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Pipe Wall Effects on Suspension Velocities of Single Freely-Suspended Spheres and on Terminal Velocities of Single Spheres in a Pipe*

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This paper presents the following two velocities in terms of non-dimensional factors:

- (a) The mean suspension velocities of single freely-suspended spheres in a pipe v_1 ,
- (b) The terminal velocities of single spheres falling through still water in a pipe v_2 .

The experiments were carried out with changes in sphere diameter d , specific gravity of sphere γ_s , pipe diameter D and inclination angle of pipe θ .

The results obtained are summarized as follows:

- (1) v_1 is not always identical with v_2 ,
- (2) $v_1/v_t = 1.024 - 1.329 \ln(d/D + 1.0)$, (1)
- (3) $v_2/v_t = 1.0 - 2.130(d/D)^{2.462}$ (2)
- for the spheres falling straightly along the pipe axis, or
- $v_2/v_t = 0.817 - 2.390(d/D)^{2.462}$ (3)
- for the spheres falling with oscillation in the pipe, where v_t is the terminal velocity of single spheres in unbounded fluid,
- (4) $v_2/v_1 = 1.10 - 1.35$, in the case of Eq.(2)
- and
- $v_2/v_1 = 0.90 - 1.03$, in the case of Eq.(3).

1. Introduction

Uncertainties occur about the terminal velocities of single spheres falling through still fluid in a pipe, as mentioned later, when one deals with such velocities in investigations on flow patterns of hydraulic transport of solids in pipes or in the designs of hydraulic transport systems. Two of them are as follows:

(1) It is often considered that the terminal velocities of single spheres in a vertical pipe coincide with the suspension velocities of single freely-suspended spheres in the pipe with regard to relative velocities between the spheres and the fluid.

(2) Pipe wall effects on the two velocities mentioned above have not been examined sufficiently.

In this paper, the experimental results and their discussions are presented about these two velocities of the spheres subjected to the pipe wall effects in both vertical pipes and inclined ones.

At the hydraulic transport of solids in vertical pipes, it is supposed in general that (a) the minimum transport velocities of the particles (the suspension velocities of the spheres) refer to the decelerated terminal velocities of the spheres in vertical pipes and (b) the upward velocities of the particles in vertical pipes will be obtained from the mean flow velocities minus the decelerated terminal velocities of the particles. (1), (2)

The suspension velocities and the decelerated terminal ones have been often deemed important in the designs of hydraulic transport of solid particles in pipes, mentioned above. The experiments on single spheres were carried out to examine the first step of the research on the velocities of solid particles.

According to the results obtained, the mean suspension velocities did not always coincide with the decelerated terminal velocities, i.e. the former were often less than the latter under the different wall effects as mentioned later. If the results above can be applied directly to the designs of required flow velocities in vertical pipes, the minimum transport velocities of solid particles, based on the decelerated terminal velocities, will be practically overestimated.

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The reasons for this difference between the two velocities are explained as follows: we consider the following three cases — (I) the regime of a sphere settling through still water in a pipe; (II) the regime of another case of (I) in which a velocity of the same magnitude as that of the sphere velocity in the case (I) is added in upward direction; (III) the regime of a sphere being maintained suspension in a pipe. The cases (I) and (II) are equivalent to each other with respect to the relations among the sphere, the water and the pipe, but the case (III) is not always equivalent to the case (I) or (II), because the pipe is at rest and the water alone is flowing upwards. In other words, the the flow past the sphere in the case (I) or (II) may be uniform; while the flow around the sphere in the case (III) may vary in the pipe cross section. One is likely to be misled as in the example of (a) or (b) mentioned above probably on account of supposing that the case (II) is equivalent to (III).

2. Apparatus and experimental procedure

2.1 The scope of the experiments

The ratios of sphere diameter to pipe diameter ranged from 0.12 to 0.45 and the particle Reynolds numbers based on the relative velocities between sphere and water ranged from 30 to 6550, which were located in Allen or Newton regions in the diagram of drag coefficient of a sphere versus Reynolds number.

2.2 Spheres and pipes used

The spheres and the pipes used in the experiments are shown in Table 1. The pipes in the test section were made of methacryl plastics and four types of plastic spheres were used, whose specific gravities varied.

