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PRECISE EXPERIMENTAL RESULTS AND AN ANALYTICAL FORMULATION OF THE SUBSEQUENT PLASTIC DEFORMATION OF MILD STEEL SUBJECTED TO PRE-STRESSING*

By Yoshio OHASHI**, Koichiro KAWASHIMA*** and Susumu NAGAHIKO****

Thin-walled tubular specimens of annealed mild steel were pre-stressed plastically by tension or torsion, unloaded perfectly and re-loaded proportionally by combined loadings of axial load and torque, as fundamental examples of investigating the plastic properties of the material used in received condition. The experimental results show that the shapes of equi-strain curves for the reloading on the deviatoric stress plane differ considerably from Mises' circle for smaller equivalent strains less than 0.5 %. This means that the isotropic and kinematic hardening models can not express precisely the real material property of the material mentioned above. Then, a new expression of plastic stress-strain relation is proposed based on the general tensor equation between noncoaxial tensors. The plastic strain tensor for the reloading is expressed as a function of the tensors of the pre-stress and the reloading stress. The relation obtained approximates closely the experimental results.

1. INTRODUCTION

Most metallic materials used for structural members or plastic working have an anisotropy due to the primary cold working. When precise evaluation on the plastic behavior of these materials is required for various types of loadings, it is necessary to take account of this anisotropy. In this connection, many experimental investigations⁽¹⁾ have been carried out to clarify the effect of deformation history on the plastic behavior.

In most of these investigations, the effect of deformation history was treated in the framework of the plastic flow theory. The plastic flow theory is well established, but, it has the following difficulties for analysis of the succeeding plastic deformation with strong effect of the deformation history. Extension of the concept of the yield surface, introduced first for the ideally plastic material, to the plastic deformation of actual materials yields many subsequent yield surfaces whose shape markedly depends on the strain values to define them. Thus, in order to formulate the plastic behavior of actual materials, the subsequent yield surface should be expressed by a functional of the deformation history, which loses the advantage of the flow theory in

giving a simple and convenient formulation.

In general deformation analysis, it is sufficient to find for each material element a constitutive equation considering the effect of deformation history only on the succeeding deformation of the element. From this standpoint, Il'yushin⁽²⁾ proposed a general plastic theory to formulate the plastic behavior with strong effect of the deformation history. In his theory, the deformation history is determined only by the shape of a curve in the vector space that traces out the deformation process; for example, the shape of a strain trajectory in the deviatoric strain space or that of stress trajectory in the deviatoric stress space, and the deformation behavior at any point on the curve (stress-strain relation) may be expressed as a functional relation that includes the intrinsic geometrical parameters of the curve (viz. curvature, torsion and the like).

In the present paper, in order to investigate the effect of the deformation history on the plastic behavior, the plastic behavior of thin-walled tubular specimens of mild steel, perfectly unloaded from an assigned pre-stressed state, is measured precisely for combined proportional loadings of axial force and torque. The loading history is represented as trilinear trajectories in the deviatoric stress space, whose first branch corresponds to the pre-loading in the form of uniaxial tension or torsion, the second branch does to its perfect unloading, and the third does to the reloading. The results obtained are formulated as a stress-strain relation in the form of a tensor equation between noncoaxial ten-

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** Professor, Faculty of Engineering, Nagoya University, Nagoya

*** Lecturer, College of General Education, Nagoya University

**** Engineer, Kawasaki Heavy Industries Ltd.

sors in the plane-stress state.

2. SPECIMEN AND EXPERIMENTAL APPARATUS

2.1 Specimen

The thin-walled tubular specimen shown in Fig. 1 was machined precisely from a bar (diameter 43 mm) of annealed mild steel (furnace-cooled after soaking at 880°C for 1 hr). Chemical composition of the steel are shown in Table 1.

In order to secure specimens with identical mechanical properties, the grain size and orientation of each specimen were examined under microscope (x200). The result showed that the grain orientation was quite isotropic, pointing to the possibility of the macroscopic isotropy. The specimens whose mean grain-area laid within $1.8 \sim 2.3 \cdot 10^{-4} \text{ mm}^2$ were selected for the experiment in accordance with the grain-size dependence of the mechanical property mentioned in a previous paper⁽³⁾.

