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Influences of Collared Ribs and Core-Holes on the

Static Stiffness of Cylindrical Column*

—Study on the C.A.D for Machine Tool Structures, Part I—

by

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In the Computer Aided Design for a machine tool structure being developed, there are two important problems to be solved to compute the overall stiffness of a machine tool. One is the limitation of the core size of computer, which affects largely the computing accuracy. Another is how a machine tool structure is replaced by a suitable model available to the computation with good propriety. To make some countermeasures for these problems, one trial of CAD was carried out here with the cylindrical column having collared ribs and core-holes. From the results, it can be concluded that the cylindrical column in closed cross-section is well replaced by an one-dimensional beam-like element notwithstanding the existence of collared ribs and core-holes. This means that there may be some possibility of a new CAD technique to solve the present problems. In addition to this investigation, the effects of collared ribs and the influences of core-holes on the static stiffness of cylindrical column have been also described here.

1. Introduction

Machine tool structures are ordinarily very complicated and their appearances are quite different from one another, since machine tools have been developed and improved to fit many machining processes. The machine tools are, however, constructed fundamentally from scores of basic structural elements, such as a column, base, bed, cross rail, top beam and so on. In other words, one machine tool is constructed from several basic structural elements and these elements are assembled one with another by joints, such as bolted joints and guideways.

In the process of a design, owing to the complexity of a structure, a structure has been designed as follows even at the best case. The stiffness of structure is calculated by using the simple structural model replacing an actual one, and then a lot of empirical knowledges obtained over the long time is also taken into consideration together with this calculated result.

This method being used in machine tool industries demands long design time and also much efforts of designers.

To make not only a quick but also a full response to the recent demand for the performance of machine tools, the design

method used so far could be improved and a new one, by which the detailed and exact performance of a machine tool to be designed can be known and also altered at the design stage, should be developed. One calculation method which is considered to be available as a procedure of the new design method for machine tool structure, is the matrix methods of structural analysis aided by digital computers.

On this calculation method called usually CAD, there have been made many studies and the computing procedure of the stiffness calculation for a structure is being developed and performed. For example, how the position and the dimension of basic structural elements should be modified to fulfill the specification of machine tool to be designed and how to calculate the overall static and dynamic stiffness, have been published elsewhere⁽¹⁾.

The CAD's developed so far for a machine tool structure, were frequently carried out with the direct stiffness method (DSM)⁽²⁾ or the finite element method (FEM)⁽³⁾. Both these methods, however, have the following serious problems to be solved. In the former method, the questioned point is the procedure in which the structure to be designed is replaced by the simple beam-like structure. In the latter method, the questioned point is that the size of computer would not be enough to compute the overall stiffness of a machine tool. In this method, however, the stiffness of a basic structural element, such as a stiffness of column itself, can be computed closely in full consideration of the effects of ribs, partition plates, core-holes and so on, which complicate a basic structural element itself.

These problems being in the present CAD

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techniques should be solved, as the CAD for a machine tool structure is one of design techniques to be used practically, and its computing accuracy is always expected to be high notwithstanding the complexity of structures. One countermeasure for these problems being considered may be the method in which the stiffness of basic structural elements is computed with FEM and then using this calculated result, the overall stiffness of a machine tool is computed with DSM.

To investigate the propriety of such the computing method mentioned above, here the cylindrical column has been taken as the concrete structure to be studied and the influence of collared ribs and core-holes on the static stiffness of column has been described. The reliability of the computation for the static stiffness of this kind of basic structural element has been also investigated and from all results known from this study, the propriety of the model in which the cylindrical column having collared ribs and core-holes is replaced by a one dimensional beam-like element, has been investigated finally.

