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Fatigue Limit of Steel with Cracks*

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Fatigue tests were carried out on mild steel plate specimens with various lengths of cracks. The relation between the alternating fatigue limit σ_w and the crack length was well expressed by following equation.

$$R_{pcw} = A_e \left(\sec \frac{\pi \sigma_w}{2 \sigma_y} - 1 \right) = \text{const.}$$

where, R_{pcw} is the cyclic plastic zone size at fatigue limit, A_e is the equivalent crack length of an infinite plate, and σ_y is the yield strength of the material. The size of the persistent slip band zone on the fatigue limit is about one tenth of R_{pcw} and is nearly equal to the measured mean grain size of the material. R_{pcw} has close relations with the crack length corresponding to the fatigue limit of a plain specimen, the material constant which was already obtained in the criteria of fatigue crack initiation at notch roots, and the critical notch root radius at the branch point.

1. Introduction

Machine parts and structures may have flaws which have appeared in the process of casting, forging, welding and heat treatment, but they still have inherent strength which is sometimes enough for their operations. Their fatigue limit σ_w mainly depends on the crack length a . Frost⁽¹⁾ and Kobayashi and Nakazawa⁽²⁾ obtained the experimental relations;

$$\sigma_w^n a = \text{const.} \dots \dots \dots (1)$$

where n is a constant equal to 3 or 4. Recently, Ando⁽³⁾⁽⁴⁾, Frost et al.⁽⁵⁾ and Harrison⁽⁶⁾ analyzed the data using the stress intensity factor by linear fracture mechanics.

The authors considered that, as the condition of a fatigue crack tip is elastic-plastic, the most essential parameters are cyclic plastic zone size R_{pc} and cyclic crack opening displacement $\Delta\Phi$; and then they revealed that the fatigue crack growth rate is proportional to them⁽⁷⁾. It can be considered that the fatigue limit of a cracked material is the critical condition of fatigue crack propagation. This idea also has close relation with Ishibashi's E_0 criterion⁽⁸⁾ for fatigue crack initiation which is based on the average stress in inherent region at a notch root.

In this report, fatigue tests are carried out on mild steel plate specimens with various lengths of cracks. These results are analyzed by elastic plastic fra-

cture mechanics, and then, the relations among the characteristics at fatigue limit of a cracked material, a notched material and a plain specimen are investigated.

2. Experimental procedure

JIS SM41 structural mild steel plate is used as a test material. Its chemical compositions and mechanical properties are shown in Table 1. The mean grain size is 0.025 mm. The test specimens were taken from a plate with a thickness of 12 mm in the rolling direction of the material, and machined into side notched plates with a width of 110 mm as shown in Fig. 1(a). Then, fatigue cracks were introduced at notch roots in the length from 0.35 mm to 10 mm by an alternating stress of $\pm 6 \text{ kg/mm}^2$ using the Tokyo-koki resonance type Vibrophoe fatigue testing machine (2, 5, 10t, 150Hz). During the tests, load unbalance of the specimen was adjusted to within 5%, so the differences of crack length were held within 20% even in the case of the smallest crack, 0.35 mm in length. To minimize the fatigue damage at the crack tip, the crack growth rates were held small, from 10^{-6} to 10^{-5} mm/c when fatigue cracks were added at notch roots.

Then the notches were cut out to make uniform length of the cracks, as shown in Fig. 1(b), except for a specimen with cracks of 10 mm in length. The plain specimen was a cylindrical specimen of 5 mm in diameter as shown in Fig. 1(c). All the specimens were annealed in a vacuum at 650°C for 2 hours in order to remove the influence of machining and adding the cracks. Each plate specimens was electro polished at the cracked area. Fatigue tests were performed under reversed loading in the same way as the cracks were added.