2.3 Apparatus and experimental procedure

Table 1. The spheres and the pipes used

diameter of pipe D in mm	19, 22, 28, 40, 50.
length of pipe in m	2.
diameter of sphere d in mm	3, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20, 22.
material of sphere, specific gravity of sphere	vinyl chloride 1.420, resin for microsection 1.269, polycarbonate 1.203, polystyrol 1.065.
inclination angle of pipe θ in degree	0, 22.5, 45.0, 67.5, 90.0.

The equipment used in the experiment is shown schematically in Fig.1.

The loop used was a closed circuit with water as the working fluid. Water was drawn from a reservoir by a single-stage, single-suction centrifugal pump with 50mm pipe driven by a 1.5KW motor. The test sections were attached in turn to a 3m steel channel, which was turned till the desired angle θ around the pivot on the supporting frame. Flexible vinyl hoses with four sluice valves were connected at the top and bottom of the test section to enable the test section to turn with ease. Water delivered from the top of the test section, passing through the valves, was measured for the flow rate with a measuring cylinder. Spheres were inserted into the section by means of a T-joint at the top of the test section and suspended in testing between two fine brass nets provided at both ends of the test section.

The experiments for measuring the suspension velocities of single spheres: After insertion of single sphere into the vertical test section the pump was started and the flow was balanced with the valves, after making sure that any air in the system had been evacuated. The sphere was thus located in the mid-section. The suspension velocities of single spheres were obtained from the flow rate of the water divided by the cross sectional area of the pipes.

The experiments for measuring the decelerated terminal velocities of single spheres: After removing the vinyl hose from the top of the vertical test section the sphere was released in still water at the top of the section with no initial velocity. The decelerated terminal velocities of single spheres were measured with six pairs of light photoelectric devices, which were attached to the lower part of the section.

The tests of the spheres in the inclined pipes were carried out in the same manner as in the vertical section for conveni-

Fig. 1. Schematic diagram of apparatus

ence'sake. In these cases, however, the spheres were laid on the bottom of the pipe section, for measuring the suspension velocities, or were set rolling in contact with the pipe for measuring the terminal velocities. In the latter case, we consider the measured velocities as the "rolling down velocities".

3. Theoretical consideration

3.1 Terminal velocity of a sphere in unbounded fluid

Denoting the particle Reynolds number by Re , the terminal velocities of single spheres in unbounded fluid v_t are as well known as follows⁽³⁾:

For the terminal velocity of a sphere based on Stokes' law, $Re < 1$,

$$v_{ts} = \frac{d^2 \gamma_w}{18\mu} \left(\frac{\gamma_s}{\gamma_w} - 1 \right), \quad (1)$$

For the terminal velocity of a sphere based on Newton's law, $Re > 2.5 \times 10^3$,

$$v_{tn} = \sqrt{\frac{4}{3} \cdot \frac{gd}{C_D} \left(\frac{\gamma_s}{\gamma_w} - 1 \right)}, \quad (2)$$

For the terminal velocity of a sphere based on Allen's law, v_{tA} ,

$$\frac{1}{v_{tA}} = \frac{1}{v_{ts}} + \frac{1}{v_{tn}}, \quad (3)$$

where v_t , v_{ts} , v_{tn} , v_{tA} : theoretical terminal velocities of a sphere,

d : diameter of sphere,

μ : fluid viscosity,

γ_s , γ_w : specific gravities of sphere and fluid respectively,

g : acceleration of gravity,

C_D : drag coefficient of sphere.

Equations (1) — (3) were used for the calculations of the velocity ratios later.

3.2 Forces on a single sphere in pipes

Fig. 2. Forces on single sphere in pipes.

When one maintains suspension of a single sphere in a vertical pipe or settling the sphere in an inclined pipe, the flow around the sphere may be a kind of shear flow [Fig.2(a),(c)].