2. Computation of the static stiffness with FEM

2.1. Cylindrical column having collared ribs

The usefulness of FEM has been certified by many researchers⁽⁴⁾ who made a stress analysis for an axi-symmetric solid having an arbitrary geometry and being subjected also to axi-symmetrically external loads. Wilson⁽⁵⁾ has developed this technique of FEM furthermore by applying it to the stress analysis of axi-symmetric solid under nonaxi-symmetrically external loads, such as bending and torsion. In the computing method presented by Wilson, the external loads and displacements at nodes are to be the expansion in Fourier series in θ -direction, and then the unknown coefficients of the displacement function corresponding with each term of the external load expanded are determined by using the orthogonality properties of the harmonic functions. This means that in Wilson's method three-dimensional stress analysis in r , θ and z co-ordinates can be divided into a series of uncoupled two-dimensional stress analysis in r and z co-ordinates.

Fig.1 Dividing the column into triangular ring elements

This Wilson's technique is well available to the stiffness computation of a cylindrical column; therefore, the outlines and the application of this technique are explained briefly in the following.

As shown in Fig.1, at first the cylindrical column placed in the cylindrical co-ordinates is divided into triangular ring elements centered around z -axis, and then the nodal force P and displacement u are expanded in Fourier series in θ -direction.

$$P_r = \sum P_{rn}(r, z) \cos n\theta, \quad P_z = \sum P_{zn}(r, z) \cos n\theta,$$

$$P_\theta = \sum P_{\theta n}(r, z) \sin n\theta \quad \dots (1)$$

$$u_r = \sum u_{rn}(r, z) \cos n\theta, \quad u_z = \sum u_{zn}(r, z) \cos n\theta,$$

$$u_\theta = \sum u_{\theta n}(r, z) \sin n\theta \quad \dots (2)$$

Writing the displacement in the element with the equation of the first degree,

$$u_r = (b_{1n} + b_{2n} + b_{3n}z) \cos n\theta, \quad u_z = \sum (b_{4n} + b_{5n}r + b_{6n}z) \cos n\theta, \quad u_\theta = \sum (b_{7n} + b_{8n}r + b_{9n}z) \sin n\theta \quad \dots (2.a)$$

Meanwhile, the correlation between displacements u and strains ϵ in the cylindrical co-ordinates can be written as

$$\begin{aligned} \epsilon_{rr} &= \partial u_r / \partial r, \quad \epsilon_{\theta\theta} = 1/r (\partial u_\theta / \partial \theta) + u_r / r, \\ \epsilon_{zz} &= \partial u_z / \partial z, \quad \epsilon_{rz} = \partial u_r / \partial z + \partial u_z / \partial r \\ \epsilon_{r\theta} &= 1/r (\partial u_r / \partial \theta) + \partial u_\theta / \partial r - u_\theta / r, \\ \epsilon_{z\theta} &= \partial u_\theta / \partial z + 1/r (\partial u_z / \partial \theta) \quad \dots (3) \end{aligned}$$

By substituting Eq.(2-a) to Eq.(3), the relationships between strains $[\epsilon_n]$ and unknown coefficients $[b_n]$ yield to

$$[\epsilon_n] = [g_n][b_n] \quad \dots (4)$$

Namely

$$\begin{bmatrix} \epsilon_{rrn} \\ \epsilon_{zzn} \\ \epsilon_{\theta\theta n} \\ \epsilon_{rzn} \\ \epsilon_{r\theta n} \\ \epsilon_{z\theta n} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1/r & 1 & z/r & 0 & 0 & 0 & n/r & n & nz/r \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -n/r & -n & -nz/r & 0 & 0 & 0 & -1/r & 0 & -z/r \\ 0 & 0 & 0 & -n/z & -n & -nz/r & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_{1n} \\ b_{2n} \\ b_{3n} \\ b_{4n} \\ b_{5n} \\ b_{6n} \\ b_{7n} \\ b_{8n} \\ b_{9n} \end{bmatrix} \quad \dots (4.a)$$

