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Flow over Rectangular Cylinders Immersed in a Turbulent Boundary Layer*

(Part 2, Flow Patterns and Pressure Distributions)

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As continuation of the foregoing paper, the present paper describes the flow patterns around the rectangular cylinders immersed in a turbulent boundary layer and the pressure distribution on the surface of the cylinders. The effect of three parameters, i.e. h/δ , u_τ/U_∞ and the width-to-height ratio b/h on the pressure distribution is examined in some detail, where b and h are respectively the width and height of the cylinder, δ is the boundary-layer thickness, u_τ is the shear velocity and U_∞ is the free-stream velocity outside the boundary layer. The main results will be summarized as follows: (a) Three sets of measurements of the mean flow and turbulence are presented for the width-to-height ratios of 2.0 and 4.0 with $h/\delta = 1.13$ and 0.551. (b) The mean velocity profiles in the rear separation eddy are closely approximated by the velocity profile in a constant pressure half-jet theoretically obtained by Goertler. (c) The base pressure of the cylinders can be correlated with h in the form of a wall law. (d) The pressure distribution on the upstream face of the cylinders can be represented by a single curve which is irrespective of the width-to-height ratio and the boundary-layer characteristics.

1. Introduction

In the foregoing paper⁽¹⁾, the experimental apparatus, the characteristics of the turbulent boundary layer and the pressure drag of the rectangular cylinders were described. It was revealed that the drag coefficient can be expressed as a logarithmic function of the height of the cylinder in a similar manner to the mean velocity distribution of the boundary layer in the neighbourhood of the wall. The experimental result of the drag coefficient was rearranged in the form of an empirical formula. Accordingly, the pressure drag of a rectangular cylinder at least with a height of the magnitude of the boundary-layer thickness can be obtained in terms of the boundary-layer characteristics at the location of the cylinder.

Although the drag is admittedly one of the most important properties of a body immersed in a turbulent boundary layer, the other items such as the pressure distribution around the body and fluid-dynamic

structure of the separated eddies behind the body are also of practical importance. Especially, the structure of the near wake behind the body is closely related to the shielding effect of the body and the diffusion of heat or mass at the rear of the body. In recent years, this problem has received considerable attention in connection with the diffusion of radioactive release from the atomic-energy power plants. Moreover, the knowledge about the mean and turbulent properties in the separated eddies is also required to estimate the behavior of the exhaust gas from automobiles on the roads surrounded on both sides by a series of modern buildings.

Low-velocity reverse flow of the separated eddy is sometimes utilized to protect the river beds or banks from erosion. If a number of rectangular cylinders are arranged with appropriate spacings so that the separation streamline from a cylinder will reattach to the lateral surface of the cylinder just downstream, the river beds will be exposed only to a low-velocity reverse flow and hence will be free from erosion.

With these practical applications in mind, the present paper experimentally describes the mean flow and turbulence characteristics around the rectangular cylinder of typical shapes immersed in a smooth-wall turbulent boundary layer with zero pressure gradient. Although a group of the cylinder should be considered in some applications, the case of a single cylinder

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is nevertheless important as a fundamental information on such cases.

2. Nomenclature

The nomenclature of the foregoing paper will also be used in the present paper. In addition, the following three suffixes are employed here.

- u : upper surface of rectangular cylinder
- wf : plane wall upstream of cylinder
- wb : plane wall downstream of cylinder

3. Flow patterns

The mean-flow field around a rectangular cylinder was measured by a probe shown in Fig. 1. Two yaw-tubes were set parallel with the total-pressure and static-pressure tubes. A part of the mean velocity profiles was measured by a hot-wire anemometer of constant-temperature type.

