Cutting Force Analysis of Wood (1st Report).

Orthogonal Cutting accompanied with Split ahead of Tool)

By Osamu DOI**, and Masao TOKYAMA***

The mechanism of chip formation in wood cutting differs essentially from that in isotropic materials.

The authors propose a calculating formula of cutting forces in orthotropic material of wood considering the typical split ahead of tool in orthogonal wood cutting. The authors adopt a simplified model to make clear the basic cutting mechanism and introduce an approximate calculating formula by extending the theory of beams on elastic foundation and that of linear fracture mechanics.

The calculating formula is expressed as a function of elastic modulus of wood species, chip thickness, rake angle, tool width and frictional coefficient.

The calculated values from the authors’ formula agree well with the experimental ones with respect to the variations in wood species, chip thickness, rake angle and tool width.

1. Introduction

Voigt and McKenzie have suggested the empirical formulas for wood cutting force, but the characteristics of wood cutting such as split failure are not represented exactly and their formulas are not appropriate for practical use.

The authors have discussed experimentally the characteristics of mechanism and cutting force in wood cutting from various viewpoints considering the intense anisotropy of wood(1)(2). In this paper, the authors propose a theoretical formula to calculate the cutting force considering split failure as a representative phenomenon in orthogonal cutting of I type as shown in Fig.9 on the basis of experimental results obtained up to this time.

It is difficult to analyze exactly by applying the elastic-plastic theory for the reasons that wood is a locally non-homogenous anisotropic material and that there is not a perfect elastic range, so that the cutting mechanism and cutting force are analyzed approximately for a simplified model.

On the case of I type cutting in several models devised by the authors, they apply the theory of beams on elastic foundation to chip formation and that of linear fracture mechanics to split ahead of tool respectively, extending both theories for isotropic material to orthotropic one. Moreover the ploughing length is obtained from the equilibrium of cutting energy and the balance of moment at the tip of split failure just before and after split formation. Then the calculating formula of cutting force is introduced, and is proved to be of practical use sufficiently by confirming the agreement of the theoretical values with experimental ones.

2. Combinations of cutting direction and depth direction

Formerly the cutting directions in wood cutting have been divided into perpendicular direction and parallel one to the grain in each cross, radial and tangential section of a lumber, but this method is still insufficient to represent the characteristics of wood cutting exactly.

As the cutting mechanism and cutting force in wood working vary considerably with combinations of cutting direction and depth one, the authors treat a lumber as a polar anisotropic cylinder generally and the cutting direction and depth one are represented by relating with three geometrical axes (radial \(r\), tangential \(\theta\), longitudinal \(z\)) of a lumber cylinder, furthermore, the sign + or - is employed to distinguish the direction of wood growth for \(r\) and \(z\).

Sixteen combinations are considered as shown in Table 1. Fig.1 illustrates the combinations of the cutting and depth directions according to the authors’ classification method.

<table>
<thead>
<tr>
<th>Cutting direction</th>
<th>Radial direction</th>
<th>Tangential direction</th>
<th>Longitudinal direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial direction</td>
<td>(Y_x) (Y_x) (Y_x) (Y_x) (Y_x)</td>
<td>(\theta) (\theta) (\theta) (\theta) (\theta)</td>
<td>(Z_x) (Z_x) (Z_x) (Z_x) (Z_x)</td>
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<tr>
<td>Tangential direction</td>
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<tr>
<td>Longitudinal direction</td>
<td>(Z_x) (Z_x) (Z_x) (Z_x) (Z_x)</td>
<td>(\theta) (\theta) (\theta) (\theta) (\theta)</td>
<td>(Z_x) (Z_x) (Z_x) (Z_x) (Z_x)</td>
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Table 1 Combinations of cutting directions and depth directions

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Wood is composed of longitudinal fiber cells and the mechanism of wood cutting differs from that of a homogeneous isotropic material owing to the annual rings, the spring wood, the summer wood etc. The variation of cutting mechanism owing to the difference of the cutting and depth directions in the authors' classification is also remarkable. One of the most representative cutting phenomena is a large split ahead of tool that is not observed in metal cutting. The authors carried out an experiment in which the cutting force and cutting energy of wood (White fir, Maple, Lauen) were measured by means of lathe, and discussed the relation between the cutting force and the chip forming process recorded synchronously on a film of high speed camera.