Fig.2 Triangular element and the co-ordinate of its nodes

By substituting the co-ordinates of each node shown in Fig.2 to the terms r and z in Eq.(2-a), and also by exchanging the nodal displacement $[u_n]$ for the unknown coefficient $[b_n]$ with each other, the following equation can be obtained,

$$[b_n] = [h_n][u_n] \quad \dots (5)$$

where

$$[h_n] = \frac{1}{\lambda} \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}$$

Furthermore, $[H]$ is given by

Namely

$$\begin{bmatrix} \sigma_{rrn} \\ \sigma_{zzn} \\ \sigma_{\theta\theta n} \\ \sigma_{rzn} \\ \sigma_{r\theta n} \\ \sigma_{z\theta n} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & & & \\ & 1 & \nu/(1-\nu) & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{rrn} \\ \epsilon_{zzn} \\ \epsilon_{\theta\theta n} \\ \epsilon_{rzn} \\ \epsilon_{r\theta n} \\ \epsilon_{z\theta n} \end{bmatrix} \quad \dots (6.a)$$

The nodal force $[P_n]$ is, therefore, in the following relation to the displacement $[u_n]$.

$$[P_n] = [k_n][u_n] \quad \dots (7)$$

where

$$[k_n] = [h_n]^T \left\{ \int_A [g_n]^T [D] [g_n] dV \right\} [h_n] \quad \dots (8)$$

In Eq.(8), the symbol A means the integration within the one whole element.

The terms $[D]$, $[h_n]$ and $[k_n]$ in Eq.(8) are also very easy to calculate by using the widely known method(6); therefore, such a method is not explained here.

After the stiffness matrix of individual element $[k_n]$ expressed in Eq.(8) was assembled suitably, the stiffness matrix of an overall system $[k]$ can be known, and then the relationship between the displacement and load at the nodes can be written as

$$[u_n] = [K_n]^{-1} [P_n] \quad \dots (9)$$

From Eq.(9), the displacement relating to each value of n can be determined, and thus the displacements u_r , u_θ and u_z to be computed can be known.

The displacement and the stiffness of column are, therefore, computed as follows.

(1) In the bending load applied at the end of column.

Putting $n=1$,

$$P_r = P_1 \cos \theta, \quad P_z = 0, \quad P_\theta = 0 \quad \dots (10)$$

In this condition, the bending load is distributed such as shown in Fig.3-a.

(2) In the torsional load applied at the end of column.

Exchanging the sine terms for the cosine terms with each other and putting $n=0$,

$$[H] = \begin{bmatrix} r_j z_k - r_k z_j & r_k z_i - r_i z_k & r_i z_j - r_j z_i \\ z_j - z_k & z_k - z_i & z_i - z_j \\ r_k - r_j & r_i - r_k & r_j - r_i \end{bmatrix}$$

where

$$\lambda = r_j(z_k - z_i) + r_i(z_j - z_k) + r_k(z_i - z_j)$$

As well known, the relationship between a stress $[\sigma_n]$ and a strain $[\epsilon_n]$ can be written as

$$[\sigma_n] = [D][\epsilon_n] \quad \dots (6)$$

$$\begin{bmatrix} \sigma_{rrn} \\ \sigma_{zzn} \\ \sigma_{\theta\theta n} \\ \sigma_{rzn} \\ \sigma_{r\theta n} \\ \sigma_{z\theta n} \end{bmatrix} = \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & & & \\ & 1 & \nu/(1-\nu) & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{rrn} \\ \epsilon_{zzn} \\ \epsilon_{\theta\theta n} \\ \epsilon_{rzn} \\ \epsilon_{r\theta n} \\ \epsilon_{z\theta n} \end{bmatrix} \quad \dots (6.a)$$

$$P_r = 0, \quad P_z = 0, \quad P_\theta = P_0 \quad \dots (11)$$

In this condition, the torsional load is distributed such as shown in Fig.3-b.