When the measured magnitude and direction of the local velocity vector are respectively denoted by V and θ , the velocity component in the x-direction is given by

$$u/U_\infty = (V/U_\infty) \cos \theta$$

Further, the values of the nondimensional stream function $\psi/(U_\infty h)$ can be calculated as

$$\psi/(U_\infty h) = \int_0^{y/h} (u/U_\infty) d(y/h)$$

The streamlines thus computed are shown in Figs. 2 and 3, together with the longitudinal velocity component. The traces of flow patterns visualized by the oil-film method are also included in these figures. In the case of $b/h = 4.0$, it will clearly be seen that the flow separated from the

Fig. 1 Details of probe used for measurement of mean velocity vector

top edge A of the cylinder reattaches to the upper surface AD, while no indication of reattachment to the upper surface can be seen for $b/h = 2.0$. The surface-flow patterns also show that the reattachment to the plane wall downstream of the cylinder occurs at the distance about $7h$ for $b/h = 2.0$ and about $5h$ for $b/h = 4.0$, when measured from the rear surface of the cylinder. In this connection, it may be interesting to note that this distance is about $7h$ ($h/\delta = 0.75$ and $U_\infty = 28.0$ m/sec, Ref. 2) for the downward-facing step which will correspond to the present case of $b/h = \infty$. The reattachment point is located at the distance of about $13h$ ($h/\delta = 2.34$ and $u_T/U_\infty = 0.0358$) for the normal plate⁽³⁾ immersed in a turbulent boundary layer and about $17h$ ($U_\infty h/\nu = 5.52 \times 10^4$) for the normal plate with a splitter plate⁽⁴⁾ placed in unbounded uniform flow. Moreover, ac-

Fig. 2 Mean-flow pattern for $b/h = 2.0$ (streamline pattern corresponds to $h/\delta = 0.551$)

Fig. 3 Mean flow pattern for $b/h = 4.0$

R.P. = reattachment point

cording to Fig. 2, no appreciable difference in the velocity profiles can be observed in the two cases of $h/\delta = 0.551$ and 1.13.

The outer part of the separation eddy behind the cylinder may approximately be regarded as a region where the higher-velocity main stream is mixing with the lower-velocity stream downstream of the cylinder. The mixing process in this region will have much in common to the flow field where an upper stream of initially uniform velocity joins a lower stream of initially zero velocity obtained by Görtler⁽⁵⁾. Namely, if the higher uniform velocity is denoted by U_m , the mean velocity u in the mixing layer can be written as

$$\frac{u - (U_m/2)}{(U_m/2)} = \text{erf}\left(0.477 \frac{y - y_{1/2}}{y_{3/4} - y_{1/2}}\right) \quad (1)$$

Fig. 4 Comparison of velocity profiles through shear layer downstream of cylinder with half-jet velocity profile

where y is the transverse distance measured from an appropriate origin, $y_{1/2}$ and $y_{3/4}$ are the values of y where u attains the values of $U_m/2$ and $(3/4)U_m$, respectively. In Fig. 4, the velocity profiles for the case of $h/\delta = 0.551$ in Fig. 2 are compared with Eq. (1). A good agreement with the theoretical profile is observed in the range of $x/h \leq 7.0$. Owing to the effects of the plane wall in the lower-velocity side, the velocity profiles in the region $x/h \geq 8.0$ are not well correlated with Eq. (1).

Figures 5 and 6 show the distributions of static pressures and longitudinal turbulence intensities around the rectangular cylinder with $b/h = 2.0$. The general shape of the static-pressure distribution is similar to that reported by Good & Joubert⁽³⁾. The mean velocity is small in some region of the wake and hence the accuracy of the measurements of the turbulence intensities becomes poor. Since there are few reliable ways, at present, to measure the fluctuating-velocity components accurately in such a region, the conventional technique of measurement was adopted. As will be seen in Fig. 6, a maximum longitudinal turbulence as high as 30% of U_∞ occurs at about $y/h = 1.13$. Since the strength of the shear layer generated in the initial mixing process near the top edge A of the cylinder decreases as h/δ decreases, the turbulence level in the wake will also decrease unlike the case of higher h/δ . The experimental result shown in Fig. 6 confirms this tendency.

In the experiments of Arie & Rouse⁽⁴⁾, the maximum turbulence intensity occurred at the position $y/h \approx 2.5$ in the wake behind a normal plate placed in a uniform flow with a tail plate installed. The

Fig. 5 Static pressure distribution around rectangular cylinder

Fig. 6 Distribution of longitudinal turbulence intensities around rectangular cylinder

smaller value of y/h where the maximum turbulence is measured in the present experiments may be explained as due to the Coanda effect by which the separated shear layer is deflected towards the side of the upper surface AD of the cylinder. The existence of the Coanda effect is also evident in the fact that the maximum height of the dividing streamline is about $1.4h$ for the rectangular cylinder with $b/h=2.0$, while it is about $2.1h$ for the normal plate⁽⁴⁾ ($b/h = 0$).

