Application of the Fiber Reinforced Composite to Rotating Discs*

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Discs of uniform thickness with a hole at the center were molded by winding the prepregs of glass fiber roving with polyester resin. A circumferential crack occurred in the disc when the inner to outer radius ratio of the disc became larger than a certain critical value. It was found that this crack was due to residual stresses in the disc developed in molding process. The distribution of residual stresses in the discs were analysed by considering anisotropic shrinkage of impregnated roving during the setting and cooling processes. These theoretical results approximately coincided with experimental results. From these results, it was made possible to predict the critical value of the inner to outer radius ratio less than which the crack did not occur in molded discs. It was also shown that the autofrettaged discs with arbitrary inner to outer radius ratio could be molded by anisotropic shrinkage of resin impregnated roving.

1. Introduction

Recently, the improvement on the strength of the rotor in such cases as high speed centrifuge and super flywheel has been strongly required. There is little possibility, however, of a new homogeneous material being developed which with-stands a super high speed revolution. Even if such a strong material becomes available in future, it might be not so effective in economical and mechanical sense to design the rotor under non-uniform stress distribution with only one isotropic material. On the other hand, fiber reinforced composite materials could have arbitrary anisotropic mechanical properties of strength, stiffness and density by changing the fiber volume fractions and fiber orientations and furthermore have higher specific strength. By use of these characteristics of fiber reinforced composite materials, a high speed rotor may be most efficiently designed. From this viewpoint, authors have already estimated theoretically the possibility of

making a super high speed rotor discs by using the composite materials⁽¹⁾. However, even though the most effective reinforcement is determined theoretically for certain structures, it is not always sufficient for engineering purpose and it is necessary furthermore to find out the available manufacturing techniques for performing these theoretically optimum reinforcements. In making the fiber reinforced rotor discs, we encountered many practical problems. In this paper, some solutions to these problems are proposed. The filament wound discs with large outer to inner radius ratios which have not been molded without cracks, are accomplished by the method of piling up several filament wound rings with small radius ratio. It is found, furthermore, that the autofrettage effect is generated by molding shrinkage in the above process. Through these attempts, the feasibility of strengthening the rotor discs by utilizing not only the fiber reinforcement but also the autofrettage effect is confirmed.

2. A new filament winding machine

As a most fundamental case, we investigate the filament reinforcement method for the rotor discs with uniform thickness having a hole at the center. The requirements of this filament reinforcement method are not only the feasibility of controlling the fiber directions and fiber volume fractions most efficiently but also the easiness and the reproducibility of the manufacturing process. Then the discs are molded by a kind of the filament winding processes in which the resin impregnated glass rovings are wound from inner diameter to outer diameter of

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the disc continuously. This process is excellent in the manufacturing easiness and the identical reproducibility. On the other hand, this has the defect that the fibers can be aligned only in the tangential direction. But considering the fact that the tangential stress is higher than the radial stress in the isotropic disc with uniform thickness having a hole at the center, the above mentioned winding method is not so unsuitable reinforcement from the view point of improving the rotating strength. When we apply this filament winding method to molding the disc, there arise various problems, for instance, how to control the fiber volume fractions and how to prevent the flux of the impregnated resin by the roving tension. Fig. 1 shows a new filament winding

- ① glass roving ② resin vessel
- ③ fiber volume fractions controller
- ④ resin gelation furnace ⑤ rollers
- ⑥ mandrel for molding discs
- ⑦ driving device
- ⑧ hot blast blower
- ⑨ winding number counter

Fig.1 Filament winding machine

machine improved by finding the solutions of above mentioned problems. This machine consists of resin vessel ②, fiber volume fractions (V_f) controller ③, resin gelation furnace ④, hot blast blower ⑧, mandrel ⑥ and driving device ⑦. In resin vessel, there exist three rollers through which the roving is drawn through these rollers to impregnate the resin. Fiber volume fractions controller is shown in Fig. 2. The resin impregnated roving

Fig.2 Fiber volume fractions controller

is drawn through six (three pairs of) vertical cylinders and four (two pairs of) horizontal cylinders of this controller. The fiber volume fraction, V_f , is controlled by adjusting the clearance between two pairs of horizontal cylinders by using two micrometers. The calibration diagram of fiber volume fraction with respect to the clearance, t , is shown in Fig.3. In

Fig.3 Calibration diagram of $t - V_f$

this calibration test, we use glass roving (2.4g/m) as a fiber and polyester resin as a matrix. The directions of the arrows added at the experimental points in Fig.3 indicate the measurement conditions whether t is increasing or decreasing and the numbers added at the experimental points indicate the orders of measuring. The hysteretic deviation in increasing and decreasing of t is not found.