2.2. Cylindrical column having core-holes

The cylindrical column having such a core-hole has obviously no axis-symmetry in its own form. In this case, it is not enough to consider the finite elements of the column at only one cross-section as shown in the previous section. The column with a core-hole should be, therefore, divided into finite elements as shown in Fig.4

Fig.3 Loads applied to the column

Fig.4 Dividing the column into finite elements

and the displacements of all these nodes should be computed.

In this case, however, the size of computer comes to a problem, since the limitation of the core size leads to be limited the number of nodes which affects directly the computing accuracy. It is, therefore, easy to presume that the computing accuracy for this kind of problem is forced to fall as compared with the technique used in the previous section.

The computing method used here is very popular one and its detail has been explained in a lot of papers⁽⁶⁾; therefore, its explanation is not written here at all.

3. Experimental techniques

To investigate the propriety of the computing method mentioned in the previous chapter, the bending and torsional tests for the cylindrical column were carried out with test rigs shown in Figs.5 and 6.

In the rig for the bending test, the cylindrical column 1 to be examined is tightly held at its one end with the collet chuck 4 whose multi-jaws are opened and closed by the control screw for collet chuck 5. The fixing condition of the cylindrical column may be in sufficient, however, to ensure the good accuracy of the measurement the influence of the inclination of

- 1 Cylindrical column (Test specimen)
- 2 I-beam
- 3 Load cell
- 4 Collet chuck
- 5 Control screw for jaws of collet chuck
- 6 Clock indicator
- 7 Surface plate

Fig.5 Rigs for the bending test

Fig.7 Symbols for the detailed shape of column

collet chuck, in other words, the influence of the fixed condition of cylindrical column on the measured values has been eliminated by using the standard I-beam 2 which is fixed directly on the collet chuck. Displacements of the column under the load were measured with clock indicators 6 graduated 1/100mm or 1/1000mm on the basis of this I-beam.

The bending load is applied to the free end of column with the loading screw through the load-cell 3, by which the applied load can be detected. As shown in Fig.6, the torsional load given by the dead weights 11 is also applied to the free end of column through the torque bar 8, the adapter 9 and the straight pin for torque transmission 10. In the torsional test, the torsion angle has been investigated by measuring the displacement of the straight pin whose both ends stick out the outer-circle of column.

The chucking load of the cylindrical column to be examined is generally of importance in this kind of test rig; therefore, from the result of the pre-experiment, the preload of the control screw for collet chuck has been fixed up to 3000kg at which the natural frequency of column is almost constant.

4. Effects of collared ribs on the static stiffness of cylindrical column

The theoretical and experimental studies were carried out with the cylindrical column shown in Fig.7, and the results are shown in the following.

4.1. Effects on the local deformation due

Fig.6 Rigs for the torsional test

	Kind of machine tools ; Name of elements	t/D	Remarks
A	Manufacturer	0.06 0.025	With two ribs
B	Horizontal boring and milling machine, Column	0.027 0.037	
C	Planner, Cross rail	0.067	Double column type
D	Planner, Column	0.039	Open side type
E	Lathe, Bed	0.060	

Table 1 Ratio between the wall thickness t and the outer diameter D of basic structural elements in actual machine tools

to the distortion of cross-section.

To determine the concrete shape and dimensions of the cylindrical column to be computed, the investigation for the wall thickness t and the outer diameter D of basic structural elements has been made on actual machine tools. Some examples of investigated results are shown in Table 1, and from these results it can be known that the ratio t/D is between 0.02 and 0.06. It is, therefore, said that a basic structural element of a machine tool is the structure having very thin walls, namely a thin-walled structure.

The local deformation of column due to the distortion of cross-section near the loading point is, therefore, considered easily to be very large. In other words, the displacement δ_1 and the torsion angle ϕ_1 with regard to the circumference including the loading point may be larger than the displacement at the central axis δ_0 and the torsion angle under an uniform torque ϕ_0 respectively, if the bending and torsional loads are applied with two concentrated loads as shown schematically in Fig.8.