A typical example of the cutting force curves involving the split ahead of tool in the case of the \((z, \theta)\) cutting is qualitatively shown in Fig. 2. In the figure, the cutting force curve has a certain cycle. The outline of the relation between the cutting force curve and the chip forming process shown in Fig. 2 is as follows. At the point (1), the tool first engages the workpiece and the cellular tissue of wood is locally compressed on the tool face. When the tool arrives at the point (2), the split failure ahead of tool spreads rapidly close to the point (3) in the direction of the tool movement and the maximum cutting force appears just before the split occurs. Then the cutting force drops quickly just after the split occurs and diminishes gradually with the tool movement through the point (3). The chip fails in bending at the point (4) and the tool encounters again the beginning point of the next occurrence of failure to repeat a similar cycle. The authors have confirmed that the split failures arise at a regular interval as shown in the cutting force curve in Fig. 2. This interval changes with variation of wood species, chip thickness, rake angle; and the length of split failure is nearly equal to the cutting length between the points (2) and (4).

On the basis of the results described above, the authors adopt the workpiece of I type as an orthogonal cutting model to analyze the phenomenon of split failure on the cutting and depth directions \(x, \theta\), \(z\). Where a split tends to occur.

As chip is built up by the split failure in any direction above, the deformation in the part to be a chip before the split occurrence is considered as a bending problem of a semi-infinite beam having thickness \(t\) (chip thickness) and width \(b\) (workpiece width) on elastic foundation, and the workpiece as an orthotropic material. (Fig. 3)

The force \(P_x\) against the workpiece acts on the tool face at a distance \(x\) from the tool edge in the vertical direction as shown in Fig. 3. Employing the coordinate axis \(x\) in the direction of tool movement and \(y\) in the upward direction normally, the component \(P_x\) of \(P\) in the \(y\) direction lifts the chip onto the tool face and the component \(P_x\) in the \(x\) direction gives the moment \(M_x = tP_x / 2\) at the end of chip as a cantilever. The deflection \(\delta_x\) at the end of the beam is the sum of deflections by \(P_x\) and \(M_x\). Furthermore, tool moves forward and when the ploughing length \(\alpha\) (initial crack) reaches some value by the wedge action, a split of the length \(l\) occurs ahead of tool, and then \(P_x\) gives the bending moment to the cantilever of the chip length \((\alpha + l)\).

A certain amount of the elastic bending strain energy stored after the ploughing of tool just before the split occurrence is consumed with the split occurrence and the remaining energy is the elastic strain energy in bending the cantilever of length \((\alpha + l)\) to maintain the deflection \(\delta\) at the instant the split occurs. Assuming that the extending speed of split is extremely high and that the force \(P_x\) does no work during the split extension, furthermore the bending moment due to \(P_x\) is equal to the moment due to crack stress \(\sigma\) at the end of the split length \((\alpha + l)\) on the \(x\) axis, the relation between the ploughing length \(\alpha\) and the split length \(l\) is introduced. The value \(\alpha\) can be obtained by substituting this relation into the conditional equation of energy equilibrium described above.

The cutting forces \(P_x, P_y\) are introduced from the relation between \(\alpha\) and deflection \(\delta\) due to \(P_x\).

\[ (\alpha, \theta) \text{ cutting} \]

\[ \text{Cutting length} \]

\[ \text{Tool} \]

\[ \text{Fig.2 Example of relation between chip forming process and cutting force} \]

\[ \text{Fig.3 Cutting model and cutting force} \]
4. Analysis of cutting force

To analyze the cutting force according to the model, the following assumptions are considered.

1) Workpiece against the tool edge is a locally homogeneous orthotropic material.

2) Neglecting the local plastic deformation in the ploughing region, the chip formation caused by the tool ploughing is an elastic deformation spread out by the wedge action and the relation between stress and strain in the region ahead of an initial crack obeys Hooke's law.