When the resin impregnated rovings are wound immediately after the volume fraction, V_f , is settled, the resin will flow out by roving tension and therefore the controlling of V_f becomes meaningless. Then, the resin impregnated rovings are wound on the mandrel by drawing through the gelation furnace 250 mm long to activate the gelation of the resin. Furthermore, in order to accelerate the setting of the portion already wound, the mandrel is heated up to 80°C by the hot blast blower. The winding speed of the resin impregnated rovings depends on the gelation time which varies with the furnace temperature and fiber volume fraction of the resin impregnated roving. The mandrel for molding disc consists of a steel cylinder which is sandwiched by two duralumin discs with diameter of 260 mm. By varying the length and diameter of the steel cylinder, the discs with various thicknesses and inner diameters can be molded. In the mandrel driving device the reciprocating feed mechanism is provided to make it possible to mold plane plates and cylinders.

3. Limit of moldable radius ratio

It is well known empirically that the cracks used to occur in the thick fiber reinforced plastics (FRP). The reasons have been however scarcely investigated theoretically. When the discs with uniform thickness and uniform fiber volume fraction are molded, the cracks occur circumferentially in the discs with large radius ratio (see upper photographs of Fig.4). After curing of the molded discs,

Fig.4 Cracked discs (upper photographs) and uncracked discs (lower photographs)

these cracks occur at almost the same place in the same size discs. Thus, it can be considered that these cracks are not accidental but substantial results. The cracks are considered to be caused by the residual stresses due to the anisotropic shrinkage in the unidirectional fiber reinforced plastics during the setting and cooling processes. Following the above considerations, an analysis with respect to the crack occurrence is here and the results are compared with the experimental results.

3.1 Theoretical investigation

The anisotropic shrinkage of unidirectional fiber reinforced plastics consists of two components. First, the chemical shrinkage when the resin sets and secondly, the thermal shrinkage when the wound disc is cooled from the molding or curing temperature to the room temperature. These shrinkage strains are defined as follows.

Δ_{11} : Setting shrinkage strain* in the fiber direction

Δ_{22} : Setting shrinkage strain in the transverse direction

α_{11} : Thermal expansion coefficient* in the fiber direction

* The signs of Δ and α are defined as follows; extension is indicated by plus sign (positive) and shrinkage is indicated by minus sign (negative).

α_{22} : Thermal expansion coefficient in the transverse direction

T : Temperature difference between molding or curing temperature and room temperature. (If the curing temperature is higher than the glass transition temperature of the resin, T is the temperature difference between glass transition temperature and room temperature.)

By using these quantities, the circumferential and radial free shrinkage strains, $\epsilon_{\theta 0}$ and $\epsilon_{r 0}$, of the infinitesimal element in the molded disc (Fig.5) are shown as follows

$$\epsilon_{\theta 0} = \Delta_{11} + \alpha_{11} T \quad (1)$$

$$\epsilon_{r 0} = \Delta_{22} + \alpha_{22} T \quad (2)$$

Fig.5 Infinitesimal element in the disk

The residual stresses and strains in this disc are analysed as a plane stress problem because the thickness of this molded disc is very small compared with the diameter. And the fiber volume fraction is assumed to be constant in the radial direction. The stress-strain relations of infinitesimal element in this disc are shown by using Young's modulus, E , and Poisson's ratio, ν , as follows.

$$\epsilon_r = \frac{\sigma_r}{E_r} - \frac{\nu_r \sigma_\theta}{E_\theta} + \epsilon_{r0} \quad (3)$$

$$\epsilon_\theta = \frac{\sigma_\theta}{E_\theta} - \frac{\nu_\theta \sigma_r}{E_r} + \epsilon_{\theta 0} \quad (4)$$

The equilibrium condition and compatibility condition are shown as follows

$$\frac{d\sigma_r}{dr} - \frac{\sigma_\theta - \sigma_r}{r} = 0 \quad (5)$$

$$\epsilon_r = du_r/dr, \quad \epsilon_\theta = u_r/r \quad (6)$$

where u_r is the radial displacement. Solving Eqs.(3)~(6) under the following boundary conditions

$$[\sigma_r]_{r=r_0} = 0, \quad [\sigma_r]_{r=r_1} = 0 \quad (7)$$

the solutions are obtained as

$$\begin{aligned} u_r = & \left[\frac{\epsilon_{r0} - \epsilon_{\theta 0}}{1 - 1/K^2} \left\{ \left(\frac{1}{K} - \nu_{r\theta} \right) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} - \left(\frac{1}{K} + \nu_{r\theta} \right) \right. \right. \\ & \left. \left. \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (\nu_{r\theta} - 1) \right\} + \epsilon_{r0} \right] r \end{aligned} \quad (8)$$

$$\begin{aligned} \sigma_r = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0}) E r}{1 - 1/K^2} \left\{ \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} \right. \\ & \left. + \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} - 1 \right\} \end{aligned} \quad (9)$$

$$\begin{aligned} \sigma_{\theta} = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0}) K E r}{1 - 1/K^2} \left\{ \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} \right. \\ & \left. - \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} - \frac{1}{K} \right\} \end{aligned} \quad (10)$$

