622.612 : 532.5

A Study of a Flame in the Boundary Layer on a Porous Flat Plate*

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A two-dimensional flame in a boundary layer with uniform injection of propane gas from a porous flat plate parallel to a uniform air stream is treated in this study. The positions of the beginning of the flame in both directions parallel and normal to the plate are considered to be one of the most important factors for the stabilization of such a flame.

In order to justify the experimental results of the positions, an analysis of both the mass balance at the flame sheet and the mass balance at the surface of the plate is briefly carried out by local similarity method and an experimental equation of an air stream-velocity. The influences of injection rate, mass flux of an air stream and the surface temperature of the plate on the positions are discussed qualitatively.

1. Introduction

A two-dimensional flame in a boundary layer with uniform injection of propane gas from a porous flat plate parallel to a uniform air stream is treated in this study. The positions of the beginning of the flame in both directions parallel and normal to the plate are considered to be one of the most important factors for the stabilization or the combustion mechanism of such a flame.

First, the influences of injection rate, mass flux of an air stream and the surface temperature of the plate on the positions have been investigated in the experiment of this study, and then the experimental results concerning the positions were discussed by a simple analysis as follows. In order to make it as brief as possible, the analysis carried out by use of the local similarity method in heat and mass transfer, and an experimental equation instead of the momentum equation concerning an air stream on the plate.

2. Symbols

b : Variable defined in Eq.(19).
 $c_f/2$: Friction coefficient at wall.
 D_{im} : Diffusion coefficient of the i -th species into the mixture.
 h : Enthalpy, $dh=c_p dT$.
 h_o^* : Enthalpy defined in Eq.(24).
 P_r : Prandtl number.
 q_o : Heat flux at the surface of a porous plate.

Q_p : Constant pressure heating value.
 Q_v : Volumetric rate of energy production.
 r : Stoichiometric constant of proportionality for fuel.
 S_c : Schmidt number.
 T_o : Surface temperature of the plate.
 u : Local down stream velocity.
 U_e : Free stream velocity.
 u_+ : Local down stream velocity at flame surface.
 v : Local normal velocity.
 W_i : Volumetric rate of production of i -th species.
 x : Down stream distance from leading edge of the plate.
 x_+ : Flameholding distance from the leading edge.
 y : Normal distance.
 y_+ : Flameholding distance from the surface of the plate.
 Y_i : Mass concentration of i -th species.
 Y_{f-} : Mass concentration of fuel at entrance of porous wall.
 $\alpha_1 \sim \alpha_6$: Constant used in Eqs.(7),(8),(20).
 β : Blowing ratio, $\rho_o v_o / \rho_e U_e$.
 μ : Absolute viscosity.
 ρ : Density.
 $\rho_e U_e$: Mass flux of free stream.
 $\rho_e^* U_e$: Flow rate of free stream per area per unit time.
 $\rho_o v_o$: Mass flux of fuel at the surface of the plate.
 δ : Boundary layer thickness.
 τ_o : Shear stress at porous wall.
 ϕ : Equivalence ratio of fuel to oxygen at flame surface, r/r_{stoh} .
 ϕ_A : Mass concentration of fuel to fuel-air mixture.
 ϕ_N : Mass concentration of fuel to fuel-nitrogen mixture.

* Received 26th February, 1973 and 14th October, 1971.

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Subscripts

- e : Conditions in free stream.
 o : Conditions at porous wall surface.
 f : Fuel species.
 + : Conditions at flame sheet.
 - : Conditions inside of porous wall.
 ϕ : Oxygen species.

3. Experimental apparatus and experimental procedure

3-1. Experimental apparatus

Flow chart of the apparatus-system used in this experiments is shown in Fig.1.

1. Straightener 2. Test section
 3. Turbo-blower 4. Burner 5. Gas bomb for fuel 6. Gas bomb for nitrogen
 7. Dryer 8. Compressor 9. Reservoir for chilled water

Fig.1. Flow sheet of apparatus-system

The central piece of apparatus was set on the suction side of the turbo-blower. Downstream from the straightener, whose cross section was 530 by 580 mm, were three screens spaced at fifty centimeter intervals. A covering nozzle followed this section. The contraction area ratio was 10.3:1 from the section to the nozzle throat. Adjoining the section was the test section whose cross section was 250 by 300 mm.