3. Experimental results and discussions

Figure 2 shows the test results of

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Table 1 Chemical compositions and mechanical properties of the material

Chemical compositions					Mechanical properties				
C	Si	Mn	P	S	σ_y	σ_B	σ_T	ϕ	ψ
0.18	0.01	1.04	0.016	0.026	25.6	49.3	98.7	43.2	66.4

Fig. 1 Details of test specimens

the relation between alternating net section stress σ_n and effective crack length a_e which is the equivalent crack length of an infinite plate and expressed as follows:

$$a_e = \frac{K^2}{\pi \sigma_n^2} \quad \dots\dots (2)$$

$$= h a \left(1 - \frac{2a}{W}\right)^2 \frac{W}{\pi a} \tan \frac{\pi a}{W} \quad \dots\dots (2)$$

where, K is the stress intensity factor, W is width of the specimen, and h is a coefficient which is 1.258 when $2a/W=0$ and 1.0 when $2a/W$ is larger than 0.7. The cyclic plastic zone size R_{pc} in a modified Dugdale model⁽⁹⁾ which was proposed by the authors as a useful parameter to express the fatigue damage at crack tip, is shown as follows:

$$R_{pc} = a_e \left(\sec \frac{\pi \sigma}{2\sigma_y} - 1 \right) \quad \dots\dots (3)$$

where σ_y is yield strength and σ is stress range $\Delta\sigma$ when minimum stress

$\sigma_{\min} \geq 0$ and is maximum stress $\sigma_{\max}$ when $\sigma_{\min} \leq 0$. Above equation is based on the phenomena that the cyclic yielding strength is about a half of the static yield strength σ_y in common steels⁽¹⁰⁾.

In Fig. 2 the solid points show that the cracks propagated and the specimens were fractured, and the hollow circular point show that the cracks did not propagate any more even after 10^7 cycles of loading. One of the specimens shown in Fig. 1(a) was stressed under $\pm 4.75 \text{ kg/mm}^2$ and cracks propagated far enough from notch roots. Then, alternating stress was decreased as $\pm 4.5 \text{ kg/mm}^2 \rightarrow \pm 4.2 \text{ kg/mm}^2 \rightarrow \pm 4.0 \text{ kg/mm}^2$ each time cracks propagated 2 mm. Each crack growth rate was $4.4 \times 10^{-6} \text{ mm/c}$, $2.6 \times 10^{-6} \text{ mm/c}$ and 0 mm/c (no propagation after 10^7 cycles of loading). Their microphotographs at crack tips are shown in Fig. 3. These results are also shown in Fig. 2. From this Figure, it is clear that the critical strength (threshold) of fatigue crack propagation and the fatigue limit of a cracked material are very similar, so it seems that these are the same characteristics of materials.

A calculated curve, which means that the cyclic plastic zone size R_{pc} of Eq.(3) is constant 0.27 mm, is shown in Fig. 2. This curve is well close to the experimental values of fatigue limit. Then, the relation between fatigue limit σ_w and effective crack length a_e can be shown as follows:

$$R_{pc}/w = \text{const.}$$

$$= a_e \left(\sec \frac{\pi \sigma_w}{2\sigma_y} - 1 \right) = 0.27 \text{ mm} \quad \dots\dots (4)$$

Therefore, the cyclic plastic zone size at the fatigue limit is material constant, and the model for the fatigue limit of a cracked material can be shown as Fig. 4. Equation(4) can be simplified by using the stress intensity factor K in the small

Fig. 2 Effect of crack length on fatigue limit

Fig. 3 The microphotographs at the crack tips ($a_e \approx 8.5 \text{ mm}$)

Fig. 4 Schematic model for fatigue limit of material with a crack

scale yielding condition as $\sigma_n/\sigma_y \leq 0.3$.

$$K_w = \sigma_w \sqrt{\pi a_e} = \text{const.} \\ = 21 \frac{\text{kg}}{\text{mm}^{\frac{3}{2}}} \dots\dots\dots(5)$$

Equation(4) can also be expressed, for convenience's sake, with modifying the stress intensity factor modified by the hypothetical crack length which consists of the effective crack length and R_{pcw} .

$$K_w = \sigma_w \sqrt{\pi(a_e + R_{pcw})} \\ \approx \sigma_w \sqrt{\pi a_e / (1 - \frac{\pi^2}{8} (\frac{\sigma_w}{\sigma_y})^2)} = \text{const.} \dots\dots\dots(6)$$

From Fig. 2, the crack length a_e corresponding to the fatigue limit of a plain specimen is obtained as about as about 0.22 mm. This length is nearly equal to R_{pcw} .