Many theoretical and experimental studies were made on the drag and lift of the sphere or cylinder submerged in the similar shear flows: early A. Einstein⁽⁴⁾, lately S.I. Rubinow and Keller⁽⁵⁾, M. Arie and M. Kiya⁽⁶⁾, and R. Eichhorn and S. Small⁽⁷⁾, who studied on the low Reynolds number flows around the sphere or cylinder. Without any suitable corrections, these results of the investigators may not be directly applied to the consideration of forces on the spheres in this case. One can predict, however, from the results above that the lift on the spheres may be negligibly small in comparison with the drag of the spheres, so the considerations were applied on the drag, not on the lift. In the cases of the inclined pipes, the frictional force at the bottom of the sphere was taken into consideration besides the drag of the sphere [Fig.2(e)]. When the sphere was freely falling in the vertical pipe or rolling down in the inclined pipe, the flow relative to the sphere might be uniform [Fig.2(b),(d)].

Using the nomenclature as follows, the conditions of the force balances maintaining the suspension, settling and rolling down of the sphere are described by Equations (4) — (11).

Nomenclature

- D_r : drag on sphere,
- B : buoyant force on sphere,
- W : weight of sphere,
- F : frictional force at the bottom of sphere, reset at the center of sphere [Fig.2(e)],
- G : apparent weight of sphere in fluid, available force for falling of sphere $= W - B$,
- v_1 : suspension velocity of sphere in pipe,
- v_2 : decelerated terminal velocity or rolling down velocity of sphere in pipe,
- C_{D1} : drag coefficient of sphere at suspension in pipe,
- C_{D2} : drag coefficient of sphere at decelerated falling or rolling down in pipe,
- ρ_s , ρ_w : densities of sphere and water respectively,
- P : force required to roll the sphere in pipe,
- N : frictional force perpendicular to pipe wall [Fig.2(e)],
- f : arm of N [Fig.2(e)],
- θ : inclination angle of pipe [Fig.2(c)],
- α : $(\gamma_s/\gamma_w) - 1$,
- Fr_1 , Fr_2 : Froude number based on v_1 and v_2 , $Fr = v_1/\sqrt{gd}$ and $Fr = v_2/\sqrt{gd}$ respectively.

For the vertical pipes:

$$D_r = C_{D1}(\pi/4)d^2(\rho_w/2)v_1^2, \quad (4)$$

$$D_r = C_{D2}(\pi/4)d^2(\rho_w/2)v_2^2, \quad (5)$$

$$G = W - B = (\pi/6)d^3(\gamma_s - \gamma_w). \quad (6)$$

Equating D_r in Eq. (4) to 0,

$$C_{ps1} = \frac{4}{3} (\rho_s/\rho_w - 1) \frac{gd}{v_t^2} = \frac{4\alpha}{3(Fr_1)^2}, \quad (7)$$

in the same way,

$$C_{ps2} = \frac{4\alpha}{3(Fr_2)^2} \quad (8)$$

For the inclined pipes:

$$\begin{aligned} P(d/2) &= Nf, \quad P = F, \\ N &= G \cos \theta \quad [\text{Fig. 2(e), (d)}], \quad \text{therefore} \\ F &= 2fG \cos \theta / d. \end{aligned} \quad (9)$$

In the case of settling of the sphere in the inclined pipes [Fig. 2(c)],

$$D_r = G \sin \theta + F = G \sin \theta + 2fG \cos \theta / d,$$

then

$$3v_1^2 C_{ps1} - 8g\alpha \cos \theta f - 4gd\alpha \sin \theta = 0. \quad (10)$$

In the case of rolling down of the sphere in the inclined pipes, supposing

$$D_r = G \sin \theta - F \quad [\text{Fig. 2(d)}],$$

then

$$3v_2^2 C_{ps2} + 8g\alpha \cos \theta f - 4gd\alpha \sin \theta = 0. \quad (11)$$

From Equations (7) and (8), C_{ps1} and C_{ps2} for the vertical pipes are obtained, while C_{ps1} , C_{ps2} and f are obtained for the inclined pipes from Equations (10) and (11) by the method of least squares, provided the assumption is valid, in which C_{ps1} , C_{ps2} and f will be constant in the same inclination angle of the pipes.