3) The tool edge is sharp without dullness and the friction between workpiece and flank of tool is not taken into account.

4) The effect of the cutting speed on the cutting force is ignored and the chip forming is static.

First, the authors introduce the elastic bending strain energy of a chip moved onto the tool face by the tool ploughing just before the chip formation. If the beam is deflected by the component \( P_x \) of \( P \) and the deflection angle at the working point becomes \( \phi \) by the moment \( M_2 = t \cdot E_x / 2 \) due to the parallel component \( P_y \), the elastic bending strain energy is

\[
U = \frac{1}{2} (P_x^2 + P_y^2 M_2) \quad (1)
\]

When \( P_y \) acts on the end of a semi-infinite beam of thickness \( t \) (chip thickness) on an elastic foundation having linear orthotropy as shown in Fig.4, the deflection \( \delta \) at an arbitrary point \( x \) is given from references (2),(5) as:

\[
\delta = \frac{2P_x^2}{E_x t} e^{-\alpha x} \quad (2)
\]

\( E_x \) : modulus of the foundation kg/m²

\( E_x \) : modulus of elasticity in the \( x \) direction kg/m²

I: moment of inertia of cross section of the beam mm⁴

When \( M_2 = t \cdot E_x / 2 \) acts on the end of a semi-infinite beam on an elastic foundation, the deflection \( \delta \), deflection angle \( \phi \) and bending moment \( M_2 \) at an arbitrary point \( x \) are (Fig. 4)

\[
\delta = \frac{2P_x^2}{E_x t} e^{-\alpha x} (\cos \alpha x - \sin \alpha x) \quad (3)
\]

\[
\phi = \frac{4M_2}{E_x I} e^{-\alpha x} \quad \frac{M_2}{E_x I} = \frac{M_2}{t \cdot E_x / 2} \quad (3)
\]

\( \delta \), \( \phi \) are obtained by putting \( x = 0 \) in Eqs. (2),(3) and substituting them into Eq. (1).

\[
U = \frac{P_x^2}{2} + \frac{P_y^2 M_2}{2} \quad (4)
\]

Fig.4 Bending of a beam on an elastic foundation

Considering orthotropy for the theoretical solution of the modulus of foundation \( k \) obtained by Biot, M. K.,

\[
h = 0.710 \left( \frac{E_x (b/2)^{\frac{1}{2}}}{E_y} \right) \quad (5)
\]

\( b \) : workpiece width mm

When the elastic bending strain energy in Eq. (4) reaches a certain value, the split ahead of tool occurs from the initial crack with ploughing length \( \alpha \),

As the application of Irwin's fracture mechanics for the split failure of wood done by Porter, A.W. and Wu, E.M. has been confirmed to be of practical use, the authors introduce the releasing strain energy when the split extends from the tool edge to the length \( \alpha \) by referring to this conception.

The deformation of the split failure owing to the initial crack is classified into an opening mode and an edge sliding mode (Fig.5). Only the necessary equations of stress and displacement in the opening mode are shown as follows:

\[
\sigma_y = \sqrt{2\alpha E_y \sin \left( \frac{\alpha}{2} \right) + \left( 1 + \nu \right) \cos \left( \frac{\alpha}{2} \right) \left( \frac{1}{2} \right)} \quad (6)
\]

\[
\tau_{xy} = \sqrt{2\alpha E_x \sin \left( \frac{\alpha}{2} \right) + \left( 1 + \nu \right) \cos \left( \frac{\alpha}{2} \right) \left( \frac{1}{2} \right)} \quad (7)
\]

Where, \( E_x \) : modulus of elasticity kg/m²

\( \nu \) : Poisson's ratio

\( K_1 \) : stress intensity factor kg/m²

Similarly, stress and displacement in the edge sliding mode are

\[
\tau_{xy} = \sqrt{2\alpha E_x \sin \left( \frac{\alpha}{2} \right) + \left( 1 + \nu \right) \cos \left( \frac{\alpha}{2} \right) \left( \frac{1}{2} \right)} \quad (7)
\]

Where, \( K_1 \) : stress intensity factor kg/m²

As the split extension is extremely rapid, the forces \( P_x, P_y \) do not work, and the releasing strain energy of a chip due to the split occurrence is calculated as the work done necessary to restore from the state of the split length \( \alpha + \beta \) to the original state of initial crack \( \alpha \).

