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Aerodynamic Force Acting on a Circular Cylinder

with Tangential Injection of Air*

By Yoshimasa FURUYA** and Fumio YOSHINO***

An investigation was carried out on various characteristics of a circular cylinder with tangential injection of air immersed in a uniform flow. As a result, it was found that a separation bubble took place where the radius of curvature of the cylinder was discontinuously increased even if the wall itself was smoothly connected; that the velocity profile of mixing region of jet was similar and cosine-like up to downstream of separation-point; that, when the intensity of jet was weak, the lift (drag) coefficient was reduced (increased) by injection of air when the station of the slot was upstream of the laminar separation-point and that, when the station of the slot was downstream of the laminar separation-point, the control to fix the separation-point anywhere between the laminar separation-point and the station of the slot was impossible.

1. Introduction

It has long been known that plane jet flows along a wall of a circular cylinder when air is injected tangentially to the wall of a circular cylinder and this phenomenon is called Coanda effect. There are reported many investigations⁽¹⁾ on jet in the absence of main flow, while we can hardly find any investigation related to the velocity profile of jet in the presence of main flow. On the other hand, Dunham⁽²⁾ carried out an investigation on aerodynamic forces acting on a circular cylinder with tangential injection of air to use it as a high lift device. Also it was tried⁽³⁾ to increase lift-to-drag ratio of a circular cylinder as a high lift device. There, however, exist few researches on small c_μ at various stations of a slot on a cylinder. Our investigation has been carried out more systematically than ever on aerodynamic forces acting on a two-dimensional circular cylinder and on velocity profiles of mixing region of jet when the station of a slot and intensity of jet are varied at rather small Reynolds number. In our experiment, our great concern was in the two-dimensionality of the flow. We herein discuss the relation of aerodynamic forces on the cylinder to the various conditions of jet.

2. Nomenclature

R	: radius of the circular cylinder
h	: width of the slot
y	: radial distance from the wall of the cylinder
y_m	: y at the point where $u=u_m$
$y_{m/2}$	: y at the point where $u=(u_m+U)/2$
b	: thickness of the mixing region of jet
θ	: angle measured from the leading

	edge in the clock-wise direction (Fig.1(a))
φ	: angle measured from the slot in the clock-wise direction (Fig.1(a))
ψ	= $(\pi/180)\varphi$
θ_{sep} and φ_{sep}	: points of separation expressed in θ and φ , respectively
θ_{ls} and θ_{ts}	: points of laminar and turbulent separations, respectively, in the absence of jet
θ_s	: station of the slot (Fig. 1 (a))
θ_m	: angular position of the minimum c_{pe} from which 90 degrees are subtracted
θ_{st}	: front stagnation-point
α_i	: induced angle of incidence
u	: velocity in the direction of θ at the point y
u_m	: the maximum velocity in the cross section of jet
U	: velocity at the outer edge of the mixing region
U_∞	: tunnel speed
m	: velocity ratio = U/u_m
p_∞	: static pressure in the test section of the tunnel
p_s	: static pressure on the wall of the cylinder
p_w	: pressure of compressed air inside the cylinder
c_μ	: momentum coefficient = (momentum of jet at the exit of the slot) $\cdot h / (\rho U_\infty^2 R)$
R_{off}	: Reynolds number = $\{(p_w - p_\infty) R h / \rho \nu^2\}^{1/2}$
R_{on}	: Reynolds number = $2 U_\infty R / \nu$
c_L, c_{Ls} and c_D	: lift coefficients including, not including reaction of jet and drag coefficient, respectively
c_{L0} and c_{D0}	: two-dimensional lift (c_L) and drag (c_D) coefficients, respectively
c_{l0} and c_{d0}	: intensities of circulation and source, respectively
c_p	: pressure coefficient = $(p_s - p_\infty) / (1/2) \rho U_\infty^2$
c_{pb}	: base-pressure coefficient
c_{pc}	: pressure coefficient due to circulation and source (Fig.8 (a))
$c_p - c_{pc}$	(Fig.8 (a))
ν and ρ	: kinematic viscosity and density

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** Professor, Nagoya University, Furo-cho, Chigusa-ku, Nagoya.

*** Assistant Professor, Tottori University, Koyama, Tottori.

c and k : constants of Eq.(1)

3. Experimental apparatus and method

3.1 Experimental apparatus Fig.1(a) shows a cross section of the circular cylinder used as a test model. The outer diameter of the hollow cylinder is 100 mm, which a circular cylinder of 25 mm in diameter inscribes. Compressed air is injected through a slot with spanwise uniform width 0.55 ± 0.03 mm and flows along the wall of that small cylinder as a part of the nozzle. Four stays are located at regular spanwise intervals to assure the uniformity of the width of the slot. The cylinder can be revolved about its axis to set the slot to an arbitrary station. The outer wall of the cylinder is chromium-plated to obtain a mirror-like smooth surface. The cylinder is 404 mm in axial length (span) and has 35 static-pressure taps distributed on the periphery at the center of the span. Five pressure taps are located at approximately regular spanwise intervals for measuring the static pressure inside the cylinder.