$$\begin{aligned} \epsilon_r = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0})}{1 - 1/K^2} \left\{ (1 - \sqrt{\nu_{r\theta} \nu_{r\theta}}) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (1 + \sqrt{\nu_{r\theta} \nu_{r\theta}}) \right. \\ & \left. \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (\nu_{r\theta} - 1) \right\} + \epsilon_{r0} \end{aligned} \quad (11)$$

$$\begin{aligned} \epsilon_{\theta} = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0})}{1 - 1/K^2} \left\{ \left(\frac{1}{K} - \nu_{r\theta} \right) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} - \left(\frac{1}{K} + \nu_{r\theta} \right) \right. \\ & \left. \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (\nu_{r\theta} - 1) \right\} + \epsilon_{r0} \end{aligned} \quad (12)$$

where $\gamma = r/r_0$, $\gamma_1 = r_1/r_0$ and $K = \sqrt{E_{\theta}/E_r}$. The radial and circumferential residual stresses, σ_{rR} and $\sigma_{\theta R}$, are given by Eqs. (9) and (10) respectively and therefore

$$\sigma_{rR} = \sigma_r, \quad \sigma_{\theta R} = \sigma_{\theta} \quad (13)$$

The residual strains, ϵ_{rR} and $\epsilon_{\theta R}$, are obtained by subtracting free shrinkage strains, ϵ_{r0} and $\epsilon_{\theta 0}$, from the practical total strains, ϵ_r and ϵ_{θ} , of Eqs. (11) and (12) respectively as follows.

$$\begin{aligned} \epsilon_{rR} = \epsilon_r - \epsilon_{r0} = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0})}{1 - 1/K^2} \left\{ (1 - \sqrt{\nu_{r\theta} \nu_{r\theta}}) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} \right. \\ & \left. + (1 + \sqrt{\nu_{r\theta} \nu_{r\theta}}) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (\nu_{r\theta} - 1) \right\} \end{aligned} \quad (14)$$

$$\begin{aligned} \epsilon_{\theta R} = \epsilon_{\theta} - \epsilon_{\theta 0} = & \frac{(\epsilon_{r0} - \epsilon_{\theta 0})}{1 - 1/K^2} \left\{ \left(\frac{1}{K} - \nu_{r\theta} \right) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} \right. \\ & \left. - \left(\frac{1}{K} + \nu_{r\theta} \right) \left(\frac{r_1^{K+1}}{r^{2K-1}} \right) r^{K-1} + (\nu_{r\theta} - 1) \right\} \end{aligned} \quad (15)$$

By these results we find that the residual stresses and strains in the discs with

uniform fiber volume fractions are proportional to the difference of free shrinkages, $\epsilon_{r0} - \epsilon_{\theta 0}$, and are dependent on γ_1 and γ but independent of r_0 . The calculated results of σ_{rR} , $\sigma_{\theta R}$ are shown in Fig. 6. The elastic constants used in

Fig. 6 Distributions of residual stresses ($V_f=0.55$)

these calculations are shown later in Eqs. (18)~(20). The maximum values of $\sigma_{\theta R}$ and σ_{rR} exist along the inner surface and near the middle radius circle of the disc respectively. The crack in the molded disc should occur at the position where the magnitude of residual stresses satisfies the failure condition of the unidirectional fiber reinforced plastics. As shown in Fig. 4, the crack forms a concentric circle with about the middle radius. Hence, it can be assumed that the crack occurrence is principally affected by the maximum radial residual stress. The location of the crack is investigated on the basis of the failure criterion that the failure occurs when the normal stress in a given direction reaches the breaking strength of the composite in that direction. Comparing the tensile strength of unidirectional fiber reinforced plastics in the fiber direction, σ_{11}^B , (Fig. 10) and transverse direction, σ_{22}^B , (Fig. 11) with maximum residual stresses, $\sigma_{\theta Rmax}$ and σ_{rRmax} , respectively, the following relation can be obtained

$$(\sigma_{\theta Rmax} / \sigma_{11}^B) < (\sigma_{rRmax} / \sigma_{22}^B).$$

According to the above failure criterion, the crack occurs when

$$\sigma_{rRmax} = \sigma_{22}^B.$$

The maximum radius ratio of the disc without crack in the molding process is defined as "limit of moldable radius ratio",