Figures 5 and 6 show microphotographs at the tips of the cracks whose lengths are 0.35 mm and 1.0 mm respectively. The persistent slip band zone at the fatigue limit R_{pcw}^* is from 0.02mm to 0.03 mm on both length of cracks. This value is close to the measured average grain size of the material. Namely:

$$R_{pcw}^* = \text{const.} \\ \approx \text{grain size} \dots\dots\dots(7)$$

When the cyclic plastic zone size R_{pc} is large enough, the grain size of the material does not affect the crack growth rate(11) and the two are proportional to each other(7), but when R_{pc} becomes small, the crack growth rate is steeply decreased(12) by the resistance of the grain boundaries, and at last, the crack does not propagate any more when the actual cyclic plastic zone (persistent slip band zone) R_{pcw}^* is restricted to a grain boundary. Equation(7) supports the criterion of Eq. (4) on the crystallographic aspect of the material's structure. Figure 7 shows the persistent slip band zone R_{pcw}^* in relation to σ_n/σ_y . From this figure, we can obtain the next relation:

$$R_{pc}^* = C R_{pc} \dots\dots\dots(8)$$

where C is a proportional constant which is 0.11 in this case. Taira and Tanaka also obtained similar relations in JIS S15C steel(13) and 0.03%C mild steel(14).

4. Analysis of experimental results obtained by other authors

4.1 Fatigue limit of materials with crack-like defects

Figure 8 shows other examples rearranged in the same way as in Fig. 2. In Fig. 8(a) and (b) the values n of Eq.(1), which were obtained experimentally by each author(2)(1), are shown as the slope. Figure 8(c) shows the results(15)(16) on different kinds of specimens, i.e., are the plate specimens and cylindrical specimens. Since both results are almost the same, their cyclic plastic conditions at

Fig. 5 The microphotographs at the crack tips ($a \approx 0.35 \text{ mm}$).

Fig. 6 The microphotographs at the crack tips after test ($a \approx 1.0 \text{ mm}$)

the crack tips must be the same.

The rearranged results for other data (3)(16)~(36) are further summarized in Table 2. Figure 9 shows their alternating stress intensity factors at fatigue limit in small scale yielding condition in relation with yield strength or 0.2% proof strength σ_y . These values slightly tend to lower in high hardness steel, but all of them fit in the next narrow region.

$$K_w = 10 \sim 22 \frac{\text{kg}}{\text{mm}^{3/2}} \dots\dots\dots (9)$$

On the other hand, R_{pcw} changes from 0.3 mm to 0.001 mm with an increasing yield strength as shown in Fig. 10. The straight line in this figure can be expressed as follows:

$$R_{pcw} = 90 \sigma_y^{-2} \dots\dots\dots (10)$$

This relation looks like Petch's equation (37) on the relations between yield

strength and grain size of materials. It suggests that R_{pcw} is a material constant relating to the structure of material, and also supports the contents of Eqs (4) and (7). From the fact that R_{pcw} is equal to $\pi/8 \cdot (K_w/\sigma_y)^2$ in small scale yielding condition, it is apparent that the meanings of Eqs. (10) and (9) are the same.