Fig.1(b) shows the cylinder mounted in the test section of the tunnel. The cylinder has end-plates at both ends, the dimensions of which are 62 mm thick, 362 mm high and 726 mm long, and the end-plates are fixed to the hollow struts through which the compressed air is supplied to the inside of the cylinder.

(a) Cross section of the cylinder

(b) Assembly drawing of the cylinder
Fig. 1

The tunnel⁽⁴⁾ is one of the type of Goettingen and its test section has the dimensions of 1 m in width, 0.7 m in height and 1.5 m in length. The contraction ratio of the nozzle is 6 and the maximum tunnel speed is about 52 m/s.

A pitot-tube, used to measure the velocity profile of jet, is a circular tube of diameter of 1 mm whose tip was changed into a rectangular form. This pitot-tube is fixed to a micrometer on a circular-arc board as a prop which is concentric with the cylinder and along which the pitot-tube together with the micrometer can be moved in a circumferential direction of the cylinder. The cross section of the circular-arc board has the shape of an airfoil.

3.2 Experimental method The measurement was made mostly at rather small Reynolds number, $Re_m = 3.2 \times 10^5$. Some measurements were also made at $Re_m = 2.6 \times 10^5$ and 1.9×10^5 to check the influence of Reynolds number on the lift coefficient, drag coefficient etc. According to the authors' preliminary experiment and Tanner's one⁽⁵⁾, the flow becomes less two-dimensional when it is turbulent. This is why the measurement was made at rather small Reynolds number. Accordingly, the main parameters are the momentum coefficient of jet c_μ and the station of the slot θ_j . To determine the value of c_μ , the velocity of jet at the exit of the slot was calculated from pressure and temperature of compressed air inside the cylinder.

A measurement of circumferential distribution of static pressures was made at six stations of the slot, namely $\theta_j = -40^\circ, -20^\circ, 0^\circ, 10^\circ, 20^\circ$ and 30° . The value of c_μ was varied from 0 to approximately 0.46 at each station of the slot.

The measurements of velocity profiles were made in two cases, i.e., in the absence of and in the presence of main flow. These measurements were carried out mainly at many circumferential stations in the cross section at the center of the span and at several circumferential positions in cross sections at a few spanwise stations to check the two-dimensionality of jet. The velocity profile of jet alone in the absence of main flow was measured at $Re_j = 1.61 \times 10^4, 3.49 \times 10^4$ and 8.11×10^4 while the slot was stationed at $\theta_j = 0^\circ$. In the case of jet in the presence of main flow, the measurement of velocity was made at $c_\mu = 0.085, 0.313$ and 0.544 while $\theta_j = 0^\circ$. The velocity of the flow, in this case, was calculated by using the static-pressure on the wall of the cylinder. When the velocity was high, the error due to compressibility was estimated⁽⁶⁾ and accordingly the velocity was corrected. The point of separation of flow was determined from the static-pressure distribution on the wall of the cylinder in both cases of jet in the presence of and in the absence of main flow.

4. Experimental results

4.1 Characteristics of jet

4.1.1 The jet in the absence of main flow In this case, the thickness of boundary layer and jet and the maximum velocity in a cross section of jet at a station are all approximately equal to the thicknesses and the maximum velocity, respectively, at the other station on the same generatrix of the cylinder regardless of Reynolds number. The velocity profiles are similar to each other at any station on the cylinder. In other words, the jet is two-dimensional. Fig.2 indicates a circumferential distribution of the static pressures in the cross section at the center of the span. The solid line in the figure is an experimental curve by Newman⁽⁷⁾. The point of separation of jet in our experiment is seen to be at $\varphi_{sep} \approx 190^\circ$ from the pressure distribution. The experimental values of $y_{m/2}/R\psi$ and $y_{m/2}/R$ are shown in Fig.3. The relation of $y_{m/2}/R\psi$ to $y_{m/2}/R$ for $x_{m/2}/R > 0.08$ ($\varphi > 26.6^\circ$) is approximately linear and is expressed as follows;

$$\frac{y_{m/2}}{R\psi} = c \left(1 + k \frac{y_{m/2}}{R} \right) \quad (1)$$

where $c = 0.115$ and $k = 2.4$. The ratio of the thickness of boundary layer to that of jet is determined from the velocity profiles to give an approximately constant value, $y_m/y_{m/2} \approx 0.2$, on the periphery.