Burner, whose cross sectional view is given in Fig.2, was placed in the space of the test section.

1. Chilled water 2. Fuel gas 3. Nitrogen gas 4. Supporting rods

Fig. 2 Burner details

The burner consisted of an approaching plate, a trailing plate, three porous plates with permeability of $10\mu\text{g}$, made of bronze particles bonded to one another and a 30 mm thick brass block. The approaching plate was employed for fairing the boundary layer on the plate. Meanwhile, two porous plates for nitrogen were employed for preventing the diffusion of oxygen in z-direction (ref. Fig.3). Throughout the combustion test, the injection rate of nitrogen from the porous plates was held identical with that of propane gas.

The temperature of the plate was held constant by circulation of a chilled water. Nine thermocouples were embedded in the porous plates. The locations of thermocouples are shown in Fig.3.

1. Porous plate for fuel.
 2. Two pieces of porous plates for nitrogen.

* The embedded location of thermocouple is represented by symbol x.

Fig. 3 Porous plates

In the preliminary measurements, no temperature profile of the plate in z-direction was found. Since the "real" surface temperature of the plate could not be measured, we tried to guess the temperature from the temperature profiles of the plate in y-direction. And, we found the variation of the guessed temperature to agree with that of the measured temperature in the plate. Therefore, the temperature measured in the plate might be well accepted as a typical temperature of the plate. The measured tempera-

ture of the plate described later in this paper means always the temperature measured by the thermocouple embedded at 13 mm downstream from leading edge of the plate and 1 mm below the plate surface.

3-2. Experimental procedure

Cold flow measurements of an air stream on the plate were made by a hot-wire anemometer with $5\mu\phi$ W-probe.

Combustion tests were made in three cases of mass injection: injection of fuel only, fuel-oxygen mixture and fuel-nitrogen mixture. The positions of the beginning of the flame on the plates, x_+ and y_+ were measured by a microscope. Meanwhile, the positions of the flame, x'_+ and y'_+ were measured by a thermocouple consisting of $50\mu\phi$ Pt/Pt-Rh wires.

4. Experimental results

4-1. Cold flow measurements

In cold flow measurements, several data were obtained under the conditions that the free-stream velocity (U_e) varied from 1 to 3 m/s and the blowing ratio varied from 0 to 0.038. Each of these experimental data was arranged in nondimensional profiles by plottings of u/U_e vs y/δ . From these plottings, we obtained the following results: for $\beta \leq 0.02$, all dimensionless velocity profiles are reduced to an identical profile, independently of U_e , v_o and x . Within the limits of our experiments, an experimental equation concerning the local downstream velocity as shown in Eqs.(1) and (2) was obtained.

$$u/U_e = 1 - \exp(-4.87 \cdot y/\delta) \quad (1)$$

$$\delta = (2.36\sqrt{x} + 14.76)/\sqrt{U_e} - 0.0547\sqrt{x} - 0.222 + (1.93/\sqrt{U_e} + 0.0276\sqrt{x} - 0.089)v_o \quad (2)$$

where, y, δ, x are in mm and U_e, v_o are in m/s.

4-2. The positions of the beginning of the flame

4-2-1. Flameholding distance from the surface of the plate, y_+

Fig.4 shows the experimental results of the relations between flameholding distance from the surface of the plate, y_+ and blowing ratio, β . The results shown in Fig.4 were obtained under conditions that the temperature of an air stream was 21.5°C and that only propane gas was injected. Fig.4 shows that normal flameholding distance, y_+ depends heavily on the temperature of the plate, T_o , independently of $\rho_e^* U_e$ and β . Under conditions that fuel-air mixture and fuel-nitrogen mixture are injected through the porous plate, we can obtain the same results as shown in Fig.4. Thus, the

following conclusion can be drawn: normal flameholding distance y_+ does not depend on the hydraulic factors such as $\rho_e^* U_e, \beta$ but on the factor of heat balance such as T_o .