Figure 11 shows the stress intensity factor at fatigue limit in $\sigma_w/\sigma_y \ll 1$ rearranging Frost's experimental results (22)(38) under various minimum stresses. Although the stress intensity factor range ΔK_w gradually decreases following the change in the minimum stress from zero to 6 kg/mm², the dominant factor is the maximum stress intensity factor $K_{max,w}$ when the minimum stress is negative and the stress intensity factor range ΔK_w when the minimum stress is positive. This fact suggests that the cyclic plastic zone size R_{pc} can fairly be expressed as

Table 2 Summary of Rearrangement for the alternating fatigue limit of steel with cracks

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No.	Steel	σ_y kg/mm ²	σ_B kg/mm ²	σ_{wo} kg/mm ²	Kind of cracks	Loading condition	a_o mm	$R_{p(cw)}$ mm	K_{w3} kg/mm ^{3/2}	ρ_o mm	Reference
1	SM41	25.6	49.3	18.0	Fatigue crack	Tension for plate	0.22	0.27	21.0	-	This experiment
2	SS41	24.5	46.2	20.5	Acute notch	Tension for plate Rotating bending	0.11	0.30	21.0	2.0	(15), (36)
3	SF55	31.6	55.1	25.0	"	Rotating bending	0.055	0.15	19.5	0.8	(19)
4	SF60	37.8	62.4	29.5	Pressed crack	"	0.018	0.033	11.5	-	(16)
5	13Cr	44.5	69.5	34.0	"	"	0.030	0.048	15.5	-	(17)
6	Ni-Cr	82.3	95.9	45.0	"	"	0.035	0.019	17.5	-	
7	SM41	27.9	45.2	22.3	Acute notch	Tension for plate	0.047	0.095	13.5	0.25	
8	HT55(R)	33.6	54.9	30.2	"	"	0.011	0.065	11.5	0.25	(20)
9	HT55(QT)	42.4	56.6	30.0	"	"	0.14	0.048	15.0	0.3	
10	SF45	28.4	46.5	20.5	"	Rotating bending	0.20	0.25	22.0	0.5	
11	SN CM	49.8	70.3	32.5	"	"	< 0.040	< 0.035	< 15.0	< 0.13	(18)
12	SF60	39.6 40.8	67.1 64.2	26.5	"	"	0.11	0.10	20.0	0.4	
13	S35C	31.1	56.9	-	Fatigue crack	"	-	0.13	17.5	-	
14	S55C	38.7	71.5	-	Pressed crack	"	-	0.030	10.5	-	(3)
15	Cr-Mo	200.4	205.1	80.0	"	"	0.0046	0.0011	10.5	-	
16	Ni-Cr-Mo (Hv=296)	78.1	90.2	48.0	"	"	0.018	0.013	14.0	-	
17	" (Hv=419)	122.9	136.1	66.0	"	"	0.013	0.0051	13.5	-	(21)
18	" (Hv=530)	159.9	174.2	75.0	"	"	0.0075	0.0020	11.5	-	
19	" (Hv=646)	191.4	213.1	80.0	"	"	0.0070	0.0014	12.0	-	
20	" (Hv=723)	160	170	69.0	"	"	0.0065	0.00145	10.0	-	
21	0.22%C	29.5	44.1	20.2	Fatigue crack Acute notch	Tension for plate Rotating bending	0.13	0.15	18.0	-	(22), (1)
22	0.05%C	23.3	29.5	-	Acute notch	Tension for plate	-	0.22	17.5	-	(23)
23	SN C1	78.8	90.5	48.5	Fatigue crack	Rotating bending	0.017	0.013	14.5	-	(25)
24	Ni-Cr	78.7	94.5	49.6	Fatigue crack Acute notch	Tension for plate	0.027	0.022	18.5	-	(24)
25	0.53%C	29.8	61.4	26.8	Fatigue crack	Rotating bending	0.023	0.105	15.5	-	(26)
26	0.08%C	20.0	31.7	19.2	"	"	0.013	0.15	12.5	-	
27	0.12%C	26.0	39.7	23.0	"	"	0.020	0.094	13.0	-	
28	S35C	33.9	58.7	26.5	Acute notch	"	0.025	0.052	12.0	0.3	(27)
29	Eutectic steel	36.9	72.3	29.0	Fatigue crack Acute notch	"	0.023	0.045	12.0	> 0.2	(2)
30	S50C	35.3	63.7	27.0	"	"	0.031	0.050	12.5	> 0.1	
31	S20C	28.5	47.9	21.5	Acute notch	"	0.040	0.071	12.0	0.5	
32	S25C	30.3	50.4	26.0	"	"	0.027	0.10	15.0	0.5	(30), (31)
33	S35C	34.3	61.2	28.0	"	"	0.036	0.093	16.5	0.4	(29)
34	S10C	20.7	38.0	18.8	"	"	0.028	0.12	12.0	0.6	(34)
35	S30C	31.5	54.8	24.9	"	"	0.030	0.060	12.0	0.4	
36	S50C	35.4	68.7	27.4	"	"	0.028	0.050	12.5	0.25	
37	SNC3	86.5	97.1	47.0	"	"	0.050	0.026	22.0	0.4	(28)
38	SN CM26	116.3	141.7	57.5	"	"	0.031	0.012	21.0	~0.05	(32)
39	SCM3 (QT)	83.0	97	41.0	"	"	0.055	0.022	19.5	0.15	(35)
40	SCM3(R)	38.4	67.4	28.0	"	"	0.045	0.060	15.0	0.2	