Fig. 2 Static-pressure distributions on the wall in the flow direction

Fig. 3 Variation of the thickness of jet, $y_{m/2}$, in the flow direction

4.1.2 The jet in the presence of main flow In this case, the velocity is seen to be reduced and eventually reversed at a place near the end-plates in comparison with that at a place far from those. The jet is nearly two-dimensional in the region near the center of the span except in the region near the point of separation of flow. The velocity of the mixing region of jet shows a similar profile in almost the whole span though the profiles near the end-plates are a little deformed.

Fig. 4 Velocity profiles of mixing region at various circumferential stations

Fig.4 shows the velocity profiles of the mixing region of jet at various circumferential stations in the cross section at the center of the span in two cases, namely, at $c_\mu = 0.085$ and 0.544 . It is seen from the figure that the velocity profiles of the mixing region are nearly similar regardless of the circumferential stations and of the value of c_μ . Also the profile is seen to preserve its similarity even after the point of separation ($\varphi = 102.6 \sim 120.6$), which is obvious from the fact that $\varphi_{sep} = 96.6^\circ$ for $c_\mu = 0.544$. The broken and solid lines in the figure represent the curves according to Eqs.(2) and (3), respectively;

$$\frac{u - U}{U_m - U} = \text{sech}^2(0.88 \frac{y - y_m}{y_{m/2} - y_m}) \quad (2)$$

$$\frac{u - U}{U_m - U} = \frac{1}{2} (1 + \cos \pi \frac{y - y_m}{b}) \quad (3)$$

Equation (3) estimates a little too small velocity at large y but the curve of Eq.(3) better agrees with experimental profiles than that of Eq.(2). Therefore the velocity profile of the mixing region of jet in the presence of main flow can be represented by Eq. (3) of a cosine function.

Figs.5 (a) and (b) show the variations of y_m/R together with $y_{m/2}/R$ and m in the flow direction, respectively. The figure indicates that the value of m does not reach unity up to the point of separation of flow but approaches unity rapidly downstream of the separation point.

Fig. 5 Variation of characteristics of jet in the flow direction

4.2 Lift and drag coefficients Figs.6 (a) and (b) show c_L and c_D calculated from the pressure distributions, respectively, where c_μ is the abscissa and θ_j is a parameter. It is shown in the figure that c_L and c_D vary in a complicated manner. The behaviors of c_L and c_D seem to depend on whether the station of the slot is $\theta_j \leq -20^\circ$ or $\theta_j \geq 0^\circ$ when c_μ is small. In the case of $\theta_j \leq -20^\circ$, c_L (c_D) decreases (increases) as c_μ increases from 0, reaches the minimum (the local maximum) in the vicinity of $c_\mu \approx 0.02$ and increases (decreases) afterwards. As c_μ increases more, c_D maintains nearly a constant value of 0.34 when $\theta_j = -40^\circ$ but of 0.46 in the range of $c_\mu = 0.1 \sim 0.23$ and afterwards increases by degrees when $\theta_j = -20^\circ$ whereas c_L monotonously increases. In the case of $\theta_j \geq 0^\circ$, on the other

hand, as c_{μ} increases from 0, the larger θ_j is, at the higher rate c_L increases when c_{μ} is small but does less rapidly for larger than $c_{\mu} \approx 0.06$. c_D 's for $\theta_j = 0^\circ, 10^\circ, 20^\circ$ and 30° , however, increases after they attain the minima at $c_{\mu} \approx 0.07, 0.03, 0.013$ and 0.013 , respectively. The larger θ_j is, at the higher rate c_D monotonously increases as c_{μ} increases when c_{μ} is large. According to the results of measurement by means of a three-component balance, though they are not shown here, the manners in which c_L and c_D vary with c_{μ} at R_{on} 's smaller than 3.2×10^5 are the same as they were at $R_{on} = 3.2 \times 10^5$ whereas c_L and c_D are a little larger at R_{on} 's smaller than 3.2×10^5 . Fig. 7 shows the lift-to-drag ratio c_L/c_D calculated from the static-pressure distribution. The distinguished variations of c_L and c_D are seen when c_{μ} is small. The smaller θ_j is, the larger the maximum of c_L/c_D is and, for $\theta_j \geq 10^\circ$, the value of c_{μ} for c_L/c_D to attain the maximum decreases as θ_j increases.