3.3 Dimensional analysis for the falling and suspension of single spheres in pipes

Selecting the following twelve factors from the quantities mentioned above, dimensional analysis was made to obtain nine nondimensional factors, $\pi_1, \pi_2, \dots, \pi_9$. Twelve factors: $d, D, \rho_w, \rho_s, \mu, v_t, v_1, v_2, g, f, C_{ps}$ and θ .

Nine nondimensional factors:

$$\begin{aligned} \pi_1 &= d/D, \quad \pi_2 = \gamma_s/\gamma_w, \\ \pi_3 &= v_t d/\nu \equiv Re, \quad \pi_4 = v_t/\sqrt{gd} \equiv Fr, \\ \pi_5 &= v_1/v_t \equiv \xi_1, \quad \pi_6 = v_2/v_t \equiv \xi_2, \\ \pi_7 &= f/d, \quad \pi_8 = C_{ps} \quad \text{and} \quad \pi_9 = \theta. \end{aligned}$$

From another viewpoint, the Reynolds numbers and the Froude numbers with v_1 and v_2 substituted for v_t in π_3 and π_4 , and the Froude numbers involving α or π_2 are important in certain cases:

$$\begin{aligned} \pi_3' &= v_1 d/\nu \equiv Re_1, \quad \pi_3'' = v_2 d/\nu \equiv Re_2, \\ \pi_4' &= v_1/\sqrt{gd} \equiv Fr_1, \quad \pi_4'' = v_2/\sqrt{gd} \equiv Fr_2, \\ Fr_1^* &= v_1/\sqrt{gd(\gamma_s/\gamma_w - 1)}, \\ Fr_2^* &= v_2/\sqrt{gd(\gamma_s/\gamma_w - 1)}. \end{aligned}$$

In the present paper, $\xi_1, \xi_2, d/D, Fr_1^*, Fr_2^*, C_{ps1}, C_{ps2}$ and θ are selected from the nondimensional factors and the combinations of these are made for comparison.

4. Experiments and results

4.1 Experiments

Several facts observed during the tests are stated in this section.

In the tests for maintaining the suspension of the sphere in the pipe, the sphere was suspended at a distance R from the pipe axis in many instances and sometimes was turning on a horizontal orbit with R radius around the pipe axis. No spinning of the sphere occurred when the forces acting on it were sufficiently balanced. (It was reported that the radius of the orbit was approximately a half of the pipe radius in Poiseuille pipe flow⁽⁸⁾.)

A sphere settling in the mid-section with an upward flow in the inclined pipe was in an approximately stable condition, although it oscillated sometimes on the pipe wall like a pendulum when the sphere had a larger diameter ratio (d/D) or a smaller density ratio (ρ_s/ρ_w).

In the tests for obtaining the decelerated terminal velocity of the sphere, it fell along the vertical pipe axis straightly in many instances [Fig. 2(b)], although it oscillated sideways in the pipe in such instances as mentioned above.

In the tests for the inclined pipe, the behavior of the sphere was the same as

Fig. 3. Suspension velocities (v_1) versus pipe inclination ($\sin \theta$) and pipe diameter (D) for vinyl chloride spheres ($\gamma_s = 1.420$, $d = 3 - 18$ mm).

that in the vertical pipe. The data on the oscillating sphere plotted in the figures shown later were distinguished from others by a special notation.

4.2 Results

Typical examples of the results obtained are presented in Figs.3 and 4 for the suspension velocities of the vinyl chloride spheres v_1 and the decelerated terminal velocities of the spheres v_2 respectively. The results obtained for the other spheres were similar to those in these figures. Comparing these figures with each other, it is evident that v_1 and v_2 varied in different manners, although both were affected by the changes of D and θ .

4.3 Relations among the nondimensional factors and discussion on them

The significant relations among the nondimensional factors in the results obtained are shown in Figs.5 — 9.