Fig.4 Experimental results of y_+ at the beginning of the flame

4-2-2. Flameholding distance from the leading edge of the plate, x_+

The influences of the temperature of the plate, T_o on flameholding distance from the leading edge of the plate, x_+ were also investigated under conditions that fuel-air mixture and fuel-nitrogen mixture are injected through the plate.

Fig.5 Experimental results of x_+ and x'_+ ($Y_f=1$)

Fig.5 shows the plotting of x_+ vs x'_+ vs β in the case of $Y_f=1$. From Fig. 5, it is found that the flameholding distances measured by both microscope (x_+) and thermocouple (x'_+) decrease with an increase of blowing ratio, β at a value of $\rho_e^* U_e$ and the values of both x_+ and x'_+ increase with a decrease of $\rho_e^* U_e$ at a value of β .

Although the value of x_+ is different from the value of x'_+ , it might be well considered that the relation between x_+ and β is almost the same as the relation between x'_+ and β . Fig.6 shows mainly the relations between x_+ and T_o . From Fig.6, the influence of the temperature of the plate, T_o on the value of x_+ is recognized: the values of x_+ increase with a decrease of T_o .

Fig.6 Experimental results of x_+
(Influence of T_o on x_+)

Figs.7 and 8 show the plottings of x_+ or x'_+ vs β in the case of fuel-air mixture injection and fuel-nitrogen mixture injection, respectively. From these figures, it is found that the values of x_+ decrease with an increase of β at a value of $\rho_e U_e$.

5. Analysis

5-1. Basic equation

Fig.9 shows a combustion model which has been provided for the analysis by Emmons⁽¹⁾. Let us apply also the model to this analysis.

Fig.9 Combustion Model

As shown in Fig.9, the regions I and II correspond to the region between free stream (e) and flame sheet (+), and to the region between flame sheet (+) and porous surface (o), respectively. The following assumptions are incorporated in the equations which appear in this section: (1) There is a two-dimensional steady laminar flow with zero downstream pressure gradient. (2) All the physical simplifications of boundary layer theory are valid.

Fig.7 Experimental results of x_+ and x'_+ (Fuel-air Injection)

Fig.8 Experimental results of x_+ and x'_+ (Fuel-nitrogen Injection)

For example, $\partial u/\partial y \gg \partial v/\partial x$, $\partial u/\partial y \gg \partial v/\partial y$.

(3) Radiative energy transport is comparatively negligible. The equation governing the model will now be stated.

Global conservation of mass:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (3)$$

Conservation of momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (4)$$

Conservation of energy:

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\mu}{Pr} \left(\frac{\partial h}{\partial y} \right) \right] + Q_v \quad (5)$$

Conservation of the i -th species:

$$\rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\mu}{Sc_i} \left(\frac{\partial Y_i}{\partial y} \right) \right] + W_i \quad (6)$$

If $P_{r1}=1$, $Q_v=0$ and $S_{ci}=1$, $W_i=0$ are substituted into Eqs.(5) and (6), respectively, general similarity among velocity (u), enthalpy (h) and concentration profiles (Y_i) at any given location will be found. Then, the following equations hold for h and Y_i .

$$h = \alpha_1 u + \alpha_2 \quad (7)$$

$$Y_i = \alpha_3 u + \alpha_4 \quad (8)$$

where, $\alpha_1 \sim \alpha_4$ are constants.

5-2. Mass balance

The following assumptions are provided for consideration of the concentration profile with flame:

Assumption I: There is a flame which is two-dimensional and can be defined by the flame-sheet model.

Assumption II: The flame burns at such a position in the boundary layer that it gets the chemical proportions of fuel and oxygen by means of various transfer mechanisms which are acting. Furthermore, both the fuel and oxygen concentrations vanish at the combustion zone. Above assumptions are identical with those used in Eschenroeder's analysis⁽²⁾.