4. 3 Static-pressure distribution and others
Figs. 8(a) and (b) show the circumferential distributions of the pressure coefficient c_p when the various values of c_{μ} are taken as a

Fig. 7 Relation of lift-to-drag ratio to c_{μ}

parameter for $\theta_j = -40^\circ$ and 30° , respectively. It is seen in both cases that there exists a large negative pressure Δc_p (which will be described later) at right downstream of the slot in case of $c_{\mu} \neq 0$ regardless of θ_j . As c_{μ} increases while θ_j is fixed to a constant, the absolute value of negative pressure becomes large and, at the same time, the point of the local minimum of c_p (described later) on the upper side of the cylinder (the side of the slot, i.e., $\theta = 0^\circ \sim 180^\circ$) a little moves rightward in the figures. When $c_{\mu} = 0$, the negative pressure on the upper side is large in the case of $\theta_j = -40^\circ$ (Fig. 8(a)) in comparison with the case of $\theta_j = 30^\circ$ (Fig. 8(b)). This implies that the boundary layer on the upper side is turbulent when $\theta_j = -40^\circ$ and that those on the upper and lower sides are both laminar when $\theta_j = 30^\circ$.

Fig. 9 shows the relation of the base pressure, c_{pb} , to c_{μ} when θ_j is a parameter. The manner in which c_{pb} varies with c_{μ} when it is small is like that of c_L (Fig. 6(a)). c_{pb} has a maximum of about 0.7. The points of separation of flow on the upper side, θ_{sep} , obtained from the static-pressure distributions, are plotted against c_{μ} in Fig. 10 where θ_j is a parameter. In this case too, the manner in which θ_{sep} varies with c_{μ} when it is small resembles that of c_L . Fig. 11 shows the front stagnation-

Fig. 8 Static-pressure distributions on the wall at various θ_j 's and c_{μ} 's

-point, θ_{st} , obtained from the static-pressure distribution. The manners of variations of θ_{st} and θ_{sep} are nearly symmetrical with each other with respect to the abscissa.

Fig. 9 Relation of base-pressure coefficient to c_μ

Fig. 10 Relation of the point of separation on the upper side to c_μ

Fig. 11 Relation of the front stagnation-point to c_μ

5. Discussion

5.1 Characteristics of jet

5.1.1 The effect of the structure of the slot on the thickness of jet. The feature of the slot in this experiment is that the radius of curvature of the wall discontinuously increases from 12.5 mm to 50 mm (Fig.1(a)). The static pressure on the wall too almost discontinuously rises due to this discontinuity of the wall. When it is assumed that the jet is inviscid and flows along the wall keeping its thickness equal to the width of the slot, the following relation holds;

$$\frac{P_\infty - P_s}{P_\infty - P_w} = 2 \frac{h}{R} + \left(\frac{h}{R} \right)^2 \quad (4)$$

The pressure coefficient expressed by this equation changes from the value 0.09 to 0.022 at the point of discontinuity in our cylinder because R jumps from 12.5 mm to 50 mm. The rapid pressure-rise in the vicinity of $\varphi = 0^\circ$ in Fig.2 supports the preceding theory. It is, however, thought due to the separation bubble such as occurs near the leading edge of an airfoil⁽⁹⁾ that the pressure measured once rises and thereafter falls. Since the existence of this kind of separation bubble seems to intensify the turbulence in the mixing region of jet, the jet may grow rather fast and separate from the wall comparatively early when the structure of our slot is made use of. There is Newman's experiment⁽⁷⁾ (the solid line in Fig.2) as an example that air is injected tangentially to the wall the curvature of which is constant. In our experiment we obtained $y_m/y_{m/2} \approx 0.2$, $\varphi_{sep} \approx 190^\circ$ and $c = 0.115$ and $k = 2.4$ in Eq.(1) while Newman⁽⁷⁾ obtained $y_m/y_{m/2} \approx 0.14$, $\varphi_{sep} = 220^\circ$ and $c = 0.11$ and $k = 1.5$. Since the discrepancy between h/R 's of our experiment and Newman's can be ignored, the difference between both experimental results is considered to come from the difference in the shape of the wall, namely from the presence of a separation bubble.

So far the discussion has been made on the case of jet in the absence of main flow, the effect of the thickness of the upper lip of the slot too as well as the curvature of the wall, however, should be taken into account in the case of jet in the presence of main flow. This effect, however, can be neglected when the ratio of the thickness of the upper lip to the width of the slot is as small as approximately 0.2 for our slot and moreover when the upper lip is tapered off toward its tip. Therefore a large negative pressure immediately downstream of the slot in Figs.8(a) and (b) and a separation bubble downstream of the point of discontinuity are considered to occur solely due to the fact that the radius of curvature of the wall discontinuously increases.