2.2 Experimental apparatus

The testing machine used is an automatic combined loading type described in the previous paper⁽³⁾. It permitted a combined application of tensile or compressive load, torque and internal pressure to the above specimens with a precise control system.

Induced deformation of the specimen was measured precisely by differential transformers as relative axial and angular displacements between knife-edges of the specimen and a circumferential relative displacement as well, from each of which the corresponding strain components could be estimated. This measuring device can detect precisely not only a large displacement but also a minute one without re-setting.

The data of load and deformation could be recorded automatically and continuously over a time interval.

Table 1 Chemical composition of the specimen (%)

C	Si	Mn	S	P
0.10	0.24	0.44	0.009	0.018

Fig. 1 Specimen

3. EXPERIMENTAL PROCEDURE

Two kinds of loading programs on the deviatoric stress plane (Fig. 2) were applied to the specimen by combined loadings of axial stress σ_{11} and shear stress σ_{12} . The loading group A consists of the tensile pre-stressing corresponding to the pre-strain 2 % or 4 %, its perfect unloading and subsequent proportional reloadings with various values of angle θ . The loading group B differs from A only in torsional pre-stressing corresponding to the equivalent strain 2 % or 4 %.

The values of angle θ , directions of proportional reloadings measured from the direction of the pre-stress, are entered in Fig. 2.

The reason why the torsional pre-stressing is considered besides the tensile one is that the torsional stress-strain curve of mild steel appears lower than the tensile one, and thus the effect of the torsional pre-stressing may be expected to differ from that of the tensile one.

Pre-strain 2 % and 4 % lie in the strain-hardening range of the stress-strain curves.

A load increment corresponding to the equivalent stress $0.5 \sim 1.2 \text{ kg/mm}^2$ was applied step-wise after the average equi-

(a) tensile pre-strain

(b) torsional pre-strain

No.	1	2	3	4	5	6	7
θ	0°	30°	60°	90°	120°	150°	180°

Fig. 2 Loading paths in deviatoric plane

valent strain-rate over each loading step had attained $3 \times 10^{-6}/s$.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Experimental results are expressed in terms of the true stress and the plastic strain measured from the beginning of the proportional reloadings, which followed perfect unloading of the pre-stressing. A plastic strain occurring during the unloading process was very small, i. e., about 0.02 %.

In the following, the results for the case of pre-strain 4 % are mainly presented for example.

4.1 Equi-strain curve

An equi-strain curve for the proportional reloading is obtained by connecting the point having the same magnitudes of the equivalent plastic strain ϵ_p [$= \sqrt{(e_{11}^p)^2 + 4(e_{12}^p)^2}/3$, e_{11}^p : axial plastic strain, e_{12}^p : shear plastic strain) on each proportional reloading path in the deviatoric stress plane. Figures 3 to 5 show these equi-strain curves, where the abscissa and ordinate denote the dimension-

Fig. 3 Equi-strain curves (tensile pre-strain 4 %)

Fig. 4 Equi-strain curves (torsional pre-strain 4 %)

Fig. 5 Equi-strain curves (tensile pre-strain 2 %)

less stresses reduced by the pre-stress $\bar{\sigma}_{11}$ or $\sqrt{3}\bar{\sigma}_{12}$ ($\bar{\sigma}_{11} = 29.3 \text{ kg/mm}^2$ for tensile pre-strain 4 %, $\sqrt{3}\bar{\sigma}_{12} = 27.0 \text{ kg/mm}^2$ for torsional one 4 %, and $\bar{\sigma}_{11} = 23.9 \text{ kg/mm}^2$ for tensile one 2 %).

As shown in Fig. 3, for the case of tensile pre-strain 4 %, equi-strain curves with $\epsilon_p^p \leq 0.5$ % differ clearly from Mises' circle corresponding to the pre-stress $\bar{\sigma}_{11}$ (shown by dash-dot line) and are flattened opposite to the pre-stress. This trend can not be expressed by the conventional isotropic or kinematic hardening model or by their simple combinations.

Equi-strain curves for the case of torsional pre-strain 4 % (Fig. 4) are almost similar to those for the tensile one, however, the former with small values of ϵ_p^p is slightly extended in the normal direction to the pre-stress. This trend may be attributed to the fact that the tensile stress-strain curve appears higher than the torsional one.