Therefore the strain energies of the opening mode and of the sliding one are summed up as follows:

\[
U = \frac{1}{2} \int \left( \sigma_y \phi + \tau_{xy} \right) d\alpha \quad (8)
\]

\( \sigma_y, \nu \) in the above equation represent the displacements in the \( x \) and \( y \) directions of an arbitrary point on the split face produced by vanishing tensile stress \( \sigma_x \) and shearing stress \( \tau_{xy} \) on the \( x \) axis.

Applying the theory of the opening mode to the authors' model, the stress \( \sigma_y \) on the split line (\( x \) axis) before the split extension in Fig.5 is given by putting \( \gamma = 0, \rho = x \) in Eq. (6) and the displacement \( \gamma \) after the split extension is obtained by substituting \( \gamma = \pi, \quad \rho = \alpha - x \) into Eq. (6), both considering the anisotropy of them.

Fig.5 Ploughing of tool
\[
\sigma_v = \frac{K_t(a)}{\sqrt{2}\pi}\frac{1}{r' - y}
\]
(9)

In the same manner, the shear stress \(\tau_{xy}\) and the displacement \(u\) in the edge sliding mode are obtained from Eq. (7).

\[
\tau_{xy} = \frac{K_t(a)}{\sqrt{2}\pi} \sqrt{r' - y} \frac{K_{s+1}}{K_s} \frac{1}{E_s}
\]
(10)

Therefore, from Eq. (8),

\[
U_1 = \frac{1}{2} \left( \frac{K_t(a)K_{s+1}}{K_s} \frac{1}{E_s} - \frac{K_t(a)}{\sqrt{2}\pi} \frac{1}{E_s} \right)
\]
(11)

If the ploughing length is small in comparison with the chip thickness \(t\) and the ploughing face is not vertical to the \(x\) axis in the actual chip forming process as shown in the model of Fig. 6, the stress intensity factors \(K_s\), \(K_t\) of a semi-infinite crack in Fig. 6 may be adopted approximately as:

\[
K_s = \frac{P_a}{\sqrt{2\pi}t}
\]
(12)

Applying Eq. (12) to the model,

\[
K_t(a) = \frac{P_a}{\sqrt{2\pi}t} \sqrt{t(a+1) + \frac{P_a}{\sqrt{2\pi}t} (a+1)}
\]
(13)

Where \(P_a\) and \(P_m\) in the above equation are the forces that give moments to the chip of the split length \((a+1)\) and cause the deflections \(\delta_s, \delta_t\) at the end of the chip.

The deflection \(\delta_s\) of the beam on an elastic foundation due to \(P_a\) at the instant of the split occurrence is obtained by putting \(x=0\) in Eq. (12), and this deflection \(\delta_s\) is equal to the maximum deflection of a cantilever having a length \((a+1)\) after the split extension, so that the acting force \(P_m\) is:

\[
P_m = \frac{6P_aE_2D}{(a+1)^3}\]
(14)

Similarly, as the deflection \(\delta_t\) due to \(P_m = \frac{x^2R_2}{2}\) before the split extension is equal to that due to the moment as a cantilever after the split extension, the acting force \(P_m\) is:

\[
P_m = \frac{6P_aE_2D}{(a+1)^3}\]
(15)

Substituting Eqs. (14), (15) into Eq. (13),

\[
K_t(a) = \frac{P_a}{\sqrt{2\pi}t} \sqrt{t(a+1) + \frac{6P_aE_2D}{(a+1)^3}}
\]
(16)

Then, putting Eq. (16) into Eq. (11),

\[
U_1 = \frac{1}{2\sqrt{2\pi}t} \left( \frac{6P_aE_2D}{(a+1)^3} \right) \frac{3P_a^2}{2P_m^2} \frac{1}{E_2}
\]
(17)

The elastic bending strain energy \(U_t\) stored in the cantilever of \((a+1)\) by the acting forces \(P_m\) is expressed as the sum of components due to \(P_m\) and \(P_m\).

