An Analysis of the Lateral Hunting Motion of a Two-Axle Railway Wagon by Digital Simulation
(2nd Report, The Results of the Simulation Experiments)
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Making use of the mathematical model of a 2-axle wagon running on the track with irregularities described in the 1st report, the influences of the track irregularities and of the wagon parameters on the dynamics of the 2-axle wagon are investigated. An index is also proposed which might be employed as a measure of the magnitude of the influence of the track irregularities on the hunting motion of the 2-axle wagon.

1. Introduction

In the 1st report**, a mathematical model of a 2-axle wagon running on the straight track was proposed and it was modified so as to be applicable on the curved track. With this model, even the phenomenon of near-derailment can be considerably well simulated.

In this report, the factors which may influence the derailment due to the lateral hunting motion are studied by the simulation experiments with this model. Concerning the track, the influences of the irregularities on the derailment ratio are investigated, and an index of the irregularities of the track which might be used roughly to estimate the adverse effect on the 2-axle wagon is proposed showing some examples of applications to actual tracks. Effects of variations in the parameters of the 2-axle wagon on the derailment ratio are also examined.

2. Effects of the Track Irregularities

The motion of a 2-axle wagon is considered to be particularly susceptible to the track irregularities and the main cause of its derailment may be the lateral hunting motion induced and enlarged by the track irregularities. In this chapter, the influences of variations in the wave length and in the number of waves in the sinusoidal track irregularities on the lateral hunting motion of the 2-axle wagon are investigated with simulation experiments.

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Fig.1 Effects of the wave length of the track irregularities on the derailment ratio.
waves in the track is set at 3, while the wave length is changed as 10 m, 15 m, 20 m and 25 m. The result of the simulation experiments is shown in Fig. 1, where only the maximum values of the derailment ratio for each wheel are indicated. At 40 km/h the effect of variations in the wave length on the derailment ratio is not so significant, though the derailment ratios are slightly larger in the cases where the wave lengths are 10 m and 15 m. On the other hand, at 60 km/h the derailment ratios are significantly larger in the cases of the wave length 15 m and 20 m than in other cases. In these cases, the cause of larger derailment ratios may lie in the fact that the wave length of the station-ary hunting motion of the 2-axle wagon is around 15 m, almost coinciding with that of the track irregularities, and in the fact that the rolling motion of the body is almost in resonance with the exciting frequency which is determined by the wave length of the track irregularities and the running speed of the vehicle.

2.2 Effects of the Number of Waves of the Track Irregularities

In this section, the effects of the number of consecutive waves in the track irregularities of the dynamic responses of the 2-axle wagon will be discussed. Simulation experiments are carried out under the same conditions as before. The curves in Fig. 2 show the maximum derailment ratios $Z_{	ext{max}}$ and the maximum fluctuation ratios $D_{	ext{max}}$ of the wheel load (the load decrease divided by the wheel load, %) in each half wave of the response curves for each wheel at 40 and 60 km/h. At 40 km/h the value of $Z_{	ext{max}}$ remains almost constant in spite of an increase of the wave number $n$ in the track irregularities for $n \geq 3$. At 60 km/h the value of $Z_{	ext{max}}$ increases with an increase of the wave number $n$ in the track irregularities with an abrupt increase to 1 at $n = k$ showing the occurrence of undesirable hunting. Thus at 60 km/h the existence of three or four consecutive waves of the track irregularities, even under the condition that the amplitude of the alignment irregularity is 20 mm and that of the cross-level irregularity is 5 mm, may exert undesirable effects of magnifying the rolling motion and of reducing the wheel load.

2.3 Evaluation of the Track Irregularities

So the phenomenon of the derailment has been analyzed from a point of view of the wagon dynamics. In this section, a quantitative index is proposed, which might be employed as a measure of the magnitude of the potential influence of the track irregularities on undesirable motions of 2-axle wagons in conjunction with the derailment. The track irregularities are usually measured with a chord 10 m in length as the standard. The track irregularities thus measured might not be an adequate measure of expressing their potential influences on the motion of a 2-axle wagon. Two-axle wagons of the type treated here are considered to be susceptible to a component of the track irregularities of approximately 20 m wave length. If this is true, for this type of 2-axle wagon it may be preferable to use some other index which is more closely related to the magnitude of such a component, rather than to the absolute values of the irregularities in usual sense.