Equi-strain curves for the case of tensile pre-strain 2 % (Fig. 5) are similar to those for pre-strain 4 % (Fig. 3), but, the former is slightly extended outside in comparison with the latter.

4.2 Stress-strain curve

Relations between the equivalent stress $\sigma_e (= \sqrt{\sigma_{11}^2 + 3\sigma_{12}^2})$ and the equivalent plastic strain ϵ_p in the proportional reloadings are shown in Figs. 6 and 7 for the tensile and torsional pre-strain, respectively.

In Fig. 6, the experimental curves depend considerably on θ , but beyond a certain transient range in ϵ_p , every curve runs parallel to the uniaxial tension

Fig. 6 Stress-strain curves (tensile pre-strain 4 %)

Fig. 7 Stress-strain curves (torsional pre-strain 4 %)

curve (dashed line). The length of the transient range increases with θ : that is, the length in ε_p is 0.1, 0.2, 0.4, 0.8, 1.0 % for $\theta = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ$ and it is 1.5, 1.7 % for $\theta = 150^\circ, 180^\circ$, respectively. After having become parallel to the uniaxial tension curve, each curve appears lower than that except for $\theta = 0^\circ$. In particular, the curve for $\theta = 180^\circ$ appears lowest (Bauschinger effect) and the stress value at $\varepsilon_p = 2\%$ appears lower by 8 % than that of the uniaxial tension curve.

For the same stress value, the difference between strain values on the curves with different values of θ is fairly remarkable, therefore, the difference due to the direction of reloading can not be neglected without serious error in estimating the plastic behavior in the reloading.

In the case of the torsional pre-stressing (Fig. 7), the same trends mentioned above are also found. However, the experimental curves for $30^\circ \leq \theta \leq 120^\circ$, in particular one for $\theta = 90^\circ$, appear higher than the pure torsion curve (dashed line). This may be due to the fact that the tensile stress-strain curve appears higher than the torsional one. The transient range mentioned above is longer than that for the tensile pre-stressing: that is, the length in ε_p is 0.1, 0.3, 0.7, 1 % for $\theta = 0^\circ, 30^\circ, 60^\circ, 90^\circ$, and it is 1.9, 2.3, 2.5 % for $\theta = 120^\circ, 150^\circ, 180^\circ$, respectively.

4.3 Deviation of plastic strain vector from deviatoric stress vector

The direction of the deviatoric stress vector remains constant during each proportional reloading process (Fig. 2). Then, let us introduce a local plane of plastic strain ($e_{11}^p - 2e_{12}^p/\sqrt{3}$) at each point on the

Fig. 8 Deviation of plastic strain vector from deviatoric stress vector (tensile pre-strain 4 %)

Fig. 9 Deviation of plastic strain vector from deviatoric stress vector (torsional pre-strain 4 %)

stress trajectory and denote the deviation of the plastic strain vector from the deviatoric stress vector by $\Delta\theta$ in Fig. 2.

Figures 8 and 9 show the relations between $\Delta\theta$ and ε_p , where the results for $\theta = 0^\circ$ and 180° are not entered because $\Delta\theta = 0^\circ$ always holds in these cases. These figures show that the plastic strain vector is not coaxial with the deviatoric stress vector even in the proportional reloading process, if the material has been subjected to pre-stressing. For every θ , $\Delta\theta$ takes a large value for small values of ε_p and decreases quickly to a constant value with an increase in ε_p . If two vectors are regarded as coaxial for $\Delta\theta \leq 6^\circ$ as in the previous paper⁽³⁾, the coaxiality holds always for $\theta = 30^\circ$ and 60° . The coaxiality holds also when $\varepsilon_p \geq 0.6, 1.0, 1.6\%$ even for $\theta = 90^\circ, 120^\circ, 150^\circ$ (Fig. 8). For the case of torsional pre-strain, it holds also when $\varepsilon_p \geq 0.4, 2.4, 3.3\%$ for the above values of θ (Fig. 9).

The trends mentioned above also appear in the cases of pre-strain 2 %. However, the values of ε_p corresponding to $\Delta\theta \leq 6^\circ$ diminish to 50 to 80 % of those for pre-strain 4 %.