\[
U_m = \frac{1}{2E_1} \int_0^t (P_m + P_m)^2 dx
\]
(18)

Substituting Eqs. (14), (15) into the above equation,

\[
U_m = \frac{2E_1}{(a+1)^3} \left( \frac{P_a^2}{4P_m^2} + \frac{P_m^2}{4P_m^2} \right)
\]
(19)

Accordingly, from the relation of the strain energy \(U_1 = U_1 + U_2\) and Eqs. (4), (17), (19),

\[
P_m = \frac{E_1}{2E_2} \left( \frac{P_a^2}{4P_m^2} + \frac{P_m^2}{4P_m^2} \right)
\]
(20)

Assuming that the split ahead of tool extends to the point where the moments due to \(P_a\) and the crack stress \(\sigma_s\) on the x axis balance, the relation between \(a\) and \(t\) is reduced from Fig. 3.

\[
P_a(a+1) = \int_0^t (t-x)^2 \sigma_s dx
\]
(21)

From Eqs. (9), (13),

\[
\sigma_s = \frac{K_t(a)}{\sqrt{2\pi}t}, \quad K_t(a) = \frac{P_a}{\sqrt{2\pi}t}
\]
(22)

Substituting the above two equations into Eq. (21),

\[
9\pi a^2(s+1)^2 = 4t
\]
(23)

A real root of Eq. (22) is

\[
t = 24.08 a
\]
(24)

Substituting \(t\) of Eq. (22), Eqs. (5), (23), \(P_a = E_2\sin \alpha\) and \(P_m = E_2\cos \alpha\) into Eq. (20),

\[
\sigma_s = 1.5 \times 10^{-3} t \left( \frac{E_2}{E_1} \right)^{1/2}
\]
(25)

\[
\sin \alpha = \frac{1.4 \times 10^{-3} t \left( \frac{E_2}{E_1} \right)^{1/2}}{2 \tan \alpha + 1.39 \left( \frac{E_2}{E_1} \right)^{1/2}}
\]
(26)

Eq. (24) is a cubic equation with respect to \(a\) and is a function of chip thickness \(t\) and the moduli of elasticity \(E_2, E_1\). A real root of Eq. (24) is as follows:

\[
\alpha = \left( \frac{A_1 \cos \alpha + t}{A_2(\cos \alpha - A_3 \sin \alpha)^{1/3}} \right)^{1/3}
\]
(27)

Where

\[
A_1 = 2 \times 10^{-2} \tan \alpha + 1.39 \left( \frac{E_2}{E_1} \right)^{1/2}
\]
(28)

\[
A_2 = \frac{1.40 E_2}{E_1} \left( \tan \alpha + 0.38 \left( \frac{E_2}{E_1} \right)^{1/2} \right)
\]
(29)

\[
A_3 = \frac{1.956}{E_2} \left( \tan \alpha + 0.38 \left( \frac{E_2}{E_1} \right)^{1/2} \right)
\]
(30)

Then so as to obtain \(P_a\) and \(P_m\), the relation between \(a\) and \(\delta_s\) may be written approximately as follows (Fig. 3).

\[
\delta_s = \frac{t}{2a(\cos \alpha + \mu \sin \alpha)}
\]
(31)

Therefore

\[
P_a = \frac{t}{2a(\cos \alpha + \mu \sin \alpha)}
\]
(32)

Where \(P_a = P_0\) and \(P_m\) are the frictional force between tool and chip acts in actual cutting as shown in Fig. 3, the sum of Eq. (26) and the x, y components of the frictional force are measured as the parallel cutting force \(P_x\) and the normal cutting force \(P_y\). The sign of the frictional force in the direction in Fig. 3 is negative, so that \(P_x\) and \(P_y\) are:

\[
P_x = P_0(\cos \alpha + \mu \sin \alpha)
\]
(33)

\[
P_y = P_0(\sin \alpha - \mu \cos \alpha)
\]
(34)
\( \mu \): coefficient of friction between tool face and chip

Substituting \( \alpha \) of Eq. (2) and Eq. (5) into Eq. (26), Eq. (27) is reformulated.