As the index suitable for such a purpose, the standard deviation of the values of the irregularities over a certain length of the track might be employed. In Japan, the track irregularities are usually measured at every five meters moving the standard along the track. Let the values of irregularities thus measured at every 5 m be $X_1, X_2, X_3, \ldots, X_n$. Then, the mean value $X$ and the standard deviation $S_n$ of the section of $5(p-1)$ m in length are given by

$$X = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$S_n = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - X)^2}$$

where $n = p + 1, \ldots, p$ is the number of the data used for the
calculation of the standard deviation of this section. For example, when \( p=5 \), with the length of 20 m, the component of the irregularities having the wave length of 20 m is most strongly reflected in this standard deviation.

Fig. 3 shows the standard deviation which are computed for various values of \( p \) based upon the data obtained from the track of the Tokaido Line between Tsurumi and Yokohama at the time of a derailment accident which occurred in November 1963. As shown in this figure, the standard deviations of both alignment and cross-level irregularities increased near the point where the derailment occurred. The fact that these values are likely to be large and flat in the case of \( p=5 \) and \( 3 \) may suggest that the component of the wave length ranging from 16 m to 20 m was predominant in the irregularities of the track near this point.

Fig. 4 shows the results of the computation of the standard deviations using the data of the track where derailment accidents occurred in the past. Assuming that the track irregularities having the wave length of 20 m have a predominant effect upon the derailment, the computations are carried out by setting the length of the section at 20 m, i.e. taking \( p=5 \). In this figure, the full lines represent the standard deviations \( s_a \) of the alignment irregularities and the broken lines represent the standard deviation \( s_r \) of the cross-level irregularities. Every curve indicates that the value increased near the point where the derailment occurred. This implies that the irregularities in these sections contained much of the component of the wave length of about 20 m. The sum \( s \) of \( s_a \) and \( s_r \) could be also suggested as an example of the index of the resultant irregularities, which is shown also with chain lines in Fig. 4.

3. Effects of the Wagon Parameters

The effects of the wagon parameters upon the derailment are examined by varying the wagon parameters of the mathematical model. With the parameters of the 2-axle wagon of WARA I type with full load running on the straight track with 3 waves of sinusoidal irregularities at 60 km/h (the amplitude of the alignment irregularity is 20 mm and that of the cross-level irregularity is 5 mm) changed by \( \pm 50 \% \) from the original values, the maximum values of the derailment ratios are computed and the results are shown in Fig. 5. Chain lines plotted with \( x \) marks indicate the maximum values of the derailment ratios in the case of the standard wagon, full lines and dotted lines indicate those when the value of one of the parameters is increased and decreased by 50 \% respectively.

As shown in Fig. 5(a), the values of the derailment ratios of the front wheels decrease with a decrease in the lateral stiffness of the suspension of the wagon. This result implies that the decrease in the lateral stiffness of the suspension in a double link suspension system may be effective in alleviating the hunting motion. Fig. 5(b) shows that the effect of the friction of the suspension in the lateral direction is small.

As to the vertical restoring force of the suspension, Fig. 5(c) and (d) show that the frictional force plays a more important role than the restoring force in absorbing the energy and alleviating the hunting. The natural frequency of the rolling vibration of the body depends on the stiffness of the suspension spring, and it coincides with the forced frequency, in-
Introducing a resonant vibration. The friction in the suspension system has the effect to alleviate this vibration. This effect of the decrease in the amplitude of the vibration is small when the frictional force is either too small or too large. The value of this friction force of the standard wagon is considered to be near the optimum.

Fig. 5(e) and (f) show that the effect of the stiffness and friction of the suspension in the longitudinal direction is insignificant.

Fig. 5(g) shows that the derailment ratio becomes larger when the mean clearance between rail and flange of wheel is either too small or too large. The mean clearance of 5 mm of the standard wagon seems to be adequate and this result suggests the importance of the maintenance of the track gauge.

Fig. 5(h) shows that the derailment ratio becomes larger for an extremely small equivalent spring constant between rail and flange at collision which also depends on the lateral stiffness of the rail. This tendency may be explained by the reasoning that the amplitude of the hunting motion becomes larger owing to the small restoring force and leads to a larger wagon movement.

The effects of the variations in the adhesion between rail and wheel are shown in Fig. 5(i), where the creep coefficient and the friction coefficient are changed simultaneously. When the adhesion is large, the slip between rail and wheel hardly occurs. This may increase the possibility of the climb-on derailment. Thus decreasing the adhesion by greasing may be effective in preventing the climb-on derailment.