Fig. 4. Decelerated terminal velocities (v_2) versus pipe inclination ($\sin \theta$) and pipe diameter (D) for vinyl chloride spheres ($\gamma_s = 1.420$, $d = 3 - 18$ mm)

4.3.1 ξ_1 — (d/D) — Fr_1^* — θ and ξ_2 — (d/D) — Fr_2^* — θ

The diagrams for ξ_1 versus (d/D) are presented in Fig.5 (a) — (e), those for ξ_2 versus (d/D) in Fig.6 (a) — (e) with the parameter of pipe inclination angle θ for the sphere materials. The curves drawn in the figures show the mean values of the data obtained. The pipe wall effects i.e. (d/D) on ξ_1 were very different from those on ξ_2 , mentioned above. In the vertical pipes, excluding the approximately zero (d/D) region,

(a)

(b)

(c)

Fig. 5. ξ_1 versus (d/D) for various spheres.

$$\xi_1 < \xi_2$$

(excepting the data obtained from the oscillating spheres), but in the inclined pipes,

$$\xi_1 > \xi_2.$$

It is supposed that the differences arose from the variations of flow patterns around the spheres⁽⁹⁾, as mentioned in the introduction. The mean flow velocities in pipes of hydraulic transport of solid particles are in general based on the decelerated terminal velocities of the solid particles, such as v_2 in this paper, in the designs of the transport systems. This does not sufficiently fit because the suspension velocities of solid particles, such as v_1 here, are supposed more adequate for the basis of mean flow velocities in pipes than the decelerated terminal velocities from considera-

tion of results presented in this paper. The flow velocities in pipes are likely to be overestimated in the vertical pipes and underestimated in the inclined ones, if v_2 is used as one of the basic velocities in planning the hydraulic transport systems.

From Fig.5 (a) — (d) and Fig.6 (a) — (d), it is evident that ξ_1 and ξ_2 versus (d/D) is in a similar relation, regardless of the sphere materials. It is supposed that the effects of the sphere

(d)

(e)

(a)

(b)

(c)

Fig. 6. ξ_2 versus (d/D) for various spheres.

weights on the velocities were eliminated by (γ_s/γ_w) involved in ξ_1 and ξ_2 .

For ξ_1 from Fig. 5 (a) — (d), the following holds:

$$\xi_1 \equiv \frac{v_1}{v_t} = a_1 - b_1 \ln \left(\frac{d}{D} + 1.0 \right). \quad (12)$$

For ξ_2 from Fig. 6 (a) — (d), the following holds:

$$\xi_2 \equiv \frac{v_2}{v_t} = a_2 - b_2 \left(\frac{d}{D} \right)^n. \quad (13)$$

These are the experimental formulas for ξ_1 and ξ_2 versus (d/D) , where a_1 , a_2 , b_1 , b_2 and n are the constants to be obtained from the experiments for every θ .

Now try to decide the constants in the formulas only for the vertical pipes. ξ_1 and ξ_2 versus (d/D) including four types of the sphere materials is plotted in Fig. 5 (e) and Fig. 6 (e) respectively. The basic formula for mean ξ_1 obtained by the method of least squares in the form of Eq. (12):

$$\xi_1 = 1.024 - 1.329 \ln(d/D + 1.0). \quad (14.a)$$

To explain the scattering of the plotted points, try to introduce Fr_1^* (see section 3.3) into Eq. (14.a), then the equation is reduced to

$$\xi_1 = 1.024 - \frac{1.329}{0.5347 Fr_1^* + 0.3988} \times \ln(d/D + 1.0) \quad (14.b)$$

The curves for the various values of Fr_1^* are shown in Fig. 5 (e). The formulas for ξ_2 obtained in the same way in the form of Eq. (13) are reduced to two equations as follows, unlike ξ_1 , according to the falling state of the sphere:

- (1) For the sphere falling straightly ($1.8 \geq Fr_2^* \geq 1.1$),

$$\xi_2 = 1.0 - 2.130(d/D)^{2.462} \quad (15.a)$$
- (2) For the sphere falling oscillating sideways ($1.5 > Fr_2^* \geq 0.8$),

Fig. 7. Suspension velocity ratio (ξ_1) versus pipe inclination ($\sin \theta$) and diameter ratio (d/D) for vinyl chloride spheres.

$$\xi_2 = 0.817 - 2.390(d/D)^{2.462} \quad (15.b)$$

There can be two phases of (1) and (2), therefore, at $1.5 > \text{Fr}_2^* \geq 1.1$, and ξ_2 may not depend on (d/D) alone.