The addition of collared ribs may have large effects on the prevention of such a local deformation of column, and thus one computed result for this problem is shown in Fig.9. From the results shown in Fig.9, it is well known that the characteristic λ under both bending and torsional loads is to be very small by adding the collared rib at the loading point. To add the rib at the end of column means that the structure will be formed similar to that having a closed cross-section at its end. Meanwhile, the characteristic λ indicates the measure of the local deformation and the small value of λ is in correspondence with the small local deformation of the column (please refer C and D type specimens in the figure).

The height of cylindrical column, i.e., the cantilever length of test specimen L , has also large effects on the local deformation. For example, the local bending deformation of column having low height is about 20% larger than the displacement at its central axis,

Fig.8 Example of local deformations of column

and the local torsional deformation is also about 50% larger. Moreover, the influence of the wall thickness t on this local deformation is to be larger as the height of column is to be short. It can, therefore, be shown that the column consisting of thinner walls is easy to display the local deformation.

4.2. Effects on the displacement of column as a beam-like element (the displacement of a cylindrical column at its central axis)

As described in the previous section, it is possible to get rid of the distortion of cross-section near the loading point by adding the collared rib at the end of cylindrical column. Under this condition, the bending deformation of column can be expressed only with the displacement at its central axis δ_0 ; therefore, the effects of collared ribs on the bending stiffness of such a column have been computed.

The computed results are shown in Fig.10, in which the length of column L and the wall thickness t are variables. In this figure, the distance between both adjacent collared ribs p and also the numbers of collared ribs z are graduated in the horizontal axis.

Fig.9 Prevention of the local deformation by adding collared ribs

Fig.10 Effects of the number of ribs on the bending stiffness

It is easy to know that the increase of stiffness is in nearly proportional relation to the increasing number of the collared ribs, and that the improvement of stiffness is to be more effective on the column having shorter length L and also thinner wall thickness t . In addition to this result, it can be also known that the column in which the distortion of its cross-section is eliminated, can be stiffened only less than 10% by adding collared ribs.

These results mentioned here and also in the previous section mean that the collared rib has effects on the prevention of a shear deformation of a column.

In the next place, let us consider the effects of collared ribs on the torsional stiffness of column whose wall thickness is nearly equal to the width of collared rib.

The torsion angle per unit length at the ribbed part Ω_{rib} is small as compared with that at the non-ribbed part Ω_0 .

By putting

$$\Delta = (\Omega_0 - \Omega_{rib}) / \Omega_0$$

and making some simple calculations, the torsion angle at the end of column with collared ribs is written as

$$\phi_{rib} = \phi_0 \{1 - \Delta(s/L)\} \quad \dots (12)$$

$$= \phi_0 (1 - \Delta \cdot \beta) \quad \dots (12.a)$$

where $\phi_0 = \Omega_0 L$ (the torsion angle of column without collared ribs)
 $\beta = s(z/L)$

Here, at first the column with collared ribs is replaced by the equivalent column whose wall thickness is uniform in its overall length, and in next the torsion angle per unit length of this equivalent column is considered. Then, the torsional stiffness of column with collared ribs K_{rib} can be written as

$$\Omega_R = \phi_{rib} / L = \phi_0 (1 - \Delta \beta) / L = \Omega_0 (1 - \Delta \beta) \quad \dots (13)$$

$$K_{rib} = T / \Omega_R$$

Generally $\Delta \beta \ll 1$, and then the ratio between both torsional stiffnesses K_{rib}/K_0 yields to

Fig.11 Effects of the number of ribs on the torsional stiffness

$$K_{rib}/K_0 = \Omega_0/\Omega_0 (1 - \Delta \beta) = 1 - \Delta \beta \quad \dots (14)^*$$

As the result, providing that the height of rib is constant, the behaviours of the torsional stiffness for the column with collared ribs can be well realized by using the characteristic β which is the ratio between the height of column L and the overall width of ribs.