5.1.2 Comparison of the experiment with the theory on the thickness of jet. When the velocity profile of mixing region is similar (Fig. 4), the thickness of jet is dependent on the angular position ψ measured from the slot and the velocity ratio $m (= U/u_m)$ in the cross section of jet, from which the following differential equation⁽³⁾ is derived:

$$\frac{dY_{m/2}}{d\psi} = \frac{1+m}{1-ck\psi+m(1+ck\psi)} \frac{dY_m}{d\psi} - \frac{c(1-m)}{1-ck\psi+m(1+ck\psi)} (R + kY_{m/2}) \quad (5)$$

c and k in the above equation are the same as those in Eq.(1). The value of m should be 0.5 for $m \geq 0.5$ in this case.

In order to confirm Eq.(5), it was numerically integrated by means of an electronic computer by using the experimental values of y_m (Fig.5(a)) and $m = 0.5$. The initial value of $y_{m/2}$ was obtained as an intersection of the line of $\varphi = 0^\circ$ with the extension of the curve which smoothly connects the experimental values of $y_{m/2}$ (Fig.5(a)). The results of calculation are shown by thick solid lines and broken lines in Fig.5(a). The solid lines are results obtained by using our constants $c = 0.115$ and $k =$

= 2.4 while the broken lines are those by using Newman's constants $c = 0.11$ and $k = 1.5$. It is obvious from the figure that the results by either c and k fall within the range of experimental errors near the slot but that the results by our c and k better agree with the results of experiment than those by Newman's c and k near the point of separation of flow. Therefore the thickness of jet can be predicted from Eq.(5) by making use of the values $c = 0.115$ and $k = 2.4$ when the structure of our slot is used. On the other hand, $Y_{m/2}$ will be calculated by using $c = 0.11$ and $k = 1.5$ when air is injected tangentially to the wall of the cylinder whose curvature is constant.

5.2 Lift and drag coefficients

5.2.1 The optimum values of c_μ and θ_j As already stated in the chapter of experimental results, c_L and c_D vary with c_μ quite differently depending on whether $\theta_j \leq -20^\circ$ or $\theta_j \geq 0^\circ$. Therefore it is not necessarily good from the view-point of the lift-to-drag ratio c_L/c_D that the station of the slot is near the point of natural turbulent separation of flow θ_{ts} . Owing to this fact, the station of the slot should be suitably determined so as to answer the purpose when the airfoil like our circular cylinder is designed. In this case, Fig.6 and Fig.7 will be referred to.

The maxima of lift-to-drag ratios are all alike for $\theta_j \geq 0^\circ$ while the maximum of c_L/c_D and the value of c_μ at that maximum increase as θ_j increases for $\theta_j < 0^\circ$. Hence $\theta_j = -40^\circ$ is appropriate when the power to produce compressed air is sufficient. In this case, $c_L/c_D \approx 10$ and $c_L \approx 3.3$ are obtained at $c_\mu = 0.4$. $c_L/c_D > 10$ can be attained as c_μ is further increased and c_L/c_D will reach the maximum somewhere at $c_\mu > 0.4$. It must be kept in mind in this case that c_L/c_D has a minimum, which is small for $c_\mu < 0.1$ (Fig.7). On the other hand, $\theta_j = 30^\circ$ is suitable when high c_L is desired even at the cost of the value of c_L/c_D . In this case, $c_L = 3.67 \sim 7.2$ is attained for $c_\mu = 0.1 \sim 0.45$.

$\theta_j = 30^\circ$ is desirable to obtain large values of both c_L and c_L/c_D when the power is not sufficient. c_L/c_D attains the maximum 4.7 at $c_\mu = 0.065$ in this case. This value of c_L/c_D hardly changes up to $c_\mu = 0.25$. Also an appreciable lift coefficient ($c_L \approx 2.7$, $c_L/c_D \approx 3.4$) can be obtained by a little injection of air ($c_\mu = 0.01$) in the case of $\theta_j = 30^\circ$.

$\theta_j = 0^\circ \sim 10^\circ$ is suitable when it is desired for c_L/c_D not to appreciably vary over a wide range of c_μ . $c_L/c_D = 4.3 \sim 4.8$ and $c_L = 2.6 \sim 5.8$ are obtained for $c_\mu = 0.1 \sim 0.4$ when $\theta_j = 10^\circ$. The range over which c_L/c_D does not comparatively change its value is wider, since $c_\mu = 0.1 \sim 0.48$, when $\theta_j = 0^\circ$ than $\theta_j = 10^\circ$.