4. Pressure distributions

Figure 7 shows the general shape of the pressure distribution on the surface of the rectangular cylinder and the plane wall for the case of $b/h = 2.0$. The pressure distribution will be governed by the three parameters, i.e. b/h , h/δ and u_τ/U_∞ ,

as explained in the foregoing paper⁽¹⁾. The effects of these parameters on the pressure distribution will now be described.

4.1 Pressure on plane boundary upstream of cylinder

Figure 8 shows the variation of the pressure distribution on the plane boundary upstream of the cylinder with respect to the parameter h/δ . The boundary layer separates from the boundary at about $x/h = -2.0$ owing to the adverse pressure gradient produced by the cylinder. As a result,

a region of smaller longitudinal pressure gradients appears in the range $-2 < x/h < 0$ for the higher values of h/δ . This region seems to disappear for the smaller values of $h/\delta = 0.392$ and 0.196 . Moreover, an abrupt increase of pressure occurs in the vicinity of $x/h = 0$ in the case of $h/\delta = 1.372$. This phenomenon was also observed by Bradshaw & Galea⁽⁶⁾ for the forward-facing step 57 mm high. Although it is reported by Good & Joubert⁽³⁾ that the relative extent of upstream influence of the normal plate increases as h/δ is reduced, such a tendency is not clearly observed in the present experiment, except in the case of $h/\delta = 1.372$.

The effects of the parameters b/h and u_τ/U_∞ are shown in Figs. 9 and 10, respectively. The upstream pressure level decreases only slightly as b/h is increased. It should be noted in Fig. 10 that the value of h/δ also changes simultaneously with the change of u_τ/U_∞ and hence the effects of these two parameters are not separated in this case.

Fig. 7 General shape of static pressure distribution on rectangular cylinder and plane wall

4.2 Pressure on boundary wall downstream of cylinder

The effects of the three parameters on the pressure distribution along the downstream boundary are shown in the upper parts of Figs. 8, 9 and 10. The variation of the pressure distribution with each of the parameters is almost the same as would physically be expected. Plate(3) obtained the result that the nondimensional pressure distribution plotted in the manner shown in these figures was independent of h/δ and u_τ/U_∞ . Although this conclusion may be true when $h/\delta \gg 1$, it cannot be applied to the case of $h/\delta < 1$.

4.3 Pressure on front surface of cylinder

Figure 11 shows the effects of the parameter b/h on the pressure distribution along the front and rear surfaces of the cylinder. The pressure level on the front surface decreases slightly as b/h is increased. However, the rate of decrease is almost the same for various values of h/δ .

Fig. 8 Effects of h/δ on pressure distribution along plane boundary upstream and downstream of cylinder

Fig. 9 Effects of b/h on pressure distribution along plane boundary upstream and downstream of cylinder

Fig. 10 Effects of u_τ/U_∞ on pressure distribution along plane boundary upstream and downstream of cylinder

The effects of h/δ and u_τ/U_∞ are shown in Figs. 12 and 13. As will physically be expected, the pressure level is seen to increase with the increase of u_τ/U_∞ .

The pressure distributions have similar forms, with maximum values somewhere in the range $0.5 \leq y/h \leq 0.6$. In Fig. 14, the nature of similarity is examined by plotting $(p_f - p_\infty)/(p_f - p_\infty)_{\max}$ versus y/h . Although a considerable scatter of the experimental data is observed, the pressure distribution can be expressed by a single curve passing through the center of the data, provided that some error of the order of $\pm 10\%$ is tolerated.