\[
\begin{align*}
P_x &= \frac{ka}{2z} \cot \alpha (\cos \alpha + \mu \sin \alpha) \\
&= -0.73aE_s \cot \alpha (\cos \alpha + \mu \sin \alpha) \\
P_y &= \frac{ka}{2z} \cot \alpha (\sin \alpha + \mu \cos \alpha) \\
&= -0.73aE_s \cot \alpha (\sin \alpha + \mu \cos \alpha)
\end{align*}
\]

(28)

If the chip thickness \( t \), rake angle \( \alpha \), workpiece width \( b \) and moduli of elasticity \( E_x, E_y \) are given, \( \alpha \) can be obtained from Eq. (25) and then \( P_x, P_y \) are evaluated from Eq. (28).

5. Experiment

5.1 Apparatus and procedure of experiment

A milling machine of knee type is used for the experimental apparatus as shown in Fig. 7. The measuring device of cutting force (parallel cutting force and normal cutting force) equipped with a tool is fixed on the arbor support of the milling machine, and the workpiece is fastened with a vice fixed on the table and is moved by the longitudinal feed of the table with a slow cutting speed of 0.2 m/min. The cutting force measurement is done by combining the bending strain of 4 strain gauges stuck on both sides of a thin circular plate clamped at the outside edge as shown in Fig. 8. Cutting forces are recorded on an electromagnetic oscillograph through a dynamic strain meter. The maximum forces just before the first split occurs as the result of the tool ploughing are adopted as the parallel and normal cutting forces \( P_x, P_y \).

5.2 Workpieces and cutting conditions

Workpieces are selected from the sapwood in the north part of Glehn's spruce, Painted maple and White fir of diameters from 40 to 50 cm. The shape of workpieces is I type as illustrated in Fig. 9 and the moisture content of them is of almost dry condition as shown in Table 2.

The test pieces for measurement of elasticity are taken out from the part adjacent to the workpiece, and the moduli of elasticity are obtained by compression tests in \( r, \theta, z \) directions according to JIS.

Table 2 Properties of workpiece

<table>
<thead>
<tr>
<th>Species</th>
<th>Moisture content</th>
<th>( E_x )</th>
<th>( E_y )</th>
<th>( E_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>White fir</td>
<td>8.7</td>
<td>62.2</td>
<td>42.2</td>
<td>1030</td>
</tr>
<tr>
<td>Glehn's spruce</td>
<td>15.0</td>
<td>85.8</td>
<td>48.2</td>
<td>1350</td>
</tr>
<tr>
<td>Painted maple</td>
<td>10.0</td>
<td>135.0</td>
<td>105.0</td>
<td>1140</td>
</tr>
</tbody>
</table>

I. Strain gages for parallel cutting force
II. Strain gages for normal cutting force

Fig. 8 Cutting force indicator

The tool used in the experiment is of 355C steel hardened and tempered, the clearance angle 10 degrees and the rake angle over the range of 5, 25, 45, 60 degrees.

6. Theoretical and experimental results

6.1 Chip thickness

Figs. 10 to 13 show the comparison of the experimental cutting forces and the theoretical ones in the \( r, \theta, z \) directions. Theoretical cuttings of White fir in which the split ahead of tool occurs. The points in the figures show the experimental values of \( F_x, F_y \) and the full lines are theoretical values from Eqs. (25), (28), where \( E_x, E_y \) in the equations are found from Tab. 2 corresponding to the cutting and depth directions, and the coefficient of friction between workpiece and tool face is adopted as \( \mu = 0.17 \) in the calculation. \( F_x, F_y \) are linear functions of chip thickness \( t \) as illustrated in Eqs. (25), (28), so that the theoretical values of \( F_x, F_y \) increase linearly with chip thickness.

The experimental values agree approximately with the theoretical values, but the experimental parallel cutting forces are higher somewhat than the theoretical ones and the experimental normal cutting forces tend to be lower than the theoretical ones.

Fig. 9 Shape of workpiece I type

Cutting direction
It seems to be one of the reasons that the friction force between tool flank and workpiece is not taken into account.