γ^* . This value is determined by using the transverse tensile strength, σ_{22}^B , (Fig.11) and the maximum value of radial residual stress, σ_{rRmax} , (Eq.17) which occur at the position, γ , given by

$$\gamma = \frac{2K \sqrt{(K+1)(\gamma_1^{K-1}-1)(\gamma_1^{2K}-1)}}{\sqrt{(K-1)(\gamma_1^{K-1}-1)(\gamma_1^{2K}-1)}} \quad (16)$$

$$\sigma_{rRmax} = \frac{(E_f \sigma - E_m \sigma_0) E_f \left[\left(\frac{\gamma_1^{K+1}-1}{\gamma_1^{2K}-1} \right)^{\frac{1+1/K}{2}} \left\{ \frac{(K+1)(\gamma_1^{K+1}-1)}{(K-1)(\gamma_1^{2K}-1)} \right\}^{\frac{1-1/K}{2}} + \left(\frac{\gamma_1^{K+1}-1}{\gamma_1^{2K}-1} \right)^{\frac{1-1/K}{2}} \left\{ \frac{(K-1)(\gamma_1^{K+1}-1)}{(K+1)(\gamma_1^{2K}-1)} \right\}^{\frac{1+1/K}{2}} - 1 \right]}{1 - 1/K^2} \quad (17)$$

3.2 Experimental investigation

3.2.1 Elastic constants and strength

The material properties such as elastic constants and strength are determined in order to confirm experimentally the validity of the analytically obtained residual stresses and the limit of moldable radius ratio, γ^* . The tensile tests under a constant loading rate for dumbbell type of specimens of unidirectional fiber reinforced plastics with various fiber volume fractions are conducted in the fiber and the transverse directions. The tensile testing machine having a closed loop feedback control system of loading rate and temperature conditions is used. These experimental results are shown in Figs.7~11. In these figures the notations, E_{11} , E_{22} , ν_{12} , ν_{21} , σ_{11}^B and σ_{22}^B ,

Fig.7 Elastic modulus in fiber direction, E_{11}

express elastic moduli, Poisson's ratios and strengths in the fiber and the transverse directions, and are given by

$$E_{11} = V_f E_f + (1 - V_f) E_m \quad (18)$$

$$E_{22} = E_m \left\{ 1 + \frac{(1 + V_f)/(1 - \sqrt{V_f}) - (1 - \nu_m^2)}{E_m/E_f V_f + (1 - \nu_m^2)} \right\} \quad (19)$$

$$\frac{E_{11}}{\nu_{12}} = \frac{E_{22}}{\nu_{21}} = \frac{V_f E_f + (1 - V_f) E_m}{V_f \nu_f + (1 - \nu_f) \nu_m} \quad (20)$$

$$\begin{cases} \sigma_{11}^B = (1 - V_f) \sigma_m^B & (V_f < V_{fc}) \\ \sigma_{11}^B = V_f \sigma_f^B + (1 - V_f) \sigma_m^B & (V_f > V_{fc}) \end{cases} \quad (21)$$

$$\sigma_{22}^B = \sigma_m^B \quad (22)$$

Fig.8 E_{11}/ν_{12} , E_{22}/ν_{21}

Fig.9 Elastic modulus in the transverse direction, E_{22}

The heavy lines in these diagrams express the calculated results by Eqs.(18)~(22). The elastic moduli, Poisson's ratios and tensile strengths of fiber and matrix are as follows

$$E_f = 8400 \text{ kg/mm}^2, \quad E_m = 357 \text{ kg/mm}^2$$

$$\nu_f = 0.22, \quad \nu_m = 0.355$$

$$\sigma_f^B = 152 \text{ kg/mm}^2, \quad \sigma_m^B = 6.0 \text{ kg/mm}^2$$

Fig. 10 Tensile strength in the fiber direction, σ_{11}^B

Fig. 11 Tensile strength in the transverse direction, σ_{22}^B

and the stress in the matrix with the same strain when fiber break is $\sigma_m' = 4.0 \text{ kg/mm}^2$ and transverse tensile strength is $\sigma_m'' = 1.27 \text{ kg/mm}^2$. The notation V_{fc} is a critical fiber volume fraction. The validity of these equations is confirmed previously by authors⁽²⁾ for the rectangular array composite and in this investigation these equations have been applied for the square array. Close agreements between the experimental and the calculated results are obtained. Then the above equations are used to express the elastic constants and tensile strengths.

3.2.2 Measurements of residual strains

To investigate experimentally the

validity of the limit of moldable radius ratio derived in the previous section, the residual strains in molded discs are measured. The difference of free shrinkages, $\epsilon_{r0} - \epsilon_{\theta 0}$, is determined by substituting the measured results of residual strains in molded discs into Eqs. (14) and (15). Substituting these free shrinkage values into Eqs. (9) and (10), the residual stresses are obtained. The residual strains in the discs are measured by cutting these discs after the strain gages have been installed in the circumferential and the radial directions at various points along a diametrically opposed line. The measurements are performed for many discs with various fiber volume fractions, V_f , inner radii, r_0 , and radius ratio, γ_1 . The measured results are shown in Figs. 12-20. The

Fig. 12 Distributions of residual strain in the molded discs.

Fig. 13 Distributions of residual strain in the molded discs.