One of computed results arranged with the characteristic β is shown in Fig.11. From this result, it can be known that the effects of the wall thickness t on the torsional stiffness and the increasing rate of torsional stiffness due to collared ribs are inclined to show similar behaviours which are seen on the bending stiffness.

4.3. Influence of the dimension of a rib

It should be considered two dimensions, i.e., the width s and the height h of rib, to investigate how the dimension of rib influences on the stiffness of column. In the torsion, as the dimension s is included itself in the characteristic β , it is enough to examine only the influence of h .

In Fig.12-b the relationships between the height of rib and the torsional stiffness are shown. According to this figure, the torsional stiffness increases steeply where the ratio h/t is less than 1-1.5, though the increasing rate of stiffness is only 4%. Even if the rib is to be higher than this ratio, the stiffness increases gradually and reaches finally to a constant value. This means that the higher rib is not effective in a cylindrical column.

The results mentioned above are also confirmed by investigating the shear stress distribution τ at the central axis of rib. As shown in Fig.12-a, it is obvious that the shear stress near the top of rib is almost negligible when the ratio h/t is more than 2. The rib whose ratio h/t is more than 2, therefore, is thought to be of no use.

Fig.12 Effects of the height of rib on the torsional stiffness

* Replying the questions presented by Dr. M. HONDA, the expression of this equation was modified. There were also made some amendments according to his comments.

In the bending condition, however, the height of rib has a little different effect on the stiffness as shown in Fig.13, namely the increase of stiffness can be seen even where the rib is considerably higher.

Meanwhile, that the increasing rate of bending stiffness is about 5%, is quite similar to that for the torsion. As shown in Fig.14, the increase of bending stiffness is in proportional relation to the thickness of rib and this stiffened tendency of the column is also quite similar one found in the torsion.

4.4. Comparison between experimental and computed values

To investigate the propriety of computed results presented in the previous sections, one experiment was carried out with the cylindrical column whose dimensions are as follows: wall thickness $t=2\text{mm}$, outer diameter $D=50\text{mm}$, and cantilever length $L=200\text{mm}$.

One example of experimental results with regard to the bending stiffness is shown in Fig.15. According to this result, the displacement at the circumference including the loading point δ_1 is larger than the displacement at the just opposite point δ_2 , and thus the cross-section of column is largely distorted when there is no collared rib at the end of column.

This distortion of cross-section is not different between both columns, i.e.,

Fig.13 Effects of the height of rib on the bending stiffness

Fig.14 Effects of the width of rib

A and B type; however, in such a C type which has the rib near the end of column, the displacement δ_1 and δ_2 come close to each other and the distortion of cross-section is to be small.

Let us consider now the column which is in the form of closed cross-section by adding the end plate hatched in the figure. For this type of column, its deformation can be defined by the displacement at its central axis δ_0 and the effects of collared ribs are in correspondence with those shown in Fig.10.

It is, therefore, possible to make some comparison between the computed and experimental values as follows.

The increase of stiffness by adding collared ribs is 1.5% in computation; however the value to be compared is 6.0%, which is the value converted considering that the thickness of rib examined is 10mm.

On the contrary, the experimental value is about 6.7%, since at the loading point

$$\delta_{B0} = \delta_{C0} = 104\mu\text{m} \quad \text{and} \quad \delta_{A0} = 111\mu\text{m}.$$

In all conditions, the torsion angle ϕ indicates the behaviours shown schematically in Fig.16, when the torsional displacements were measured between both arbitrary points, i.e., A and B points. Taking up the values at $\beta = s/AB = 0.091$, we can get the following results:

In Ao, Bo, and Co, types, the end of column is closed (Please refer the hatched part)

Fig.15 Experimental results for the bending deformation of column

Fig.16

K_{rib}/K_0 { Computed value (Fig.11) = 1.048
Experimental value (ϕ_0/ϕ_{rib}) = 1.053

Although this result is only one example, the same results of comparison can be obtained in all cases. It can be, therefore, said that the computed values are in good agreement with the experimental values.