The authors (8)(10) presented a free-streamline theory for separated flows past two-dimensional bluff bodies attached to a plane wall accompanied by the development of a turbulent boundary layer. The theory involves three or four constants which must be determined on the basis of experimental informations. The pressure distributions around the normal flat plate and semicircular projection predicted by the theory were in good agreement with experimental measurements. The solid curve included in Fig. 14 is a pressure profile obtained by the theory (8) and is expressed by

$$(p_f - p_\infty)/(p_f - p_\infty)_{\max} = 1 - q_f^2 \quad (2)$$

(a) $h/\delta = 0.196$

(b) $h/\delta = 0.558$

(c) $h/\delta = 1.372$

Fig. 11 Effects of b/h on pressure distributions along upstream and downstream surfaces of cylinder

where

$$q_f = 1.91 \sin\theta (\cos\theta - 1.11)^{-1} + 0.457 \sin\theta \ln\{\tan^2(\theta/2)\}$$

and $y/h = \sin\theta$ ($\pi/2 \leq \theta \leq \pi$). Equation (2) is applicable in the range $y/h \leq 0.95$. The point where the pressure distribution on the front surface attains its maximum value corresponds approximately to the reattachment point of the streamline separated from the plane boundary wall upstream. The standing vortex is formed in front of the cylinder. These features are also included in this theory. Actually, however, the stagnation pressure at $y/h = 0$ is always smaller than the reattachment pressure, because the downward flow along the front surface must lose its mechanical energy due to the viscous shear stress acting on the surface. Moreover, the complicated structure of flow in the separated eddy cannot be treated by the free-streamline theory which assumes an inviscid fluid. In spite of these limitations, the theory is of practical importance in predicting the surface-pressure distribution and the geometrical shapes of the front and rear standing vortices.

When the ratio h/δ is less than unity, it is expected that the pressure distributions on the front surface can be correlated with the boundary-layer characteristics by the wall law of the form

$$c_{pfr} = (p_f - p_\infty) / \{(1/2)\rho u_\tau^2\} = f_1(hu_\tau/\nu, b/h, y/h) \quad (3)$$

In Fig. 15, the experimental data are plotted in the form suggested by Eq. (3). Since there exists a considerable scatter of the data, the wall law cannot exactly be confirmed by the present experiment.

Fig. 12 Effects of h/δ on pressure distributions along upstream and downstream surfaces of cylinder

4.4 Pressure on rear face of cylinder

The pressure at the downstream surface of rectangular cylinders is substantially constant. As shown in Fig. 11, the base pressure is considerably reduced as the ratio b/h increases. When b/h increases beyond the value of 6.0, the base-pressure coefficient will tend to a value of about -0.07 for the case of a downward-facing step⁽²⁾ ($h/\delta = 0.33 \sim 4.0$ and $U_\infty = 10 \sim 20$ m/sec, air-stream experiment). The wall law of the base pressure distribution will take the following form so far as the base pressure is conceptually accepted to be constant over the whole range of y/h on the rear surface of the cylinder:

$$c_{pb\tau} = (p_b - p_\infty) / \{(1/2)\rho u_\tau^2\} = f_2(hu_\tau/\nu, b/h) \quad (4)$$

Accordingly, the correlation (4) should be expressed in the same manner as $c_{pb\tau}$ which was given in Part 1 of the present paper:

(a) $b/h = 3.0$

(b) $b/h = 6.0$

Fig. 13 Effects of U_τ/U_∞ on pressure distribution along upstream and downstream surfaces of cylinder

Fig. 14 Similarity of pressure distribution on upstream surface of cylinder

$$-c_{pb\tau} = c_{Db\tau} = A_2(b/h) \log_{10}(hu_\tau/U_\infty) + B_2(b/h) \quad (5)$$

where the values of A_2 and B_2 are already given in the preceding paper. Moreover, the effects of the parameters h/δ and u_τ/U_∞ on the base pressure distribution are indicated in Figs. 11, 12 and 13 for the convenience of judgement.

4.5 Pressure on upper surface of cylinder

Figures 16, 17 and 18 show the effects of b/h , h/δ and u_τ/U_∞ on the pressure distribution along the upper surface AD of the cylinder. As will be seen in Fig. 16, the pressure in the neighbourhood of the top edge A decreases considerably as b/h increases, when the ratio b/h is less than 3.0. However, since the reattachment of flow to the upper surface is realized in the range $b/h > 3.0$, the pressure distributions on the upper surface have approximately similar shapes in this range of b/h . The relation between x/h and the pressure distribution in nondimensional form is shown in Fig. 19 to examine the nature of similarity. As will be seen in this figure, an excellent similarity is obtained for the case of $b/h = 4.0$ and 5.0.