Fig. 14 Distributions of residual strain in the molded discs

Fig. 17 Distributions of residual strain in the molded discs

Fig. 15 Distributions of residual strain in the molded discs

Fig. 18 Distributions of residual strain in the molded discs

Fig. 16 Distributions of residual strain in the molded discs

Fig. 19 Distributions of residual strain in the molded discs

Fig. 20 Distributions of residual strain in the molded discs

differences of free shrinkages, $-(\epsilon_{r0}-\epsilon_{\theta0})$, determined from these results are shown in Table 1. The anisotropic shrinkage strains, ϵ_{r0} and $\epsilon_{\theta0}$, are investigated theoretically in the next chapter. The results calculated by substituting $-(\epsilon_{r0}-\epsilon_{\theta0})$ of Table 1 into Eqs. (14) and (15) are shown in Figs. 12~20 by heavy lines.

Table 1

V_f	0.55	0.43	0.38
$-(\epsilon_{r0}-\epsilon_{\theta0}) \times 10^{-6}$	3320	2930	3060
$\sigma_{B2}^B (\text{kg mm}^{-2})$	1.27	1.27	1.27
γ^* (Theory)	2.45	3.10	3.35
$\bar{\gamma}^*$	2.05	2.22	2.68
$\underline{\gamma}^*$	1.90	2.09	2.42

Examples of cracked discs (upper photographs) and uncracked discs (lower photographs) with $V_f=0.55$ and inner radius $r_0=60$ mm, 80 mm and 100 mm are shown in Fig. 4. From these results, it can be seen that the limit of moldable radius ratio, γ^* , (in this case $\gamma^*=2.0$) is almost independent of r_0 as predicted in the previous section.

Figure 21 shows the experimental results of residual strains in a cracked disc. The heavy line indicates the residual strains calculated following the basic assumptions that the disc is divided into two discs and these inner and outer discs are independent of each other. Close agreement between experimental and calculated results is observed. From these facts the separated surfaces by the crack are assumed to be free surfaces. The dotted lines indicate calculated

Fig. 21 Distribution of residual strain in the cracked disc

values of σ_{rR} on the condition that the disc is uncracked. The position of $\gamma=1.44$ where crack occurred coincides well with the position of $\gamma=1.44$ where the calculated maximum value of σ_{rR} exists. It is, then, supposed that the radial residual stress is a principal factor of the cracking. The average value of the maximum radius ratios of uncracked discs, $\bar{\gamma}^*$, and the average value of the minimum radius ratios of cracked discs, $\underline{\gamma}^*$, are shown in Table 1. The limit of moldable radius ratio calculated by Eqs. (17) and (22) is denoted by γ^* . The calculated and the experimental values have a similar tendency with respect to the change of V_f , but these values do not so well agree. This disagreement may be considered due to the facts that there exists considerable scattering in the experimental values of σ_{B2}^B (Fig. 11) and that the effect of $\sigma_{\theta R}$ is neglected in the crack conditions.

4. Investigations of anisotropic shrinkage

4.1 The calculation of anisotropic thermal expansion coefficients and anisotropic setting shrinkage strains

The eifference of anisotropic shrinkage strains, $-(\epsilon_{r0}-\epsilon_{\theta0})$, obtained from the residual strains in the preceding chapter is investigated theoretically. From Eqs. (1) and (2), the anisotropic free shrinkage strains, ϵ_{r0} and $\epsilon_{\theta0}$, consist of anisotropic thermal shrinkage in the cooling process from molding or curing temperature to the room temperature and anisotropic setting shrinkage which arises from chemical shrinkage in the setting process.

4.1.1 Anisotropic thermal expansion coefficient

There is a large difference between fiber and resin thermal expansion coefficients and then the thermal distortion of

unidirectional fiber reinforced composite has a large anisotropy in the fiber and transverse directions. Although a few theoretical studies of these anisotropic thermal expansion are found^{(3),(4)}, these results are not so practical because they are given for the upper and lower limits only or are very complex. The experimental investigations on these results are scarcely performed. Then, the thermal expansions in fiber direction (α_{11}) and in transverse direction (α_{22}) are calculated here by using a combined cylinder model as shown in Fig.22. The analysis is conducted under the following assumptions

Fig.22 Combined cylinder model

that the fiber and surrounding resin are perfectly bonded and this model is long enough to consider ϵ_z uniform throughout the model. The stress-strain relations for the temperature change of T are written as follows,

$$\epsilon_R = \frac{1}{E} [\sigma_R - \nu(\sigma_\theta + \sigma_z)] + \alpha T \quad (23)$$

$$\epsilon_\theta = \frac{1}{E} [\sigma_\theta - \nu(\sigma_z + \sigma_R)] + \alpha T \quad (24)$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_R + \sigma_\theta)] + \alpha T = \text{const} \quad (25)$$

where the notations, E , ν and α , indicate the elastic modulus, Poisson's ratio and thermal expansion coefficient respectively. The notations (E_f , ν_f and α_f) indicate the values of fiber in $0 < R < R_0$ and (E_m , ν_m and α_m) indicate the values of resin in $R_0 < R < R_1$. The equilibrium and compatibility conditions in this cylinder model are shown as follows