5. Influence of core-holes on the static stiffness

Here, the columns whose diameters, heights and wall thicknesses are similar to those used in the previous chapter, have been also investigated.

From the results presented in the previous chapter, it may be thought that the wall thickness of column has less effects on the reduction of stiffness even if there is the same core-hole, when the wall is thinner and the ratio t/D is less than 0.04. Therefore, the influence of the wall thickness is not taken into consideration in all computations.

The local deformation near the loading point which is derived from the distortion of cross-section by existing core-holes, is also to be obviously large; however, the column has been dealt with as the beam-like element, since this distortion of cross-section can be prevented by stiffening the end of column with the collared rib.

5.1. Influence on the static bending stiffness

One example of computed results showing how the cylindrical column reduces its stiffness by the existence of a square hole, is given in Fig.17. The end displacement of this kind of beam is given by summing up both displacements due to the bending and the shear forces.

Namely,

$$\delta = \delta_B + \delta_S \quad \dots (15)$$

$$\delta = PL^3/3EI + kPL/AG \quad \dots (15.a)$$

where, δ_B : bending displacement

δ_S : shear displacement

I: cross-sectional second moment of area

A: cross-sectional area

k: constant

E: young's modulus

Fig.17 Influence of the position of core-hole

G: shear modulus

If the ratio L/D is large, i.e., the beam is slender, $L^3 \gg L$, and then $\delta_B \gg \delta_S$. Under this dimensional relation, therefore, as well known the stiffness of beam is almost determined by the bending deformation.

By using this simple expression, it is possible to explain well the reason why the position of core-hole changes its influence on the stiffnesses of the both A and B type columns near the point where $L/D=2.0$.*

The A type and B type columns have a hole lying in the perpendicular plane to the loading direction and in the parallel plane to the loading direction respectively. This means that the cross-sectional second moment of area reduces itself directly in the A type column, but keeps an almost constant value in the B type column. In the long column, therefore, the B type column is to be more stiff, as the bending deformation subjects the overall displacement of column.

In the short column, however, the shear deformation is to be of importance as compared with the bending deformation, and furthermore a shear stress distribution of a thin-walled beam has its maximum point at the central axis of beam. The reduction of shear stiffness is, therefore, to be larger in the B type column than in the A type column. As the result, the A type column is more stiff.

Thus, as shown in Fig.17, in the region where L/D is more than 2.0, the B type column is more stiff than the A type column, and this correlation between both columns becomes to the reverse one where L/D is less than 2.0.

For the sake of comparison, in Fig.17 one experimental value is also shown for the column whose dimensions are $d/D=0.3$ and $L/D=4.0$. It can be seen that this experimental value is in good agreement with the theoretical one.

Keeping the width in constant and changing only the length of a core-hole, the reduction of stiffness was also computed, and the results are shown in Fig.18 together with experimental values. From this result, it can be shown that the reduction of stiffness is large when the core-hole is long,

$d/D=0.3$, $L/D=4.0$, $D=50\text{mm}$, $t=2\text{mm}$
Fig.18 Influence of the length of long-hole

* Replying Dr. M. HONDA's questions, the explanation here was amended.

and that the computed value is in good agreement with experimental one where the core-hole is short. The computed and experimental values do not closely agree when the core-hole becomes long, because the computed accuracy falls due to the limitation of the number of finite elements.

In Fig.19, the influence of the size of square hole on the reduction of stiffness is shown. After the correlation between both computed and experimental results mentioned above has been well discussed, the computation was carried out with the column to which the computing method proposed here is available with enough accuracy. From the results, it can be shown that the stiffness is not so reduced till the d/D reaches to about 0.2, but the reduction of stiffness is obvious as the value of d exceeds this ratio. Where, d/D is the ratio between the side length of square hole and the outer diameter of column.

