thick dashed curves correspond to those in S-group. The chain curve is a mean curve of the $|\sigma'| - S^p$ curves in both groups. In the range of yield strain for pure torsion, the $|\sigma'| - S^p$ curves for both groups slightly deviate from the yield-step. Thus, the yield-step of the chain

Fig. 7 Variation modes of stress components (2N)

Fig. 8 Variation modes of stress components (0.47N)

Fig. 9 Variation modes of stress components (0.47S)

Fig. 10 Variation modes of stress components (0.09N)

curve is approximated by that of $|\sigma|$ - S^p curve for pure torsion because the latter is not affected by the third invariant. An error due to this approximation is within ± 3 percent. The maximum yield strain is approximated by a strain at an intersection of the yield-step and the above-mentioned averaged hardening curve.

As found from these figures, the $|\sigma|$ - S^p curves in both groups show a similar trend on the whole, only with an interchange of hump and hollow in the wavy curves.

The $|\sigma|$ - S^p curves in both groups almost coincide with each other as the result of the wavy configuration being removed, and thus may be approximated by a chain curve with sufficient accuracy. This approximation may be applied for the other values of r^p .

In such a way, for a strain trajectory with constant curvature, the relation between the magnitude of stress vector and the arc length of strain trajectory in a relation of mirror transformation, as those corresponding to the experiments of N- and S-groups, is almost the same if the effect of third invariant is eliminated.

In order to examine the effect of curvature of strain trajectory on the relation between $|\sigma|$ and S^p , averaged curves of the $|\sigma|$ - S^p curves of both groups herein obtained are collected in Fig.13. In the figure, the single and double dotted chain curves correspond to the experimental results of pure torsion and uniaxial tension, respectively. As the effect of third invariant has been eliminated, the discrepancy between the hardening curve for pure torsion and the $|\sigma|$ - S^p curve for strain trajectory with constant curvature may be regarded only as an effect of curvature on the plastic behaviour of the material.

As found from the figure, the curves for $r^p = 2$ and 4 percent coincide with each other and appear slightly higher than the hardening curve for pure torsion. The coincidence shows that the effect of curvature is saturated for $r^p \geq 2$ percent. The curve for $r^p = 1$ percent agrees with the above-mentioned curves for small values of S^p , but becomes lower than them for $S^p \geq 3.3$ percent and even lower than the hardening curve for pure torsion for large values of S^p . Every curve for $r^p = 0.47, 0.21$ and 0.09 percent appears lower than the hardening curve for pure torsion, and this trend is remarkable for small values of radius of curvature r^p .

Such a trend may be illustrated as follows. When these curves for strain trajectories of small curvature $r^p \geq 2$ percent attain $S^p = 6$ percent, strain state varies along the corresponding strain trajectory as far as quarter- or half-round, and this proc-

ess corresponds to a mild change of strain state from tension to torsion or vice versa. On the other hand, as mentioned in the previous paper⁽⁵⁾ the hardening of material depends on the change of deformation state. Thus the rise of $|\sigma|$ - S^p curves for $r^p \geq 2$ percent from the hardening curve for pure torsion may be attributed to the mild change of deformation state. When the $|\sigma|$ - S^p curves for strain trajectories with large curvature ($r^p = 0.47, 0.21$ and 0.09 percent) attain $S^p = 6$ percent, strain state follows each strain trajectory more than 2, 4 or 7 times, and the process corresponds to a cyclic variation of each stress component Figs.8, 9 and 10. On the other hand, it has been well-known for uniaxial state that the

Fig.11 Comparison of experimental results in N- and S-groups ($r = 2\%$)

Fig.12 Comparison of experimental results in N- and S-groups ($r = 0.47\%$)

Fig.13 Effect of curvature of strain trajectory on relation between $|\sigma|$ and S^p

hardening of materials is remarkably mitigated for cyclic deformation (cyclic softening effect), and this is ascribed to the Bauschinger effect. According to the experimental results for alternating torsion of constant strain amplitude, mentioned in the next section, a hardening curve for alternating torsion appears lower than the hardening curve for pure torsion, and the trend is remarkable for small amplitude. Such a trend may be expected for each component deformation in the combined loading, and may be applied to explain the above-mentioned trend; the hardening curve for strain trajectory with large curvature appears lower than that for pure torsion. The hardening curve for $r^p = 1$ percent corresponds to a transition between the above-mentioned both cases, and it agrees first with the hardening curve for $r^p \geq 2$ percent owing to the hardening due to the change of deformation state, but it gradually becomes lower afterwards on account of cyclic softening effect than the hardening curve for pure torsion.

5.1.2 DIRECTION OF STRESS VECTOR

If a difference between the direction of stress vector θ_s and that of plastic strain increment vector $\theta_{s'}$ is denoted with $\Delta\theta = \pm(\theta_{s'} - \theta_s)$, where + and - correspond to N- and S-groups, respectively, relations between $\Delta\theta$ and S^p in every experiment are obtained as shown in Fig.14. The value of $\Delta\theta$ is positive whenever the direction of stress vector is delayed from that of plastic strain increment vector in either group. In the figure, various symbols correspond to the experimental results, and the chain line shows a limit $\Delta\theta = 6^\circ$, within which the directions of stress and plastic strain increment vectors are regarded to coincide in the pre-

Fig.14 Direction of stress vector

Fig.15 Alternating torsion test for amplitude $a^p = 0.43\%$

vious paper⁽⁵⁾. As found from the figure, the experimental results of N- and S-groups show a similar trend and may be approximated sufficiently with the solid curves in the figure. This means that the relations between $\Delta\theta$ and S^p corresponding to the strain trajectories in a relation of mirror transformation appear in the same manner, analogous to the relations between $|\sigma'|$ and S^p mentioned in the previous section.

