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Experimental Study on the Growth of Strain

by Thermal Ratchet Mechanism*

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There are many cases that a permanent deformations called plastic flow take place in the materials of severely heated parts of thermal engines and etc. caused by the ratcheting superposition of thermal and mechanical stress. In this paper, the ratcheting is first classified in various types of thermal load, and then, those elongation rates are theoretically analysed. Moreover, several experiments were performed in order to check whether the theoretical explanations were reasonable or not, using thin tubes which are heated rotationally or alternatively from outside, while they are pulled in axial direction. Besides those, the existence of critical conditions for the plastic flow generation was also studied theoretically and experimentally. Recently, as ratchet problems are becoming the subject of wide interest, it is desirable that this study can be of any service to further investigations.

1. Introduction

There are so many cases of combined steady mechanical stress and cyclic thermal stress in the stress problems of thermal apparatus, where the steady mechanical stress is caused by the pressure or centrifugal force, and the cyclic thermal stress is superposed by unsteady start and stop of the heat flux.

It is well known that under the effect of the combination of these stresses of the two kinds, a plastic flow of the material takes place in one direction in many cases and fatigue rupture is necessarily accelerated by thermal ratchet effects. The analysis of the mechanism of thermal ratchet itself was first reported by Miller(1)(2), who used a model of three columns, and then, his theory was extended to the cases including creep elongation by Isshiki (3) (4), who is one of the authors of this report. Also, a predictional equation of critical heat flux for thin wall tubes with inside pressure, which gives the maximum heat flux that does not cause the successional thermal ratchet elongation

was introduced by Isshiki(5) (6).

Since 1968, experiments of thermal ratchet phenomena using several methods including rotational heating method have been made in the authors' laboratory, and in their results, the elongations by plastic flow are shown to be in good agreement with the theoretical predictions and a limitation to maximum heat flux without ratchet effect is actually recognized.

In this report, the methods and the results of the experiments are described.

2. Several Patterns of Thermal Cycles

Simply considered, the values of permissible limiting stress or heat flux are different depending on the type of cyclic heating process, namely thermal cycle pattern and also on whether the material is brittle or ductile.

However, the patterns of thermal cycles may be classified in the following three cases.

(a) A-type thermal cycle (cyclic heating from one side)
 In the convectional boilers and heat exchangers, the heating surfaces are usually heated from one side under pressure loading, so one side thermal stress is cyclically superposed on mechanical stress. This heating process is called A-type thermal cycle in this paper, and it is illustrated in Fig-1a.

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(b) B-type thermal cycle (cyclic heating from both sides)

When a body is heated from both sides alternately, the heating process is called B-type thermal cycle in this paper, and the process is illustrated in Fig-1b.

(c) C-type thermal cycle (processional cyclic heating)

If a thick body is heated from one side by an oscillating heat source, the wave of temperature oscillations proceeds into the body as shown in Fig-1c, this type of heating is classified as C-type thermal cycle here. In this case, the thermal stress wave proceeds inside the body.

Fig. 1 Various types of thermal cycles

3. Theoretical analysis of the limitation of the growth of plastic flow in ductile material by various heating processes

3.1 Case of A-type thermal cycle

For a two-column model made of a ductile material which has elastic and plastic characteristics as shown in Fig.2, the generation of plastic flow by thermal ratchet mechanism caused by A-type thermal cycle is first analysed here.

As shown in Fig-3a if two bars I and II of same sectional area S and same length are suspended from a solid wall on one side, and the temperature of the bar I is maintained at a standard temperature ($T=0$), and then, the second bar II is heated to a higher temperature $T=T_0$ ($T_0>0$), the bar II makes thermal expansion at free ends.

Then if the lower ends of the two bars are forcibly constrained at equal length, and moreover, an axial load is applied to both bars by a weight W as seen in Fig-3b, the thermal stress $\sigma_t (= \pm E \alpha T_0 / 2)$ and mechanical stress σ_m are superposed on both bars.

In this case, the absolute stress on the bar I is more severe than that on the bar II, and if the stress of the bar I becomes higher than the yield stress σ_Y , a local plastic deformation takes place in the bar I.

For this case, if the stress and strain of the bars I and II are written as $\sigma_{I1}, \epsilon_{I1}, \sigma_{II1}, \epsilon_{II1}$ respectively, the relations between them are as follows:

Fig. 2 Elastic and plastic characteristics of materials

Fig. 3 Ratcheting at two bar model

$$S(\sigma_{I1} + \sigma_{II1}) = W \quad (1)$$

$$\epsilon_{I1} = \epsilon_{II1} + \alpha T_0 \quad (2)$$

$$\sigma_{II1} = E \epsilon_{II1} \quad (3)$$

$$\sigma_{I1} - \sigma_Y = E_p (\epsilon_{I1} - \epsilon_Y) \quad (4)$$

where, E and E_p are the elastic coefficients of elastic and plastic regions as illustrated in Fig.2.