(a) Without flame (b) With flame

Fig.10 Mass concentration profile

From Eq.(8) and assumptions I and II, the concentration profiles with flame and without flame are obtained as shown in Fig. 10. In comparison of Fig.10 with Eq.(8), the concentration profiles of fuel and oxygen are obtained as follows:

Without flame:

$$Y_f = -(Y_{f0}/U_e) \times (u - U_e) \quad (9)$$

With flame:

in the region II

$$Y_f = -(Y_{f0}/u_+) \times (u - u_+) \quad (10)$$

in the region I

$$Y_\phi = \{Y_{\phi e}/(U_e - u_+)\} \times (u - u_+) \quad (11)$$

Here, reconsider the relation represented in Eq.(8). Putting $W_i=0$, Eq.(8) is obtained. By the restriction due to the substitution of W_i , Eq.(8) is not valid in the region bounded flame sheet. Eq.(8) is considered to be valid in all the regions including the region bounded flame sheet from the following facts: Flame sheet model in which the thickness of the flame is infinitesimal, is now treated and mass concentrations of both fuel and oxygen at the flame sheet are considered to be zero under the assumption II.

Here, consider the mass balance at flame sheet.

Coordinate axes of flame sheet are now defined as below:

l -coordinate----- along the flame sheet
 n -coordinate----- normal to the flame sheet

The stoichiometric balance at the flame sheet is expressed by equating the mass flux of fuel into the sheet to that of oxygen into it multiplied by a constant of chemical proportionality r . Each quantity in bracket is evaluated at the flame sheet approaching either from region I or region II as indicated by the second subscript on each bracket.

$$-[\rho D_{fm} \frac{\partial Y_f}{\partial n}]_{+II} = r [\rho D_{\phi m} \frac{\partial Y_\phi}{\partial n}]_{+I} \quad (12)$$

After the introduction of the linear dependence of species concentrations implied by Eqs.(10) and (11), the following equation can be obtained.

$$u_+ = Y_{f0}/(Y_{f0} + r Y_{\phi e}) \quad (13)$$

The two unknown concentrations at the wall surface, $Y_{\phi 0}, Y_{\phi e}$ can be obtained from species balance at the wall surface. After the introduction of β , $c_f/2$ and several rearrangements of above equations, the following equations can be obtained:

Fuel concentration without flame

$$Y_f/Y_{f0} = (1 - \bar{u})/(1 + c_f/2\beta) \quad (14)$$

Fuel concentration with flame:

$$Y_f/Y_{f0} = (\bar{u}_+ - u)/(\bar{u}_+ + c_f/2\beta) \quad (15)$$

By rewriting Eq.(11), the desired expression for the downstream velocity at the flame is obtained:

$$\bar{u}_+ = \frac{Y_{f0} - r Y_{\phi e} (c_f/2\beta)}{Y_{f0} + r Y_{\phi e}} \quad (16)$$

5-3. Heat balance

Since consumption of fuel and production of heat happen simultaneously, the

following equation can be obtained.

$$Q_p = -Q_p W_f \quad (17)$$

Adding together each side of Eqs.(5) and (6), and introducing the symbol b defined in Eq.(19), Eq.(18) is obtained.

$$\delta u \frac{\partial \delta}{\partial x} + \delta u \frac{\partial \delta}{\partial y} = \frac{\partial \delta}{\partial y} \left(u \frac{\partial \delta}{\partial y} \right) \quad (18)$$

$$\delta = b + Q_p Y_f \quad (19)$$

From the comparison of Eq.(18) with Eq.(2), the linear dependence of b on u can be obtained as below:

$$b = J_1 u + J_2 \quad (20)$$

where, both J_1 and J_2 are the constants which can be determined from the two boundary conditions in free stream and at porous wall surface.

Heat flux at the surface of the porous plate, q_0 can be represented as below:

$$q_0 = -k \left(\frac{\partial \delta}{\partial y} \right) \quad (21)$$

Heat flux represented as q_0 can also be considered to be equal to enthalpy increment of fuel on the porous wall. Then, we can write as below:

$$q_0 = \rho_e U_e (h_0 - h_-) \quad (22)$$

Equating Eq.(21) to Eq.(22) and rewriting by use of β , $c_f/2$, the following equation is obtained.

$$\frac{1}{\beta} \left(h_0 - h_- \right) = Q_p Y_f \left\{ \frac{1}{(1 + c_f/2\beta)} - \frac{1}{1 + c_f/2\beta u} \right\} \quad (23)$$