As every value of $\Delta\theta$ appears as positive one, the direction of stress vector is always delayed from that of plastic strain increment vector. The value of $\Delta\theta$ increases first with an increase of S^p and is saturated to a constant value for each value of curvature, and thus the direction of stress vector is delayed from that of plastic strain increment vector by a constant angle in the plastic deformation of this kind after saturation. The angle of delay after saturation becomes remarkable for large curvature, and attains about $\Delta\theta = 30^\circ$ for $r^p = 0.09$ percent. On the other hand, the value of $\Delta\theta$ for small curvature ($r^p \geq 2$ percent) is less than 6° , and thus the directions of stress and plastic strain increment vectors may be regarded as almost coincident.

5.2 ALTERNATING TORSION WITH CONSTANT STRAIN AMPLITUDE

Figure 15 shows an example of the results obtained for a plastic strain amplitude $a^p = 0.43$ percent, where absolute values of stress $\sqrt{3}\tau_z$ and S^p are taken on ordinate and on abscissa, respectively. In the figure, the small solid circles correspond to the experimental results, and the symbols (+) and (-) show the plus and minus sides of stress cycle. The chain curve shows the result for pure torsion for comparison.

In order to compare the hardening under alternating torsion for various strain amplitudes, curves connecting the stress value at the beginning of unloading for each cycle are collected in Fig.16 for every plastic strain amplitude. In the figure, the chain curve shows the result for pure torsion, and the dashed curves show the results for $r^p = 0.21$ and 0.47

Fig.16 Effect of strain amplitude

percent reproduced from Fig.13. As found from the figure, the curve for alternating torsion for every plastic strain amplitude appears lower than the curve for pure torsion, and the trend becomes remarkable for small values of amplitude. Especially, the hardening scarcely appears for $\alpha^p = 0.18$ percent. In the previous section, the trend was used to explain the fact that the hardening curves become lower for large curvatures.

As found from the figure, though the above-mentioned trend appears in cyclic deformation under combined loading, a remarkable quantitative difference is recognized between them if the experimental results in both cases whose plastic strains per cycle are of almost the same amount ($\alpha^p : r^p = \pi/2 : 1$), for example, $\alpha^p = 0.30$ and $r^p = 0.21$ percent, $\alpha^p = 0.66$ and $r^p = 0.47$ percent, are compared with each other. It is clear that the cyclic softening effect appears more strongly in the alternating torsion. This may be attributed to the cyclic softening effect appearing in a specific slip-plane where plastic loading and unloading are repeated for the alternating torsion, whereas the effect appears in sequence in each of different slip-planes under the complex cyclic loading.

6. CONCLUSIONS

A plastic deformation whose strain trajectory in a plane of deviatoric strain had constant curvature was investigated on mild steel by applying the combined load of axial force and torsion to thin-walled tubular specimens. The experiment was performed using a full-automatic complex loading testing machine of IS-10-TS type, and every measurement was made with sufficient accuracy to obtain reliable results. Main conclusions obtained from the experiment may be summarized as follows.

i) Relations between the magnitude of stress vector and the arc length of plastic strain trajectory for the strain trajectory with constant curvature in the relation of mirror transformation coincide with each other if the effect of third invariant of stress deviator is eliminated from the experimental result. This trend has been ascertained in the previous paper for strain trajectory consisting of two straight branches.⁽⁵⁾

ii) The above-mentioned relation depends mainly on the curvature r^p of plastic strain trajectory. Hardening curves showing the relation for $r^p \geq 2$ percent appear higher than the hardening curve for pure torsion owing to the effect of curvature of strain trajectory. On the other hand, those for $r^p = 0.47, 0.21$ and 0.09 percent appear lower than the hardening curve for pure torsion owing to the remarkable effect of cyclic softening. The hardening curve for $r^p = 1$ percent is subjected to both effects mentioned above as an intermediate example.

This is an experimental evidence that

the assumption used in the conventional plastic flow theory; i.e., hardening of materials is measured as a function of arc length of plastic strain trajectory or of an integration of the increment of plastic work irrespective of the configuration of strain trajectory, is not satisfactory for complex loading.

iii) The direction of stress vector is always delayed from that of plastic strain increment vector. The angle of delay increases first and is saturated gradually to a constant value with an increase of the arc length of plastic strain trajectory. The fixed angle of delay after saturation is large for large curvature. This is another experimental evidence that the assumption used in the conventional plastic theory, especially the Prandtl-Reuss relation; i.e., plastic strain increment vector is oriented in the normal direction on the yield surface or coaxial with the stress vector, is not satisfactory for complex loading.

iv) However, for the plastic deformation for $r^p \geq 2$ percent, the curves showing the relation between the magnitude of stress vector and the arc length of plastic strain trajectory are independent of r^p . Moreover, the angle between the directions of the stress and plastic strain increment vectors in the above-mentioned deformation is sufficiently small. Therefore, if the little difference between the curves for $r^p \geq 2$ percent and the curve for pure torsion as well as the small angle between both vectors for $r^p \geq 2$ percent is neglected, the conventional plastic flow theory may be a sufficient approximation to plastic deformations under complex loading with strain trajectory of constant curvature $r^p \geq 2$ percent.

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