$$\frac{d\sigma_R}{dR} - \frac{\sigma_\theta - \sigma_R}{R} = 0 \quad (26)$$

$$\begin{aligned} \epsilon_R &= du_R/dR \\ \epsilon_\theta &= u_R/R \end{aligned} \quad (27)$$

where u_R is the radial displacement. Solving Eqs.(23)~(27) with respect to ϵ_z and u_R under the following boundary conditions

$$[u_R]_{R=0} = 0, [u_R]_{R=R_0}^{\text{fiber}} = [u_R]_{R=R_0}^{\text{matrix}} \quad (28)$$

$$[\sigma_R]_{R=R_1} = 0, [\sigma_R]_{R=R_0}^{\text{fiber}} = [\sigma_R]_{R=R_0}^{\text{matrix}} \quad (29)$$

$$\int_0^{R_0} \sigma_z R dR + \int_{R_0}^{R_1} \sigma_z R dR = 0 \quad (30)$$

the thermal expansion coefficients in the fiber direction, α_{11} , and in the transverse direction, α_{22} , are given as follows

$$\alpha_{11} = \frac{\epsilon_z}{T} = \frac{A \left\{ \frac{E_f}{(1+\nu_f)(1-2\nu_f)} \left[2\nu_f(1+\nu_m)\alpha_m - (1+\nu_f)\alpha_f \right] - E_m \left(\frac{1}{V_f} - 1 \right) \alpha_m \right\} + \frac{E_f}{(1+\nu_f)(1-2\nu_f)} \left[(1+\nu_f)\alpha_f - (1+\nu_m)\alpha_m \right]}{A \left\{ \frac{E_f}{(1+\nu_f)(1-2\nu_f)} (2\nu_f\nu_m + \nu_f - 1) - E_m \left(\frac{1}{V_f} + 1 \right) - \frac{E_f(\nu_m - \nu_f)}{(1+\nu_f)(1-2\nu_f)} \right\}} \quad (31)$$

where

$$A = \frac{\frac{E_f}{(1+\nu_f)(1-2\nu_f)} \left[\nu_f(1-2\nu_m) + 1 \right] + \frac{E_m}{(1+\nu_m)} (1-\nu_f)}{\frac{E_f}{(1+\nu_f)(1-2\nu_f)} 2\nu_f \left[\nu_f(1-2\nu_m) + 1 \right] + \frac{E_m}{(1+\nu_m)} 2\nu_m(1-\nu_f)} \quad (32)$$

$$\alpha_{22} = \frac{[u_R]_{R=R_1}}{TR_1} = \frac{\frac{2E_f(1-\nu_m)}{(1+\nu_f)(1-2\nu_f)} \left[\alpha_{11}(\nu_m - \nu_f) + \left\{ (1+\nu_f)\alpha_f - (1+\nu_m)\alpha_m \right\} \right]}{\frac{E_f}{(1+\nu_f)(1-2\nu_f)} \left\{ (1-2\nu_m) + \frac{1}{V_f} \right\} - \frac{E_m}{1+\nu_m} \left(1 - \frac{1}{V_f} \right) - \nu_m\alpha_{11} + (1+\nu_m)\alpha_m} \quad (33)$$

Anisotropic thermal expansion coefficients, α_{11} and α_{22} , are then obtained alternatively by using a simple plate model in which the plate-like resin and fiber are bonded with each other as shown in Fig.23. The determination of α_{11} and α_{22} is performed under the following assumptions that the amount of expansion in the 11 direction is equal in resin and fiber

Fig.23 Simple plate model

for the temperature change of T . Hence the equilibrium condition in 11 direction is as follows

$$V_f E_f (\alpha_f - \alpha_{11}) T + (1 - V_f) E_m (\alpha_m - \alpha_{11}) T = 0 \quad (34)$$

The corresponding distortion in 22 direction, $\alpha_{22} T$, is shown as follows

$$\alpha_{22} T = V_f \alpha_f T + (1 - V_f) \alpha_m T + V_f \nu_f (\alpha_f - \alpha_{11}) T + (1 - V_f) \nu_m (\alpha_m - \alpha_{11}) T \quad (35)$$

Equations (34) and (35) yield, respectively

$$\alpha_{11} = \frac{V_f E_f \alpha_f + (1 - V_f) E_m \alpha_m}{V_f E_f + (1 - V_f) E_m} \quad (36)$$

$$\alpha_{22} = V_f (1 + \nu_f) \alpha_f + (1 - V_f) (1 + \nu_m) \alpha_m - \{V_f \nu_f + (1 - V_f) \nu_m\} \alpha_{11} \quad (37)$$