Equation (15.a) resembles the following equations:

$$\text{J. Finkay formula}^{(10)}; \quad v_2 = \{1 - (d/D)^2\} v_t \quad (16)$$

$$\text{and M. Mikumo formula}^{(10)}; \quad v_2 = \{1 - (d/D)^{1.5}\} v_t. \quad (17)$$

It is notable that the powers of the terms (d/D) in these experimental formulas are different from Eq.(15.a).

Fig. 8. Terminal velocity ratio (ξ_2) versus pipe inclination ($\sin \theta$) and diameter ratio (d/D) for vinyl chloride spheres.

Fig. 9.

The relations of ξ_1 — (d/D) — θ and ξ_2 — (d/D) — θ for the vinyl chloride spheres are illustrated in Fig.7 and 8 respectively. These figures show that ξ_1 and ξ_2 change in different manners with an increase of (d/D) and θ .

4.3.2 ξ_2/ξ_1 — (d/D) in the vertical pipes

Thus far, the pipe wall effects on ξ_1 and ξ_2 have been treated in terms of (d/D) independently. Now try to make comparison of v_2 with v_1 directly, that is, to describe (ξ_2/ξ_1) including the terms of (d/D) . From Eqs.(15.a) and (14.a),

$$\frac{\xi_2}{\xi_1} = \frac{1.0 - 2.130(d/D)^{2.462}}{1.024 - 1.329 \ln(d/D + 1.0)}, \quad (18)$$

and from Eqs.(15.b) and (14.a),

$$\frac{\xi_2}{\xi_1} = \frac{0.817 - 2.390(d/D)^{2.462}}{1.024 - 1.329 \ln(d/D + 1.0)} \quad (19)$$

Fig. 10.

Fig. 11.

Following these formulas, the ratio of terminal velocities to suspension ones (ξ_2/ξ_1) versus the diameter ratio (d/D) is plotted in Fig.9. Thus the numerical values of (ξ_2/ξ_1) are obtained. The former ranges from 1.10 to 1.35 and the latter from 0.90 to 1.03.

4.3.3 C_{ps1} , C_{ps2} ; f — θ

The mean values of C_{ps1} , f and C_{ps2} , f for every θ obtained from Eqs.(10) and (11) by the method given in section 3.2 are plotted in Figs.10(a),(b) and 11(a),(b) respectively. It is found from the figures that both the values of C_{ps1} and C_{ps2} decrease from values far larger than those of C_D (drag coefficient of a sphere in unbounded fluid) with an increase of θ and approach C_D asymptotically, while the values of f increase.

The method given in section 3.2 is useful for obtaining the mean values of C_{ps} and f in the inclined pipes provided with the assumption that they may be constant for the same pipe inclination angle θ . It is remarkable that the values of drag coefficients of the sphere in pipes are always larger than those in unbounded fluid.

5. Conclusions

Experiments to obtain the decelerated terminal velocities and suspension velocities of single spheres were carried out with various sphere materials, sphere diameters, pipe diameters and pipe inclination angles. The test results obtained are summarized in terms of nondimensional factors as follows:

(1) The suspension velocities of single freely-suspended spheres in vertical pipes v_1 were not always identical with the decelerated terminal velocities of the spheres in the pipes v_2 . In the vertical pipes, (v_1/v_t) and (v_2/v_t) are expressed as Eqs.(14.a),(14.b),(15.a) and (15.b) in terms of (d/D), which manifest the pipe wall effects, where v_t is the theoretical terminal velocity of the spheres. In the inclined pipes, they are greatly influenced by pipe inclination angles.

(2) (v_1/v_t) and (v_2/v_t) did not vary

significantly with variation of the sphere weight in the test range.

(3) (v_2/v_1) or (ξ_2/ξ_1) is expressed as Eqs.(18) and (19), their numerical values ranging from 1.10 to 1.35 for the terminal velocities of the straightly falling spheres versus the sphere suspension velocities; from 0.90 to 1.03 for the terminal velocities of the oscillating spheres versus the suspension velocities.

(4) The author has tried one method for obtaining the mean values of drag coefficients and arms of frictional forces of the sphere in inclined pipes and found that the drag coefficients of the sphere in pipes are always larger than those in unbounded fluid.

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