A correlation in the form of wall law was also examined for the pressure distribution on the upper surface. However, the validity of the wall law was not confirmed in this case, owing to a considerably large scatter of the experimental data.

5. Conclusions

In the present paper, the flow patterns and the pressure distributions around rectangular cylinders immersed in a smooth-wall turbulent boundary layer with zero pressure gradient were experimentally cor-

Fig. 15 Wall law for pressure distribution on upstream surface of cylinder

related with the thickness and wall shear stress of the boundary layer at the location of the cylinders. The experimental range of parameters was $h/\delta \leq 1.37$ and $b/h \leq 6.0$. The main results of the present investigation can be summarized as follows:

- (1) The distribution of the mean velocities, static pressures, longitudinal turbulence intensities and stream functions around the rectangular cylinders with $b/h = 2.0$ and 4.0 are clarified. The geometrical shape of the rear standing vortex and the reattachment points of the separated flows are also obtained.
- (2) The velocity profiles in the outer region of the rear standing vortex are well correlated with the half-jet solution obtained by Görtler, at least in a region upstream of the reattachment point on the plane boundary wall. This fact means that

Fig. 16 Effects of b/h on pressure distribution along upper surface of cylinder

Fig. 17 Effects of h/δ on pressure distribution along upper surface of cylinder

Fig. 18 Effects of u_τ/U_∞ on pressure distribution along upper surface of cylinder

Fig. 19 Similarity of pressure distribution on upper surface of cylinder

the shear flow in this region is dominated by the turbulence generated in the initial mixing process near the top edge of the cylinder.

(3) The static pressure on the downstream surface of the rectangular cylinder is substantially constant and can well be described by the wall law of the form

$$(p_b - p_\infty) / \{(1/2)\rho u_\tau^2\} = A_2 \log_{10}(hu_\tau/\nu) + B_2$$

where A_2 and B_2 are functions of b/h alone. A_2 and B_2 are graphically given in Fig. 13 of Part 1 of the present paper.

(4) In the range of $b/h \geq 4.0$ and $0.2 \leq h/\delta \leq 1.37$, the pressure distribution on the upper surface of the rectangular cylinder can be expressed as

$$(p_u - p_\infty) / \{(1/2)\rho U_h^2\} = f(x/h)$$

The reattachment of flow to the upper surface occurs in the range of $b/h \geq 3.0$.

(5) If some error of the order of $\pm 10\%$ is tolerated, the pressure distributions on the upper surface of the rectangular cylinder can be expressed by a single curve irrespective of both the shape of the cylinders and the boundary-layer characteristics.

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References

- (1) Arie, M., et al., Bull. JSME, Vol. 18, No. 125 (1975-11).
- (2) Tani, I., et al., Aeron. Res. Inst., Univ. of Tokyo, Rep., No. 364 (1961).
- (3) Good, M. C. and Joubert, P. N., J. Fluid Mech., Vol. 31, Pt. 3 (1968), p. 547.
- (4) Arie, M. and Rouse, H., J. Fluid Mech., Vol. 1, No. 1 (1956), p. 129.
- (5) Görtler, H., Z. AMM, Vol. 22 (1942), p. 244.
- (6) Bradshaw, P. and Galea, P. V., J. Fluid Mech., Vol. 27, Pt. 1 (1967), p. 111.
- (7) Plate, E. J., Aerodynamic characteristics of atmospheric boundary layers, U. S. Atomic Energy Commission, Technical Information Center, Oak Ridge, (1972).
- (8) Kiya, M. and Arie, M., J. Fluid Mech., Vol. 56, Pt. 2 (1972), p. 201.
- (9) Fukusako, S., et al., Bull. of Faculty of Engineering, Hokkaido Univ. (in Japanese), Vol. 54 (1969), p. 155.
- (10) Sakamoto, H., et al., Trans. JSME (in Japanese), Vol. 41, No. 342 (1975-2), p. 526.