σ_y : Yield strength or 0.2% proof strength, σ_B : Tensile strength, σ_{wo} : Fatigue limit of plain specimen,
 a_o : Critical crack length corresponding to σ_{wo} ,
 $R_{p(cw)}$: Cyclic plastic zone size at fatigue limit which expressed as Eq.4,
 K_w : Stress intensity factor at fatigue limit in small scale yielding condition,
 ρ_o : Critical notch root radius at branch point

Fig. 7 The persistent slip band zone size at fatigue crack tips

(a) Eutectic steel (rotating bending)⁽²⁾

(b) 0.22%C steel (axial loading for plates)⁽¹⁾

(c) JIS SS41 mild steel (acute notch)⁽¹⁵⁾⁽³⁶⁾

Fig. 8 Rearrangements of other results

Eq. (3). Similar effects of minimum stress are observed on a plain specimen of high hardness steel⁽³⁹⁾ and in corrosion fatigue⁽⁴⁰⁾.

4.2 The relations between R_{pdcw} and other characteristics at fatigue limit

In the criteria for the fatigue limit of crack initiation from a notch root, the inherent length of material is also considered to be important. These material constants^{(41)~(43)} are similar to R_{pdcw} as shown in Fig. 10. Therefore, it can be considered that the limit of crack initi-

Fig. 9 The alternating stress intensity factors at fatigue limit in small scale yielding for various steels

Fig. 10 The relation between material constants and yield strength

Fig. 11 The Effect of minimum stress on the fatigue limit of cracked material (0.22%C steel)⁽²²⁾⁽³⁸⁾

ation at a notch root and the fatigue limit of a cracked material are based on a similar mechanism.

Figure 12 shows the relations between $R_{p(c)w}$ and the equivalent crack length a_0 which corresponds to the fatigue limit of a plain specimen. The line of this

Fig. 12 The relation between $R_{p(c)w}$ and equivalent crack length of plain specimen

Fig. 13 Fatigue limit of material with a notch

Fig. 14 Relation between $R_{p(c)w}$ and notch root radius at branch point.

figure shows the next relation:

$$a_0 = R_{p(c)w} \quad \dots\dots\dots (11)$$

Above equation may relate to the facts that the plain specimen also has slip band cracks in some grains and that $R_{p(c)w}^*$ is nearly equal to the grain size of the material. When $R_{p(c)w}$ is below 0.005 mm, a_0 does not decrease any more. It means that the fracture origin changes from cyclic slip bands in grains to non-metallurgical inclusions.