5.2.2 The effects of c_μ and θ_j on lift and drag coefficients

(i) The case of small c_μ Now, c_L (Fig.6(a)), c_D (Fig.6(b)) and c_L/c_D (Fig.7) make remarkable variation with c_μ when the latter is small and these phenomena are closely related to the station of the slot. This is because the phenomena depend on whether the station of the slot is upstream of the point of laminar separation $\theta_{ts} \approx 70^\circ$ when $c_\mu = 0$ (Fig.10) or downstream of it. As already stated, the boundary layer on our cylinder is laminar and $\theta_j \leq -20^\circ$ and $\theta_j \geq 0^\circ$ correspond to the stations of the slot upstream of and downstream of θ_{ts} , respec-

tively. Consequently we can classify the cases into two, i.e., (a) $\theta_j \leq -20^\circ$ and (b) $\theta_j \geq 0^\circ$ as before.

(a) In the case of $\theta_j \leq -20^\circ$, we obtained $c_L > 0$ even at $c_\mu = 0$ (Fig.6(a)), which was due to the fact that the boundary layer on the upper side was turbulent because of the slot serving as a kind of trip-wire. This fact is seen from the curve for $c_\mu = 0$ in Fig.8(a) and from that $\theta_{sep} \approx 115^\circ \sim 120^\circ$ at $c_\mu = 0$ in Fig.10. The boundary layer on the lower side of the cylinder, however, keeps itself laminar so that the pressure distributions of the upper and lower sides are asymmetric to each other (Fig.8(a)) to give $c_L > 0$ at $c_\mu = 0$. When c_μ is very small but not zero, velocity of jet must be smaller than that of main flow and the momentum of this velocity profile near the wall is considered to be smaller than that of the velocity profile of turbulent boundary layer, which in turn seems to result in earlier separation of flow than in the case of $c_\mu = 0$. Hence the point of separation moves rather forward by a little injection of air (Fig.10) and c_{pb} is decreased (Fig.9). Consequently the negative pressure in the static-pressure distribution on the upper side is reduced. This is obviously seen by comparison of the curve for $c_\mu = 0$ with that for $c_\mu = 0.035$ in Fig.12. Then c_L (c_D) rather decreases (increases) by a little injection of air. If c_μ is further increased, the momentum near the wall begins to increase and the point of separation of flow begins to move backward (Fig.10). There, c_L (c_D) passes a point of the local minimum (the local maximum), therefore c_L/c_D too passes a point of the minimum (Fig.7).

(b) The case of $\theta_j \geq 0^\circ$ seems to be a problem of stability of the boundary layer. The station of the slot in this case is downstream of the point of natural laminar separation of flow $\theta_{ts} (\approx 70^\circ)$. The reverse flow in the separated region is held back by a little injection of air and the laminar boundary layer upstream transits to the turbulent one so that separation point jumps from the laminar to the turbulent one. Therefore the jet does not supply the boundary layer with energy to prevent it from separating but merely causes a disturbance for transition from the laminar to turbulent boundary layer. As a consequence, energy to retard the separation of flow is thought to be supplied mainly from the main flow. As a result, a control of boundary layer to fix θ_{sep} anywhere between θ_{ts} and $90^\circ + \theta_j$ is impossible. For instance, the point of laminar separation is $\theta_{ts} \approx 70^\circ$ for $c_\mu = 0$ at any station of the slot, i.e., $90^\circ + \theta_j = 90^\circ, 100^\circ, 110^\circ$ and 120° (i.e., $\theta_j = 0^\circ, 10^\circ, 20^\circ$ and 30°) on one hand but that point instantly jumps to the points of turbulent separation $\theta_{sep} = 108^\circ, 120^\circ, 127^\circ$ and 127° , respectively, when air is slightly injected on the other hand (Fig.10). Any value of θ_{sep} is larger than the corresponding $90^\circ + \theta_j$. Hence the larger θ_j is, the more rapidly c_L (c_D) increases (decreases) with c_μ and the steeper the gradient is. Therefore the larger θ_j is, the more rapidly c_L/c_D increases (Fig.7).

(ii) The case of large c_μ When $c_\mu > 0.1$, c_L is roughly proportional to c_μ if θ_j is kept constant (Fig.6(a)) and its gradient is nearly constant irrespective of θ_j . Furthermore, c_L increases as θ_j increases when c_μ is kept constant.