5.2. Influences on the static torsional stiffness

Same investigation as in the case of bending stiffness has been done for the torsional stiffness, and one example of results is shown in Fig.19 together with results relating to the bending stiffness. From this result, it is shown that the size of square hole has more influences on the torsional stiffness rather than on the bending stiffness of column.

Meanwhile, the torsional stiffness of column at the square hole can not be determined independently, since the stiffness of the core-hole's own part is influenced by the closed cross-sectional surrounding of the core-hole itself. The torsional stiffness was, therefore, determined by

$$K_h = T/\phi_h \quad \dots (16)$$

where T is the torque applied to the end of column and ϕ_h is the torsion angle at the end of column.

In Fig.20, computed results showing how the torsional stiffness reduces itself as the length of hole increases, are given together with experimental values. As same as the bending stiffness, the reduction of torsional stiffness is large when the hole; is large however, the rate of reduction is to be much larger, for example, it reaches to as much as 30% where the ratio l/L is 0.2.

Fig.19 Influence of the size of square hole

From the results, it is also known that the computed values are in good agreement with experimental values where the hole is within the short one. As the hole is prolonged, however, the difference between both values becomes to large gradually owing to the same reason mentioned in the previous section.

In addition to our results, the results of analytical calculation for the stiffness of the box-type beam with thin walls, which were presented by V.V. Kaminskaya⁽⁷⁾, are shown together in Fig.20 to make some comparison. As known clearly, the computed values with FEM is in good agreement with the experimental values rather than the analytical values presented by Kaminskaya. According to this result, it is said that the computing method for the cylindrical column with core-holes presented here is reasonable and effective.

Meanwhile, Kaminskaya has pointed out that the stiffness calculated analytically is always about 50% higher than the experimental one for the column whose dimensions are almost similar to those examined here. Her comment is also confirmed to be right from our results.

6. Conclusion

To assist the accomplishment of the computer aided design calculation for a machine tool structure, in this study one trial has been made. One computing method, by which the effects of collared ribs and core-holes on the static stiffness of cylindrical column can be calculated by using FEM, has been investigated theoretically and experimentally. In addition, from all results the possibility whether the cylindrical column can be replaced by an one-dimensional beam element in the computing process or not, has been also investigated.

As the result, it can be confirmed that the static bending and torsional stiffness of the cylindrical column with collared ribs or core-holes are computed with good accuracy by using FEM.

For the problem replacing the cylindrical column with an one-dimensional beam-like element in order to make computation for the overall stiffness of a machine tool, the following can be concluded.

As same as an actual machine tool structure, when the cylindrical column has

Fig.20 Influence of the length of long-hole

several collared ribs so that no distortion of cross-section can be seen and also the dimensions of its core-hole fulfill the conditions $d/D < 0.3$ and $l/L < 0.1$, the stiffness of column differs only within about 10% from the stiffness of the beam-like element replaced the cylindrical column adequately. In most cases, therefore, the cylindrical column can be well replaced by an one-dimensional beam-like element even though there are collared ribs and core-holes.

From the results of this study, the following effects of collared ribs and core-holes on the static stiffness can be known furthermore.

(1) On both bending and torsional stiffness of the column, the collared rib is more effective as the wall of column is to be thinner and the height of column is to be shorter. The rib, however, has not so much effects on the stiffness when its height exceeds a proper value.

(2) According to our experiments, the increase of stiffness due to the addition of collared ribs to the column, is 100% in maximum. This stiffening value is equal to those presented elsewhere⁽⁸⁾, however this improvement is not due to the increase of stiffness of the column regarded as a beam-like element, but mainly due to the reduced distortion of cross-section.

It is, therefore, thought that there have been some problems in the previous studies in which experimental results have been arranged and discussed without taking into consideration such a distortion of cross-section separately.

(3) The reduction of stiffness due to core-holes is definitely large for the torsional stiffness rather than for the bending stiffness.

(4) Both bending and torsional stiffnesses reduce themselves steeply when the length of core-hole is more than a certain value.

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