5. ANALYTICAL FORMULATION OF STRESS-STRAIN RELATION

In the following, the stress-strain relation only for the proportional reloading is considered because the plastic strain occurring during unloading may be neglected. Moreover, it is assumed that the hydrostatic pressure does not affect the plastic deformation, besides the plastic incompressibility of the material. The plastic strain components are estimated by subtracting the elastic strain corresponding to the stress value from total values of strain components obtained by measurement, by assuming that the elastic moduli are not affected by plastic deformation.

5.1 Tensor relation in the plane-stress state

In what follows, the plane-stress state is assumed, which is expected to be realized in the thin-walled tubular specimen. An orthogonal coordinate system is chosen whose axes 1, 2 and 3 are directed to the axial, circumferential and radial directions of the specimen, respectively.

The tensors of deviatoric stress and plastic strain referred to this coordinate system may be written for the plane-stress state:

$$\left. \begin{aligned} D_\sigma &= \begin{pmatrix} \sigma_{11} - \sigma & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} - \sigma & 0 \\ 0 & 0 & -\sigma \end{pmatrix}, \quad \sigma = \frac{1}{3}(\sigma_{11} + \sigma_{22}), \\ T_\varepsilon &= \begin{pmatrix} e_{11}^p & e_{12}^p & 0 \\ e_{12}^p & e_{22}^p & 0 \\ 0 & 0 & e_{33}^p \end{pmatrix}, \quad e_{11}^p + e_{22}^p + e_{33}^p = 0 \end{aligned} \right\} \quad (1)$$

Analogously, the tensor of the pre-stress deviator is written as

$$D_r = \begin{pmatrix} \bar{\sigma}_{11} - \bar{\sigma} & \bar{\sigma}_{12} & 0 \\ \bar{\sigma}_{12} & \bar{\sigma}_{22} - \bar{\sigma} & 0 \\ 0 & 0 & -\bar{\sigma} \end{pmatrix}, \quad \bar{\sigma} = \frac{1}{3}(\bar{\sigma}_{11} + \bar{\sigma}_{22}) \quad (2)$$

where pre-stress is distinguished by a bar over the symbol.

These tensors have jointly the axis-3 as the common principal axis and their components referred to this axis remain constant for a rotation of the coordinate system about this axis. Thus, it is sufficient to consider the two-dimensional tensors in the plane that consists of axes 1 and 2.

$$\left. \begin{aligned} D_s^{(2)} &= \begin{pmatrix} \sigma_{11} - \sigma & \sigma_{12} \\ \sigma_{12} & \sigma_{22} - \sigma \end{pmatrix}, \quad T_s^{(2)} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{12} & \epsilon_{22} \end{pmatrix} \\ D_r^{(2)} &= \begin{pmatrix} \bar{\sigma}_{11} - \bar{\sigma} & \bar{\sigma}_{12} \\ \bar{\sigma}_{12} & \bar{\sigma}_{22} - \bar{\sigma} \end{pmatrix} \end{aligned} \right\} \quad (3)$$

The angle between the first principal axis of $D_s^{(2)}$ and the axis-1 is denoted by ω_s and those for $T_s^{(2)}$ and $D_r^{(2)}$ are denoted by ω_r and ω_r .

Tensors in the form of Eq. (3) may be referred to the principal axes of $D_r^{(2)}$ by the following relations.

$$\left. \begin{aligned} D_s^{(2)} &= \sigma G_1 + \sigma^- \{ \cos 2(\omega_s - \omega_r) G_2 \\ &\quad + \sin 2(\omega_s - \omega_r) G_3 \} \\ T_s^{(2)} &= \epsilon G_1 + \epsilon^- \{ \cos 2(\omega_s - \omega_r) G_2 \\ &\quad + \sin 2(\omega_s - \omega_r) G_3 \} \\ D_r^{(2)} &= \tau G_1 + \tau^- G_2 \end{aligned} \right\} \quad (4)$$

where

$$\left. \begin{aligned} G_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad G_2 = \begin{pmatrix} \cos 2\omega_r & \sin 2\omega_r \\ \sin 2\omega_r & -\cos 2\omega_r \end{pmatrix} \\ G_3 &= \begin{pmatrix} -\sin 2\omega_r & \cos 2\omega_r \\ \cos 2\omega_r & \sin 2\omega_r \end{pmatrix} \end{aligned} \right\} \quad (5)$$

are orthonormal base tensors⁽⁴⁾ related to the principal axes of $D_r^{(2)}$. σ or σ^- is one half of the difference or sum of the principal values of $D_s^{(2)}$, and ϵ , ϵ^- , or τ , τ^- are the like of $T_s^{(2)}$ or $D_r^{(2)}$, respectively.