Fig. 5(j) implies that the decrease in the distance between two axles leads to the decrease in the stiffness against the yawing motion of the body and finally to the decrease in the derailment ratio.

Fig. 5(k) and (l) show that the smaller mean radius of the wheels and the larger inclination of the surface of the wheel result in a shorter wave length of the geometric hunting motion of the wheelset.

Fig. 5 Effects of the wagon parameters on the derailment ratios ($\frac{\lambda_{max}}{\lambda}$ refers to the maximum value of the derailment ratio for each wheel).
Besides, a larger hunting motion of the wagon itself is induced in such cases, because the wheelsets follow the track irregularity more exactly. On the other hand, if the mean radius is larger and the inclination is smaller, difficulties may be caused in running over a curved track with a small radius of curvature, though the effect of the track irregularity upon the hunting motion will be smaller in this case.

Recently, instead of the conventional wheel tread of a conical profile, an N-type wheel tread whose inclination is zero in the small range near the middle of the width of the wheel was proposed. The wave length of the geometric hunting motion of the wheelset with the N-type tread is greater than that of the conventional wheelset. A mathematical model which is capable of dealing with the wheel tread having a curved profile has been developed and is described in the appendix. Making use of this model, computations have been carried out concerning the ordinary tread and the N-type tread, and the results are shown in Fig.6 together with the results obtained with the ordinary tread using the previous model. Though it is seen that the new model gives somewhat smaller values of the derailment ratio, the comparison of the result of the N-type tread with that of the ordinary tread shows that the N-type tread is effective in decreasing the derailment ratio.

In this analysis, the speed of the wagon is kept constant, the condition of the track irregularities is fixed and only one of the wagon parameters is changed at a time. Besides, the results are evaluated only by the values of the derailment ratio. For more general discussions, further extensive studies will be required.

4. Conclusions

The results obtained in this study are summarized as follows.

(1) An isolated single wave in the track irregularities is relatively less hazardous, while a series of several consecutive waves may have the potentiality of magnifying the rolling and increase the possibility of a derailment.

(2) The wave length of the irregularities, which determines the period of the excitation from the track combined with the wagon speed, is an important factor of governing the behavior of the wagon, so that the index of the track irregularities proposed in this paper might be effectively utilized.

(3) The phenomenon of the derailment is largely affected by the decrease in the wheel load due to the rolling motion of the body, which depends strongly on the characteristics of the suspension.

(h) As the N-type wheel makes the wave and hence the period of the hunting motion longer, the speed at which the resonance of the body with this hunting occurs may deviate from the ordinary operating speed. If the hunting motion is thus alleviated, the generation of the track irregularity with a constant wave length might also be retarded.

A mathematical model has been developed which can give similar results to those obtained in the field experiments. It will be further improved and will possibly contribute to clarification of the derailment accidents of 2-axle wagons.

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These simulations were carried out using the computers of Tokyo University Computer Center.

Appendix:

A New Mathematical Model

A mathematical model of the 2-axle wagon taking the wheel tread profile into consideration is developed. As the flange of wheel can be considered an extended part of the wheel tread, the phenomenon of the derailment can be simulated.

In order to set up the new mathematical model, the following assumptions are additionally made.

(1) The creep coefficient of each wheel increases in proportion to the 2/3th power of the wheel load. The lateral creep coefficient is equal to the longitudinal creep coefficient and the resultant creep force does not exceed the possible frictional force.

(2) The variation $x_{R_t}$ of the radius of the wheel at the point of contact with rail due to the relative displacement of wheelset to rail is taken into account.

(3) The deflection of the track is also taken into account so as to be able to deal with the dynamic characteristics of
the track.

(4) The lateral force on each wheel is given as the sum of the lateral creep force and the side force due to the wheel load and the inclination of the profile of the wheel tread.