On the other hand, the rate of increase of c_D with an increase of c_μ is dependent on θ_j (Fig.6(b)). For example, we obtained $c_D \approx 1.18$

at $c_{\mu} = 0.22$ and $c_D \approx 1.94$ at $c_{\mu} = 0.4$ when $\theta_j = 30^\circ$ whereas $c_D \approx 0.34$ was maintained for $c_{\mu} \geq 0.22$ when $\theta_j = -40^\circ$. The discrepancy can not be explained only by induced drag and base pressure. Referring to Fig. 6(a), we can see that the value of c_{μ} to give $c_L = 3$ when $\theta_j = 30^\circ$ is about 0.06 and that when $\theta_j = -40^\circ$ is about 0.37 while the values of c_{pb} 's in both cases are about -0.35 as shown in Fig. 9. Thus the drag is to be identical in both cases because c_L 's and c_{pb} 's are equal in both cases, respectively, but we nevertheless find $c_D \approx 0.34$ at $c_{\mu} = 0.37$ when $\theta_j = -40^\circ$ and $c_D \approx 0.65$ at $c_{\mu} = 0.06$ when $\theta_j = 30^\circ$ in Fig. 7(b). This discrepancy is thought to have come from the large negative pressure right downstream of the slot. The static-pressure distributions in Fig. 8(a) can be divided into the part of static pressure (Δc_p) which is extremely low due to the separation bubble right downstream of the slot and the whole part of the rest (c_{pe}), so $c_p = c_{pe} + \Delta c_p$. Since c_{pe} is the static pressure due to circulation about the cylinder, wake and uniform flow, it is the curve to be smoothly

Fig. 12 Static-pressure distributions on the wall when c_{μ} is small

Fig. 13 Static-pressure distributions when c_{Ls} 's are identical

connected from static pressure upstream of the slot to that not including Δc_p downstream. Therefore the position θ_m at which c_{pe} has its minimum moves downstream when induced velocity exists. In contrast to this, the position of Δc_p is solely determined by the station of the slot and the value of Δc_p is larger when c_{μ} is larger. Therefore Δc_p produces thrust on the cylinder, which cancels out a part of the drag due to c_{pe} , to keep c_D constant when $\theta_j = -40^\circ$. Both Δc_p and c_{pe} , however, produce drag so that c_D increases as c_{μ} is increased when $\theta_j = 30^\circ$.

5.2.3 Two-dimensional characteristics The induced drag is considerably large in spite of the effective aspect ratio of the cylinder, due to the end-plates being approximately 57⁽¹⁴⁾. The proportion of Δc_p in the total drag is only about 7 % in the case of $\theta_j = 10^\circ$ although that depends on θ_j . This proportion is not so much dependent on c_{μ} . Therefore the most part of c_D to increase together with c_{μ} and θ_j is thought to be the induced drag.

Since the induced drag is dependent on the lift coefficient c_{Ls} (not including the reaction of jet) to be obtained from only the static-pr-

Fig. 14 Relation of the point, θ_m , at which c_{pe} is the minimum, to c_{Ls}

(a) Relation of the two-dimensional lift coefficient to c_{μ}
(b) Relation of the two-dimensional drag coefficient to c_{μ}

Fig. 15

pressure distribution, the pressure distributions c_{pe} 's excluding Δc_p 's are to lie on the same curve regardless of the values of c_{μ} and θ_j when c_{ls} 's are identical. Fig. 13 shows the static-pressure distributions when $c_{ls} \approx 3.05$. c_{pe} 's are seen to be represented by a single curve although the static pressures are a little scattered downstream of the slot since the values of c_{μ} are different. The point θ_m ($\theta_m = 0^\circ$ at $\theta = 90^\circ$) at which c_{pe} takes its minimum is determined by the induced velocity and the size of wake flow, where the wake reduces θ_m and the induced angle of incidence α_i makes it large. θ_m is to asymptotically approach α_i since the size of the wake is reduced as c_{ls} increases. Fig. 14 indicates the experimental relation of θ_m to c_{ls} . θ_m obviously asymptotically approaches α_i as c_{ls} increases. It is seen from the figure that the effect of the wake vanishes for $c_{ls} \geq 5$, in the range of which θ_m is equal to α_i . α_i is proportional to c_{ls} , which implies that the spanwise lift-distributions are nearly similar regardless of c_{μ} and θ_j . The empirical relation between α_i and c_{ls} is given by $\alpha_i \approx 2.3 c_{ls}$.

The two-dimensional values c_{le} and c_{de} can be obtained by making use of α_i as follows; $c_{le} = c_l \cos \alpha_i + c_d \sin \alpha_i$ and $c_{de} = c_d \cos \alpha_i - c_l \sin \alpha_i$, hence the larger c_l is, the larger error c_{de} includes. Figs. 15(a) and (b) show the curves of $c_{\mu} \sim c_{le}$ and $c_{\mu} \sim c_{de}$, respectively. θ_{jo} 's in the figure are values of θ_j corrected by α_i . It is seen by comparison of Fig. 15 with Fig. 6(a) that there is not any great difference between c_{le} and c_l . c_{le} , however, is not so much dependent on θ_{jo} when $\theta_{jo} \geq 10^\circ$ and $c_{\mu} \geq 0.25$. On the other hand, c_{de} is quite different from c_d except that c_{de} tends to behave just as c_d when c_{μ} is small. In the two-dimensional case, the drag coefficient decreases as c_{μ} increases and moreover the larger θ_{jo} is, the smaller c_{de} is. We can get thrust when c_{μ} is large.