After eliminating G_2 and G_3 from Eq. (4), the following tensor identity is obtained for the case of $\omega_s \neq \omega_r \neq \omega_r$.

$$\begin{aligned} \frac{T_s^{(2)}}{\epsilon^-} &= \frac{\sin 2(\omega_s - \omega_r)}{\sin 2(\omega_s - \omega_r)} \frac{D_s^{(2)}}{\sigma^-} \\ &\quad + \frac{\sin 2(\omega_s - \omega_r)}{\sin 2(\omega_s - \omega_r)} \frac{D_r^{(2)}}{\tau^-} \\ &\quad + \left\{ \frac{\epsilon}{\epsilon^-} \frac{\sin 2(\omega_s - \omega_r)}{\sin 2(\omega_s - \omega_r)} \frac{\sigma}{\sigma^-} \right. \\ &\quad \left. - \frac{\sin 2(\omega_s - \omega_r)}{\sin 2(\omega_s - \omega_r)} \frac{\tau}{\tau^-} \right\} G_1, \end{aligned} \quad (6)$$

and for the case of $\omega_s = \omega_r = \omega_r$

$$\begin{aligned} \frac{T_s^{(2)}}{\epsilon^-} &= \frac{(\epsilon/\epsilon^- - \sigma/\sigma^-)}{(\tau/\tau^- - \sigma/\sigma^-)} \frac{D_r^{(2)}}{\tau^-} \\ &\quad + \frac{(\tau/\tau^- - \epsilon/\epsilon^-)}{(\tau/\tau^- - \sigma/\sigma^-)} \frac{D_s^{(2)}}{\sigma^-} \end{aligned} \quad (7)$$

The identity (6) or (7) can be used as the equation to determine $T_s^{(2)}$ if $D_s^{(2)}$ and $D_r^{(2)}$ are prescribed. The scalar parameters ϵ , ϵ^- and ω_s may be functions of τ , τ^- , ω_r , σ , σ^- , ω_s and their histories.

In the present experiment,

$$D_r^{(2)} = \begin{pmatrix} 2\bar{\sigma}_{11}/3 & 0 \\ 0 & -\bar{\sigma}_{11}/3 \end{pmatrix} \text{ or } D_r^{(2)} = \begin{pmatrix} 0 & \bar{\sigma}_{12} \\ \bar{\sigma}_{12} & 0 \end{pmatrix} \quad (8)$$

corresponds to the tensile pre-stress or torsional one, and

$$D_s^{(2)} = \begin{pmatrix} 2\sigma_{11}/3 & \sigma_{12} \\ \sigma_{12} & -\sigma_{11}/3 \end{pmatrix}, \quad T_s^{(2)} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{12} & -\epsilon_{11}/2 \end{pmatrix} \quad (9)$$

Scalar parameters included in the identity (6) or (7) may be expressed as

$$\left. \begin{aligned} \sigma &= \sigma_{11}/6, \quad \sigma^- = \sqrt{\sigma_{11}^2 + 4\sigma_{12}^2}/2, \\ 2\omega_s &= \tan^{-1}(2\sigma_{12}/\sigma_{11}), \quad \epsilon = \epsilon_{11}^2/4, \\ \epsilon^- &= 3\sqrt{(\epsilon_{11}^2)^2 + (4\epsilon_{12}^2/3)^2}/4, \\ 2\omega_r &= \tan^{-1}(4\epsilon_{12}^2/3\epsilon_{11}^2), \\ \text{for tensile pre-stress} \quad \tau &= \bar{\sigma}_{11}/6, \quad \tau^- = \bar{\sigma}_{11}/2, \quad \omega_r = 0, \\ \text{for torsional pre-stress} \quad \tau &= 0, \quad \tau^- = \bar{\sigma}_{12}, \quad \omega_r = \pi/2 \end{aligned} \right\} \quad (10)$$

5.2 Relations between scalar parameters

The parameters σ , σ^- and ω_s may be expressed by σ_s and θ (Fig. 2) as

$$\left. \begin{aligned} \sigma_s &= \sqrt{\sigma_{11}^2 + 3\sigma_{12}^2} = \sqrt{3} \sqrt{(\sigma^-)^2 + 3\sigma^2}, \\ \sigma^- &= 3\sigma \sqrt{1 + \tan^2 2\omega_s}, \\ \text{for tensile pre-stress} \quad \tan 2\omega_s &= 2 \tan \theta / \sqrt{3}, \\ \text{for torsional pre-stress} \quad \tan 2\omega_s &= 2 \cot \theta / \sqrt{3} \end{aligned} \right\} \quad (11)$$

by using Eq. (10) and the definitions of σ_s and θ .