The motion of each axle is expressed as follows:

\[ \begin{align*}
W_s \frac{\partial Y_s}{\partial s} &= F_{iy} + Q_s - Q_{is} \\
W_s \frac{\partial \Phi_s}{\partial s} &= -BF_s \\
A(F_{iy} - F_{iy1}) + A \Phi_s (n_s P_s + n_s F_s) 
\end{align*} \]

The equations of motion of the body are given as follows:

\[ \begin{align*}
W_s \frac{\partial Y_s}{\partial s} &= F_{iy} + F_{iy1} \\
W_s \frac{\partial \Phi_s}{\partial s} &= B(F_{iy} + F_{iy1}) + L(F_{iy} - F_{iy1}) \\
W_s \frac{\partial \Phi_s}{\partial s} &= -B(F_{iy} + F_{iy1}) + H(F_{iy} + F_{iy1}) \\
\end{align*} \]

The restoring forces \( F_{iy} \), \( F_{iy1} \) and \( F_{iy2} \) of the suspension structure can be obtained by the following procedure. The deflection of the suspension structure due to the relative displacement between the body and each wheelset in each direction can be given as follows:

\[ \begin{align*}
X_{is} &= Y_i - Y_s + L \Phi_s - H (\Phi_i - \Phi_s) + \frac{dR_{iy} - dR_{iy1}}{A} \\
X_{is1} &= B (\Phi_i - \Phi_s) + \frac{dR_{iy1}}{A} \\
X_{is2} &= B (\Phi_i - \Phi_s) \\
\end{align*} \]

( - for \( i = 1 \), + for \( i = 2 \))

\[ \begin{align*}
X_{is} &= Y_i - Y_s + L \Phi_s - H (\Phi_i - \Phi_s) + \frac{dR_{iy} - dR_{iy1}}{A} \\
X_{is1} &= B (\Phi_i - \Phi_s) + \frac{dR_{iy1}}{A} \\
X_{is2} &= B (\Phi_i - \Phi_s) \\
\end{align*} \]

The restoring forces caused by these deflections of the suspension structure are assumed to be expressed as the curves shown in Fig.3 of Ref.(1).

The variation \( dR_{iy} \) of the radius of wheel at each point of contact with rail can be given by the curves shown in Fig.7. The inclination of the surface of the wheel tread at the point of contact with rail is given by

\[ \alpha_i = \frac{d}{dX_i} R_i = \frac{AR_{iy} - dR_{iy}}{A} \pm \Phi_s \]

( - for \( j = 1 \), + for \( j = 2 \))

Taking the deflection \( X_i \) of the track into consideration as shown in Fig.8, the lateral relative displacement of the wheelset rail is given as follows:

\[ K X_{i1} + C X_{i2} = Q_{is} - Q_{is1} \quad X_{i1} = Y_{i1} - (Y_s + X_{i1}) \]

It is assumed that the traveling speed of the center of gravity of the wheelset is constant and neither the tractive force nor the braking force is acting on the wheelset. The creep forces of each wheel in the lateral and longitudinal directions are

\[ F_{iy1} = -f_i C_{iy1} \quad F_{iy2} = -f_i C_{iy2} \]

where the creep ratios are given by

\[ C_{iy1} = \frac{B}{R_1 + f_i} \quad C_{iy2} = \frac{B}{R_2 + f_i} \]

( + for \( j = 1 \), - for \( j = 2 \))

\[ C_{iy1} = \frac{Y_s}{V} - \Phi_i \]

The creep coefficient is assumed to be given by

\[ f_i = K_i P_{iy}^{1/3} \]

Since the resultant creep force does not exceed the possible frictional force, if the resultant creep ratio thus calculated \((C_{iy1} + C_{iy2})^{1/2}\) exceeds the critical creep ratio \( C_{iy10} = P_{iy}/f_{iy} \), the creep forces should be substituted by the possible frictional forces which are given by

\[ F_{iy1} = -f_i C_{iy10} \left( \frac{C_{iy10}}{C_{iy10} + C_{iy20}} \right)^{1/2} \]

\[ F_{iy2} = -f_i C_{iy20} \left( \frac{C_{iy10}}{C_{iy10} + C_{iy20}} \right)^{1/2} \]

The wheel load on each wheel can be obtained as follows:

\[ \begin{align*}
P_{iy} = \frac{W_s}{2} + \frac{W_s}{2} + \frac{1}{2} W_i \left( Y_i + L \Phi_i \right) \\
- Y_i + Y_{i1} + W_i X_{i1} - B F_{iy} - R_i (F_{iy} + W_i Y) \\
\end{align*} \]

( - for \( j = 1 \), + for \( j = 2 \))

The lateral force can be expressed as

\[ Q_{is1} = \frac{a_i}{P_{iy}} \]

( + for \( j = 1 \), - for \( j = 2 \))

The derailment ratio is defined by

\[ Z_{iy} = \frac{Q_{iy}}{P_{iy}} \]

References


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