The dotted lines and points in Figs. 10, 13 show respectively the theoretical values of the split length obtained from Eqs. (23), (25) and the experimental values measured by the observation using a high speed camera.

In the (r, α) cutting, the measured values agree approximately with the theoretical ones and these values increase with the chip thickness, but in the (r, α) cutting, the measured values are larger than the theoretical ones and shows a remarkable scattering.

In the actual chip forming process, the split failure ahead of tool is extended rapidly by the tool ploughing. As the split extends still more while the chip moves up on the tool face, the values of points may appear to be somewhat larger than the true values.

Figs. 14 to 16 show the results of Glehn's spruce cutting. Glehn's spruce is a needle-leaf tree like white fir, but both annual rings and fiber cells differ in size and the modulus of elasticity of Glehn's spruce is also larger about 30 percent than that of White fir as shown in Tab.2.
Both the experimental and the theoretical cutting forces are similar to those of White fir, but the moduli of elasticity $E_t$, $E_s$ (Tab.2) of Painted maple of a broad-leaf tree take about two times higher values than White fir. So the cutting forces (Figs.17—20) are also larger than that of White fir in any direction. The experimental values agree well with the theoretical ones.

6.2 Rake angle

Figs.21, 22 show the results of White fir of chip thickness $t=1$ mm and tool width $B=4$ mm, and the rake angle is set at 5, 25, 45, 60 degrees. The experimental values of the normal cutting force agree with the theoretical ones in both the $(r_n,e)$ and the $(x,e)$ cuttings. When the rake angle approaches to 0 degree, the calculated values of parallel cutting force become larger than the experimental ones.

As band saw teeth and circular saw teeth used widely in wood working have a rake angle range of 10 to 40 degrees and the rake angle of wood working lathe tool is in a range of 10 to 30 degrees, therefore the authors calculating equation can cover a sufficient range of practical use.

6.3 Tool width

The results of White fir cutting are shown in Fig.23, where tool width (work-piece width) is changed as 2, 4, 8, 16 mm and the other cutting conditions are fixed. These values increase linearly with the tool width, but the experimental values are somewhat higher than the theoretical ones.

Considering that the shape of annual rings, the size of fiber cell and the amount of moisture content are different generally depending on the part of a lumber and that the experimental values scatter in the same cutting and depth directions, the authors calculating equation seem to be of practical use sufficiently.

![Fig.18 Effect of chip thickness on cutting force](image_url1)

![Fig.19 Effect of chip thickness on cutting force](image_url2)

![Fig.20 Effect of chip thickness on cutting force](image_url3)

![Fig.21 Effect of rake angle on cutting force](image_url4)

![Fig.22 Effect of rake angle on cutting force](image_url5)
7. Conclusions

In the authors’ theory described above dealing with the cutting force as a resistance to the split failure, the cutting force is derived from the energy equilibrium at the instant just before and after split occurrence and from the moment balance at the tip of split failure. Because the authors analyze the resistance to the split failure with elastic theory, their analysis has no relation with mechanical properties such as strength and hardness in the plastic region of workpiece. In practice, the plastic region in the vicinity of initial crack tip may have to be taken into account and the releasing energy may have to be corrected, Irwin has reported that if the plastic region in the tip (the microscopic circular zone) of initial crack is very small in comparison with the initial crack $a$, so that effect on the releasing energy can be ignored.

The main conclusions of this paper might be summarized as follows.

1) To make clear the basic cutting mechanism of wood, the authors proposed a simplified model of I type in orthogonal cutting of wood in the cutting directions in which a distinctive split failure occurs.

2) The authors introduced a theoretical formula to calculate the cutting forces in orthogonal wood cutting accompanied with split failure by analyzing with the theories of a beam on an elastic foundation and the linear fracture mechanics. The cutting force is represented as a function of the chip thickness, rake angle, workpiece width, modulus of elasticity of workpiece and the coefficient of friction. If these values are known, the cutting force can be obtained from their formula.

3) It is confirmed that the experimental values of cutting force agree well with the theoretical ones for the variations of chip thickness, rake angle, workpiece width and wood species to a sufficient extent for practical use.

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