Then, the parameters ϵ , ϵ^- and ω_s (of $T_s^{(2)}$) may be considered functions of σ_s and

Fig. 10 Relation between ϵ and σ_s (tensile pre-strain 4 %)

θ as well as τ , τ^- and ω_r . In determining these functional relations, it may be reasonable to take θ as the prime variable in comparison with pre-stress, judging from their effect on the experimental results shown in the previous Chapter.

The values of ϵ , ϵ^- and ω_r were thus calculated according to Eq. (10) and (11) from the experimental results with various values of θ . Figures 10 to 12 show for σ_r the results of calculation for tensile pre-strain 4% by various symbols, for example. In order to approximate the relation between ϵ or ϵ^- and σ_r for each value of θ as precisely as possible, the following expressions may be selected.

$$\epsilon = \kappa [(A_1 \sigma_r)^{n_1} \times 10^{-2} + B_1 \times \exp\{-(\sigma_r - m_1)^2 / 2S^2\}] \times 10^{-2} \quad (12)$$

where $\kappa = -1$ for $\theta \geq 90^\circ$ in the tensile pre-strain, and $\kappa = 1$ for other cases.

$$\epsilon^- = [(A_2 \sigma_r)^{n_2} \times 10^{-2} + B_2 \times \exp\{-(\sigma_r - m_2)^2 / 2S^2\}] \times 10^{-2} \quad (13)$$

In the above equations, S is selected as 2.2 kg/mm^2 .

Fig. 11 Relation between ϵ^- and σ_r , (tensile pre-strain 4%)

Fig. 12 Relation between ω_r and σ_r , (tensile pre-strain 4%)

In Eqs. (12) and (13), the first term on the right hand side approximates well the results shown in Fig. 10 or 11 in the range where the value of σ_r is either small or large, and the second term modifies the difference between the results shown by the symbols and the values given by the first term.

As shown in Fig. 12, the value of ω_r approaches uniformly a constant value with an increase in σ_r for each value of θ . Such a trend may be approximated by the following relation.

$$\omega_r = K + A_3 \exp\{-B_3(\sigma_r - m_3)^2\}, \quad \sigma_r \geq m_3 \quad (14)$$

5.3 Coefficients of scalar parameters as functions of reloading direction

The coefficients A_1 , n_1 , B_1 , ..., K , A_2 , B_2 and m_2 depend on the pre-stress besides the angle θ designating the direction of proportional reloading. In order to express the variations of coefficients in relation to θ , the coefficients A_2 and n_2 in Eq. (13) are plotted for θ consider-

Fig. 13 Relation between A_2 and θ

Fig. 14 Relation between n_2 and θ

ing the pre-stress as a parameter, for example. These results are shown in Figs. 13 and 14 by various symbols. As seen in these figures, for the same kind of pre-strain, the relation between each coefficient and θ for pre-strain 2 % is almost similar to that for pre-strain 4 %. Such a feature is also seen with other coefficients. This feature may be expressed by a linear relation

$$Y(2) = \alpha Y(4) + \beta \quad (15)$$

where $Y(2)$ denotes the value of a certain coefficient for pre-strain 2 %, $Y(4)$ does the value of the same coefficient for pre-strain 4 %, and α, β are constants. Therefore, if the relation between $Y(2)$ and θ is known, the relation between $Y(4)$ and θ is easily found from Eq. (15), and vice versa.

The relation (15) is not valid for the yield step and just after that because the effect of pre-strain has known to be complicated^{(3),(5)}.

The coefficients of the scalar parameters for pre-strain 4 % may be expressed by functions of θ as follows.