4.1.2 Anisotropic setting shrinkage strain

Thermosetting shrinkage by polymerization is found in the thermosetting resin. Due to this shrinkage, the unidirectional fiber reinforced plastics have the anisotropic shrinkage in the same manner as thermal expansion. This anisotropic shrinkage causes the residual stresses in molded discs. The setting shrinkage of the resin takes place continuously in a wide range of states from the liquid to the perfectly set state. Therefore, it is difficult to analyse the setting shrinkage in the composites. In the beginning of gelation, however, the resin flows perfectly by external forces. Therefore in this period there occurs no residual stress due to anisotropic shrinkage. After the resin has some extent of elasticity, the shrinkage yields the residual stresses in molded discs. In this chapter, the shrinkage strains in the fiber direction and the transverse direction, Δ'_{11} and Δ'_{22} , are calculated by using the imaginary shrinkage strain*, Δ' , for the resin which has the elastic constants corresponding to these of the perfectly setting resin. These shrinkage strains are analysed by using a combined cylinder model which is the same as one used in the analysis of anisotropic thermal expansion coefficients. Hence the imaginary setting shrinkage strain of resin, Δ' , corresponds to the thermal expansion of resin, $\alpha_m T$, in the previous analysis. Substituting $\alpha_m T = \Delta'$, $\alpha_f T = 0$ in the stress-strain relations

defined by Eqs. (23) ~ (25) and using the same equilibrium, compatibility and boundary conditions given by Eqs. (26) ~ (30), the imaginary setting shrinkage strains of composite are thus obtained as

$$\Delta'_{11} = \epsilon'_x = \frac{\left[A \left(\frac{2\nu_f(1+\nu_m)E_f}{(1+\nu_f)(1-2\nu_f)} - E_m \left(\frac{1}{V_f} - 1 \right) - \frac{(1+\nu_m)E_f}{(1+\nu_f)(1-2\nu_f)} \right) \Delta' \right]}{A \left(\frac{E_f}{(1+\nu_f)(1-2\nu_f)} (2\nu_f\nu_m + \nu_f - 1) - E_m \left(\frac{1}{V_f} - 1 \right) \right) - \frac{(\nu_m - \nu_f)E_f}{(1+\nu_f)(1-2\nu_f)}} \quad (38)$$

$$\Delta'_{22} = \frac{(U_R)_{R=R_1}}{R_1} = \frac{2E_f(1-\nu_m)}{(1+\nu_f)(1-2\nu_f)} \left\{ (\nu_m - \nu_f) \Delta'_{11} + (1+\nu_m) \Delta' \right\} - \frac{E_f}{(1+\nu_f)(1-2\nu_f)} \left\{ (1-2\nu_m) + \frac{1}{V_f} \right\} - \frac{E_m}{(1+\nu_m)} \left(1 - \frac{1}{V_f} \right) - \nu_m \Delta'_{11} + (1+\nu_m) \Delta' \quad (39)$$

Using the simple model (Fig. 23), the following results are obtained in the same manner as above.

$$\Delta'_{11} = \frac{(1 - V_f) E_m \Delta'}{V_f E_f + (1 - V_f) E_m} \quad (40)$$

$$\Delta'_{22} = (1 - V_f) (1 + \nu_m) \Delta' - \{V_f \nu_f + (1 - V_f) \nu_m\} \Delta'_{11} \quad (41)$$

4.2 Comparison with experimental results

The anisotropic thermal expansion coefficients are measured by heating from 30°C to 60°C the plate specimens on which the strain gages are mounted in the fiber and the transverse directions. The values of $\alpha_{22} - \alpha_{11}$ are measured by the two gages method in which the strain gages in the fiber direction and the transverse direction are put on adjacent sides of bridge circuit. The values of α_{22} are measured by using the strain gages in the transverse direction and the strain gage mounted on the steel of which thermal expansion coefficient is known as $\alpha = 11 \times 10^{-6} / ^\circ\text{C}$. Experimental results are shown in Fig. 24.

Fig. 24 Anisotropic setting shrinkage strains

* The sign of Δ' is defined as follows; expansion is positive and shrinkage is negative.

In this figure, the heavy line and dotted line show the calculated results from Eqs.(31),(33) and (36), (37), respectively. In these calculations, the elastic constants shown in the previous chapter are used and the thermal expansion coefficients of fiber and resin are

$$\alpha_f = 5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, \quad \alpha_m = 70 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

The experimental results shown by circle agree better with the calculated results from Eqs.(31) and (33) than those from Eqs.(36) and (37).

Anisotropic setting shrinkage strains are measured by embedding the strain gages in the process of filament winding in the fiber and the transverse directions. The difference of shrinkage strains, $-(\Delta'_{22} - \Delta'_{11})$, is measured by the two gages method in the same way as measuring of $-(\alpha_{22} - \alpha_{11})$. In this method, the readings of strain gages do not include the shrinkage in the beginning of gelation where the resin shows fluidity and indicates the shrinkage after the resin has some extent of elasticity. Figure 25 shows the experimental results for various fiber volume fractions. By using these experimental results and Eqs.(38) and (39), the imaginary setting shrinkage of resin is calculated as $\Delta' = 5000 \times 10^{-6}$ and the calculated results of Eqs.(38) and (39) by using above value of Δ' are shown by heavy line in Fig.25. In the same way, the calculated results from Eqs.(40) and (41) by using same value of Δ' are shown by dotted line.