5.3 Static-pressure distribution The authors have shown that the characteristics and the point of separation of jet on the wall of the cylinder can be calculated if the static-pressure distribution is known. In general, it is extremely difficult to theoretically obtain the static-pressure distribution of a flow including a separated region. It is, however, not so hard to make a theoretical calculation of the static-pressure distribution in the case of a particular shape like a circular cylinder.

The flow about a circular cylinder is simply expressed as the sum of potentials due to the uniform flow, doublet, circulation, source and sink. The equation of the static-pressure distribution is as follows;

$$c_{pe} = 1 - \left\{ 2 \sin \theta + \frac{c_{le}}{2\pi} - \frac{c_{de}}{4\pi} \cot \frac{\pi - \theta_{st} - \theta}{2} \right\}^2 \quad (6)$$

, where c_l , c_{de} and θ_{st} are strengths of a circulation, source and a front stagnation-point, respectively.

The solid line in Fig. 13 is a curve by the preceding equation into which $\theta - \alpha_i = \theta - 7^\circ$ instead of θ , $c_{le} = 3.58$, $c_{de} = 1.86$ and $\theta_{st} = -17^\circ$ are substituted. It is obvious that Eq. (6) well agrees with experimental results. Since Eq. (6), however, has a singular point at $\theta = \pi - \theta_{st}$, the pressure by the equation in the separated region is replaced with an empirical one in the region. It is already shown in Fig. 14 that $\alpha_i = \theta_m$ for $c_{ls} \geq 5$, which implies that the region of separated flow can be ignored over this range of c_{ls} and which in turn implies

that we may take into account only the term of circulation (c_{le}) in Eq. (6), neglecting the term of source ($c_{de} = 0$). The broken line in Fig. 13 is a curve by Eq. (6) into which $\theta - \alpha_i = \theta - 12.6^\circ$ instead of θ , $c_{le} = 0$ and $c_{le} = 5.52$ are substituted. It is obvious from the figure that Eq. (6) well agrees with experimental results even if $c_{de} = 0$ is used in the case of $c_{ls} \geq 5$.

Thus, the static-pressure distribution of our experiment too can well be expressed by Eq. (6) when c_{le} , c_{de} and θ_{st} are suitably chosen. Although we can calculate c_l and c_d etc. of a circular cylinder using the method of calculation by the authors and Eq. (6), we will not make comparison of the results of calculation with the present results of the experiment but will report on it at the next opportunity.

6. Conclusions

As a consequence of the investigation on static-pressure distribution and velocity profile of jet about a circular cylinder, the following conclusions were obtained:

- (1) On the characteristics of jet: (i) Steep pressure-rise and separation bubble take place, regardless of whether a main flow is present or not, at a place where the radius of curvature of the wall of the cylinder discontinuously increases. (ii) Velocity profile of mixing region of jet is similar even after passing the point of separation of flow and is expressed by Eq. (3). (iii) Thickness of jet can be calculated by Eq. (5).
- (2) On the lift and drag coefficients: (i) In the case of small c_{μ} , (a) for $\theta_j \leq -20^\circ$, we get $c_l > 0$ even at $c_{\mu} = 0$ and c_l once decreases and increases after passing through the point of the minimum as c_{μ} increases. In the meantime, c_d decreases after passing through the point of the local maximum. (b) For $\theta_j \geq 0^\circ$, the boundary-layer control like $\theta_{st} < \theta_{sep} < 90^\circ + \theta_j$ is impossible. (ii) In the case of large c_{μ} ($c_{\mu} > 0.1$), the rate of increase of c_l with an increase of c_{μ} is nearly constant regardless of θ_j . The larger θ_j is, the larger c_l is when c_{μ} is maintained constant and, on the other hand, the rate of increase of c_d with an increase of c_{μ} increases as θ_j increases. (iii) The larger θ_j is, at the smaller value of c_{μ} , c_l/c_d attains its maximum and the smaller the maximum is. (iv) The two-dimensional lift coefficient c_{le} is not so different from c_l , but c_{de} , unlike c_d , is smaller when θ_j is larger and decreases as c_{μ} increases and thrust arises when c_{μ} is large.
- (3) On the static-pressure distribution: (i) The static-pressure distribution of our cylinder too is expressed by Eq. (6). (ii) $\alpha_i = \theta_m$ for $c_{ls} \geq 5$ and Eq. (6) into which $c_{de} = 0$ is substituted can be used to express c_{pe} . (iii) c_{pe} has the maximum and it is about 0.7.

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