(1) Coefficients for ε :

$$\left. \begin{aligned} A_1, n_1 &= \sum_{i=0}^{12} C_i \cos(i\theta), \\ B_1, m_1 &= \sum_{i=0}^6 C_i \cos(i\theta) \\ A_1 &= C_0 + \sum_{i=0}^{12} C_i \sin(i\theta), \\ B_1 &= C_0 + \sum_{i=0}^9 C_i \sin(i\theta) \end{aligned} \right\} \quad (16)$$

Fig. 15 Stress-strain curves (tensile pre-strain 4 %)

Fig. 16 Plastic strain paths in deviatoric plane (tensile prestrain 4 %)

(n_1, m_1) are expressed in the same form as above, for torsional pre-stress.

(2) Coefficients for ε^- :

$$A_2, n_2, B_2, m_2 = \sum_{i=0}^6 C_i \cos(i\theta) \quad (17)$$

for both pre-stresses.

(3) Coefficients for ω :

$$\left. \begin{aligned} A_3 &= \sum_{i=0}^9 C_i \cos(i\theta), \\ B_3, m_3 &= \sum_{i=0}^6 C_i \cos(i\theta) \end{aligned} \right\} \quad (18)$$

for tensile pre-stress,

$$m_3 = C_0 + \sum_{i=0}^6 C_i \sin(i\theta)$$

(A_3, B_3) are expressed in the same form as above, for torsional pre-stress.

Numerical values of the coefficients C_i 's, which vary from case to case, are omitted for limitations of space.

5.4 Validity of the formulation

The results on $\varepsilon, \varepsilon^-$ and ω for tensile pre-strain 4 % calculated by using Eqs. (12) to (18) together with coefficients C_i 's mentioned above are entered in Figs. 10 to 12 as curves. These curves agree well on the whole with the corresponding experimental results.

The stress-strain curves calculated by using Eqs. (6) and (7) together with the empirical relations (12) to (18) are entered in Fig. 15. The calculated trajectories of plastic strain in the deviatoric strain space are shown in Fig. 16 by those curves. The dashed lines in this figure denote the directions of proportional reloading. As seen in these figures, the calculated results agree well with the corresponding experimental result on the whole.

For the case of torsional pre-strain 2 %, 4 % and tensile one 2 %, the same degree of approximation as mentioned above was obtained.

6. CONCLUSIONS

The plastic behavior of thin-walled tubular specimens of mild steel was observed in detail under the combined proportional reloadings of axial load and torque, which followed the tensile or torsional pre-stressing and its perfect unloading. The effect of the pre-stress on the plastic deformation in the proportional reloading process was discussed.

Experimental results for the proportional reloading were formulated as a stress-strain relation, based on a two-dimensional tensor relation which can take account of the effect of the deformation history. The stress-strain relation derived could express well the experimental results.

The main results obtained are summarized as follows.

(1) The equi-strain curves with $\varepsilon_p \leq 0.5$ %, obtained for the proportional reloading, differ evidently from Mises' circle and are flattened opposite to the pre-stress. This fact means that the plastic deforma-

tion of actual materials can not be expressed precisely by the conventional isotropic or kinematic hardening model or their simple combinations.

(2) The stress-strain curves in the proportional reloading depend markedly on its direction θ to the pre-stress in the deviatoric vector space. However, the curve for every θ runs parallel to the extended curve of pre-stressing beyond a certain range in ϵ_p (transient range). The amount of ϵ_p in the transient range is less than 1 % for $\theta \leq 90^\circ$, whereas it is 2.5 %, for $\theta = 180^\circ$. The stress-strain curve for $\theta = 180^\circ$ appears lower than the extended curve of the pre-stressing. Since the values of plastic strain for the same value of stress on the reloading stress-strain curve vary in a wide range in relation to θ the effect of pre-stress on the plastic behavior in the reloading can not be neglected without serious error for structural analysis.

(3) The plastic strain vector in the deviatoric plane is not coaxial with the devi-

atoric stress vector even in the proportional reloadings, due to the effect of pre-stress. The former deviates from the latter in opposite direction to the pre-stress. If the above two vectors are regarded as coaxial when the angle between them is less than 6° , the coaxiality holds always for $\theta \leq 60^\circ$. However, for $\theta = 120^\circ$ and 150° , the coaxiality is expected to hold after ϵ_p attains 1~3 %.

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