A notched material has two kinds of fatigue limit as shown in Fig. 13. One is the fatigue limit for crack initiation and the other is the fatigue limit for crack propagation. The branch point of them is important to evaluate the strength of the material which has flaws or notches. It is appropriate to indicate the branch point with the root radius ρ_0 of the notch at this point, because the stress distribution depends mainly on the radius of notch roots⁽³³⁾. Figure 14 shows the relation between $R_{p(c)w}$ and ρ_0 . They approximately hold the next relation:

$$\rho_0 = 5 R_{p(c)w} \quad \dots\dots\dots (12)$$

From Eq. (10), we can obtain the relation:

$$\rho_0 = 450 \sigma_y^{-2} \quad \dots\dots\dots (13)$$

We can not explain the phenomena of non-propagation of a crack at a notch using

Fig. 15 $R_{p(c)w}^*$ at crack tips emanating from a circular hole

the stress intensity factor because the stress intensity factor increases monotonically with an increasing crack length from the notch. So, we will investigate this phenomenon with the relation of Eq. (7). For convenience's sake, we use the equation for a circular hole for stress distribution near a notch root on tensile loading. The stress $\sigma(a)$ at the distance a from the notch root is shown as follows:

$$\sigma(a) = \frac{\sigma_{\max}}{3} \left\{ 1 + \frac{1}{2} \left(\frac{1}{1+a/\rho} \right)^2 + \frac{3}{2} \left(\frac{1}{1+a/\rho} \right)^4 \right\} \dots (14)$$

where, $\sigma_{\max}$ is the stress at the notch root and ρ is the notch radius. The stress intensity factor of a crack which emanates from a notch root is fairly well expressed by using the crack length a and the stress $\sigma(a)$ (44). And R_{pcw}^* is small enough comparing with the notch radius as shown later. Then we can obtain the expression R_{pcw}^* , using the value C of Eq. (8) as 0.1 and ignoring the effect of the slope of stress.

$$R_{pcw}^* = 0.1258 a \left(\sec \frac{\pi \sigma_{\max}}{2 \sigma_y} - 1 \right) \dots (15)$$

Figure 15 shows the calculated results of above equation. When the maximum stress at a notch root is larger than the yield strength of the material, R_{pcw}^* has a maximum value. If R_{pcw}^* is smaller than R_{pcw}^* , a crack can not propagate any more even after the crack initiates from the notch root. Above critical condition that on the branch point the maximum stress at a notch root is close to the yield strength of material, is the same as formerly obtained by Ishibashi (45). From Fig. 15, when $\sigma_{\max}$ is σ_y , the minimum value of R_{pcw}^*/ρ_0 is obtained as about 0.022, namely, $\rho_0 \approx 46 R_{pcw}^*$ = 4.6 R_{pcw} . This relation is close to Eq. (11) obtained by experiment.

5. Conclusions

From the test results on the fatigue limit of mild steel which cracks, it was tried to bridge the gap between the continuum mechanics aspects and the metallographic aspects and the relations among various characteristics at fatigue limit were investigated. The main results obtained are as follows.

- (1) The relation between alternating fatigue limit and crack length was well expressed by assuming that the cyclic plastic zone size R_{pc} is a material constant R_{pcw} .
- (2) The persistent slip band zone size R_{psw}^* is about one tenth of R_{pc} , and is close to the grain size at fatigue limit.
- (3) Under the small scale yielding condition, the fatigue limit of a cracked material can be expressed by the stress intensity factor, which is not so much influenced by the variety of steels.

- (4) R_{pcw} is close to the length corresponding to the fatigue limit of a plain specimen and the material constant which was already obtained in the criterion for the fatigue limit of crack initiation at a notch root.
- (5) The characteristics of non-propagation of a crack in a sharp notched cyclic plastic zone size at a crack tip, and the critical notch root radius at the branch point is about five times of R_{pcw} .

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