6. Results of calculation

6-1. Flame position

Here, the present analysis will be carried out by use of Eqs.(1) and (2), as described in Section 1. Substituting the flameholding distance (y_+) to y in the right hand side of Eq.(1), the local downstream velocity at flame surface (u_+) is formulated. Then, the relation between x_+ and y_+ can be obtained by equating u_+ obtained from Eq.(1) to Eq.(16). Since the value of y_+ is considered to be independent of ϕ and $\rho_e^* U_e$, as shown in Fig.4, the relation among x_+ , β and $\rho_e^* U_e$ will be able to be obtained by the substitution of a given value of y_+ in the above equation concerning x_+ and y_+ . Representative results of calculation are shown in Figs. 11 and 12. Since the value of y_+ at the beginning of the flame can be considered to be invariant, the results which were obtained in the case of the invariant value of y_+ and are shown in Fig.11(a), may be also considered to be comparable with those shown in Fig.5. After the comparison, it is found that the results obtained in Fig. 11(a) are similar to those in Fig.5. Fig. 11(b) shows that the value of x_+ increases with the value of y_+ at a value of β . Remembering the results shown in Fig.4, the following conclusion can be drawn: An increase in the temperature of the plate T leads to a decrease in the value of y_+ and then leads to a decrease in the value of x_+ .

Fig.12 shows the results in the case of nitrogen-mixture injection. The results shown in Fig.12 are also considered to be comparable with those in Fig.8. From the comparison, the following conclusion can be drawn: At a value of β , the value of x_+ decreases with an increase in fuel concentration to injectants, ϕ .

(a) Influence of $\rho_e^* U_e$ (b) Influence of y_+
Fig.11 Calculated example of x_+

Fig.12 Calculated example of x_+
(Fuel-nitrogen Injection)

6-2. The influence of the temperature of the plate on the positions of the flame

The influence of the temperature of the plate, T , on the flameholding distance x_+ has already been described in the above section. Here, we shall discuss the influence of T on the flameholding distance y_+ . In other words, the relation shown in Fig.4 will be explained through the equations obtained in Section 5.

First, the term h_0^* is defined as below:

$$h_0^* = (h_0 - h_-) / Q_p \quad (24)$$

In this study, the following facts may be supposed: The variation of h_- or h_e is

comparatively negligible. And, the variation of h_o corresponds to the variation of T_o because of enthalpy based on sensible heat. Thus, the variation of h_o^* corresponds to that of T_o . Substituting Eq.(23) in Eq.(24) and differentiating h_o^* with respect to y at $y = y_+$, we get

$$\left[\frac{\partial h_o^*}{\partial y} \right]_{y=y_+} = -Y_f \times \left[\frac{(C_f/2\beta) \times 1/(\bar{u}_+)^2}{(1 + (C_f/2\beta\bar{u}_+)^2)} \right] \times \left[\frac{\partial \bar{u}_+}{\partial y} \right]_{y=y_+} \quad (25)$$

Under given conditions, each quantity in right hand side of Eq.(25) is considered to be positive. Then, the following relation can be obtained:

$$\left[\frac{\partial h_o^*}{\partial y} \right]_{y=y_+} < 0 \quad (26)$$

From the above relation, an increase of h_o^* is considered to correspond to a decrease of y_+ . In other words, the above relation means that an increase of T_o corresponds to a decrease of y_+ and a decrease of T_o corresponds inversely to an increase of y_+ .

These facts agree well with the experimental results shown in Fig.4.

7. Conclusions

From the facts described above, we may conclude as follows: The positions of the beginning of the flame on the porous flat plate were measured experimentally.

The flameholding distance normal to the plate surface are found to depend mainly on the temperature of the plate. And, the flameholding distances parallel to the plate surface are found to depend on not only such hydrodynamic parameters as mass flux of free stream and blowing ratio but also the temperature of the plate. Meanwhile, the supporting evidences to the flameholding distances being parallel to the plate are obtained and the influences of the temperature of the plate on both flameholding distances also can be explained qualitatively by the simple analysis described here.

8. Acknowledgement

We are indebted to Takeshi Takei and Sachio Gotoh who contributed their experimental work. We are also deeply indebted to Chikahiro Kawai for his considerable assistance with the computer programmes.

9. Literature

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