As mentioned previously, in order to prevent the flux of resin, the resin impregnated rovings are drawn through furnace and wound under the condition of gelation on the mandrel. Therefore, the setting shrinkages, Δ_{11} and Δ_{22} , which

induce actual residual stresses must be evaluated by subtracting the setting shrinkage before winding from the imaginary setting shrinkages, Δ'_{11} and Δ'_{22} . Since the mandrel and winding disc are kept at high temperature of 80°C in the process of molding, the residual stresses are somewhat relaxed by viscoplastic deformation of resin. For these reasons, the setting shrinkage strains are actually reduced in appearance. The reduction ratio of setting shrinkage determined from the molding conditions is defined as follows

$$K_1 = \frac{\Delta_{11}}{\Delta'_{11}} = \frac{\Delta_{22}}{\Delta'_{22}} = \frac{(\Delta_{22} - \Delta_{11})}{(\Delta'_{22} - \Delta'_{11})} \quad (42)$$

Table 2 shows this reduction ratio, K_1 , in actual molding conditions. As the gelation of resin impregnated rovings must be performed more thoroughly in the case of lower fiber volume fractions, the values of K_1 decrease more as the value of V_f becomes lower as shown in Table 2.

5. Molding of the autofrettaged discs

From the above investigations, it can be said that if the discs are molded by the semi-setting filament winding method directly, the circumferential crack occurs in the discs of which radius ratios are larger than some limit value. This is an obstacle to designing of the fiber reinforced discs freely. As the limit of the moldable radius ratio γ^* is independent of the inner radius, the discs with arbitrary radius ratios can be molded, however, by combining discs with various inner radii. Furthermore, the autofrettaged effect can be induced by utilizing the molding shrinkage of discs in combining discs with various inner radii.

Fig.25 Anisotropic thermal expansion

Fig.26 Layered disc

The manufacturing method of this autofrettaged discs with arbitrary radius ratios is as follows. Firstly, the inner

Table 2

V_f	molding condition			$-(\Delta_{22}' - \Delta_{11}') \times 10^{-6}$	$-(\Delta_{22} - \Delta_{11}) 10^{-6}$	K_1
	gelation furnace temp.	molding temp.	winding speed			
0.55	30 °C	80 °C	0.8~0.5 m/min	2423	1745	0.72
0.43	100 °C	80 °C	0.5 m/min	3090	955	0.31
0.38	160 °C	80 °C	0.41 m/min	3361	910	0.27

disc (I) of which radius ratio is less than the limit of the moldable radius ratio, γ^* , is molded; then, after perfect setting of this disk, the outer layer disc (II) is wound out of it and this outer layer is perfectly set (Fig.26). To generate the autofrettaged effect the inner radius of the outer layer disc (II) must shrink when the outer layer disc sets. Figure 27 shows the calculated results of

Fig.27 Free distortion of inner radius of molded discs

free distortion of inner radius of outer layer disc by using Eq.(8) for $V_f=0.55$, 0.43 and 0.38. It is seen that for the disc with low radius ratio the inner radius shrinks but for the disc with larger radius ratio than a certain value which is called "limit of layerable radius ratio" γ^{**} , the inner radius of the disc enlarges. Accordingly the autofrettaged effect can be generated by layering discs with lower radius ratio than γ^{**} . In the similar way by layering discs with lower radius ratios than γ^{**} successively, discs with arbitrary radius ratio can be molded without crack. Furthermore, by the autofrettaged effect, the radial tensile residual stresses are reduced so that a high strength rotor disc can be obtained by these method. Figure 28 shows one example of an autofrettaged disc molded by means of the above method.

6. Conclusions

In this paper, we investigate a funda-

mental problem of the application of fiber reinforced plastics to high speed rotor discs. A filament winding machine equipped with a control device of fiber volume fractions is produced. Uniform thickness discs can be molded by this machine. In the discs molded by this method, circumferential cracks occur when the radius ratio is larger than a certain value. It is found that the cause of these cracks is the residual stresses due to the anisotropic shrinkage in the setting and cooling process of the composite. This anisotropic shrinkage consists of anisotropic setting shrinkage and anisotropic thermal shrinkage when cooled from molding temperature to room temperature. The amounts of these shrinkages are theoretically analysed by using a combined cylinder model. The results agree well with experimental results. It becomes possible to predict the radius ratio of the disc in which the crack does not occur. Here is proposed a molding method of autofrettaged discs with arbitrary radius ratio by means of shrinkage. There exist good prospects of improving the rotational strength by use of the fiber reinforced composite.

Fig.28 Autofrettaged disc (five layers, $r_0=60$ mm)

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