From these equations, the next nondimensional expressions are introduced here.

$$\epsilon_{I1} = \frac{\sigma_Y}{E + E_p} \{ 2m + 2t - (1 - r) \} \quad (5)$$

$$\sigma_{I1} = \frac{E \sigma_Y}{E + E_p} \{ 2mr + 2tr + (1 - r) \} \quad (6)$$

In these equations, the following nondimensional notations are used,

$$\sigma_m = W/2S, \quad \gamma = E_p/E,$$

$$m = \sigma_m / \sigma_Y, \quad t = E \alpha T_0 / 2 \sigma_Y$$

Then, keeping σ_m at same value as above, if the thermal stress σ_t is removed by unloading of heat flux, the stress and strain of both bars ($\sigma_{I.2}, \epsilon_{I.2}$), ($\sigma_{II.2}, \epsilon_{II.2}$) reach a new equilibrium with the next relations,

$$\sigma_{I.2} + \sigma_{II.2} = 2 \sigma_m \quad (7)$$

$$\epsilon_{I.2} = \epsilon_{II.2} \quad (8)$$

Here in this new status if the stress of the bar II still remains in the original elastic region, no more plastic strain growth takes place in both bars with any next heating cycles.

But, if $\sigma_{II.2}$ becomes higher than σ_Y , some new plastic deformation should take place in the bar II, and the deformation can not be removed by any next heating cycles, but a new deformation is superposed again by next heating.

In the latter case, the following relations should be established,

$$\sigma_{II.2} - \sigma_Y = E_p (\epsilon_{II.2} - \epsilon_Y) \quad (9)$$

$$\sigma_{I.2} - \sigma_{I.1} = E (\epsilon_{I.2} - \epsilon_{I.1}) \quad (10)$$

So, using the equations from (5) to (10), the strain of the bar II under this new situation is calculated as,

$$\begin{aligned} \frac{\epsilon_{I.2} - 1}{\epsilon_Y} &= \frac{\epsilon_{II.2} - 1}{\epsilon_Y} \\ &= \frac{1}{(1+\gamma)^2} \{ 4m + 2(1-\gamma)t - 4 \} \end{aligned} \quad (11)$$

As well known, the superposing of the plastic deformation as above is called plastic flow, and this mechanism is called thermal ratchet.

By the equation (11), the limit condition of the generation of plastic flow is,

$$2m + (1-\gamma)t - 2 \leq 0$$

$$\text{or } m + (1-\gamma) \frac{t}{2} \leq 1 \quad (12)$$

and if $\gamma=0$, the condition is

$$m + \frac{t}{2} \leq 1 \quad (12)'$$

By the cyclic heating, the plastic deformation proceeds, but strain gets its saturated value ϵ_∞ by stress hardening effect if $\gamma > 0$ as follows:

$$E_p (\epsilon_\infty - \epsilon_Y) + \sigma_Y = \frac{E \alpha T_0}{4} (1+\gamma) + \sigma_m \quad (13)$$

so, the following non-dimensional equation is introduced.

$$\frac{\epsilon_\infty}{\epsilon_Y} - 1 = \frac{1}{\gamma} \left\{ \frac{t}{2} (1-\gamma) + m - 1 \right\} \quad (14)$$

3.2 Two-column model with B-type thermal cycle

In this case of B-type heating, only the heating of bar I to the temperature T_b should be taken in to account in the calculation of A-type thermal cycle, and the relation between $\epsilon_{I.2}$ and $\epsilon_{II.2}$ in this case becomes the following instead of the equation (8) of the A-type thermal cycle case.

$$\epsilon_{I.2} + \alpha T_0 = \epsilon_{II.2} \quad (15)$$

So, the size of plastic flow in this case becomes,

$$\frac{\epsilon_{II.2} - 1}{\epsilon_Y} - 1 = \frac{4}{(1+\gamma)^2} (m + t - 1) \quad (16)$$

And, the limiting condition for the generation of plastic flow is,

$$m + t \leq 1 \quad (17)$$

which is more severe than that in the case of A-type heating.

3.3 The analysis of the growth of the deformation in C-type thermal cycle

In the case of C-type thermal cycle, it is very difficult to get the plastic deformation $\Delta \epsilon$ for one cycle heating by analytical method rigidly, so $\Delta \epsilon$ is calculated by an electronic computer or analytical method with rough approximation.

For computer calculation, a heated body is divided into many vertical columns with uniform temperature in each column, but their temperature changes sinusoidally from one step to another.

From the results of the computer calculation, it is clear that the stress and strain distribution in a body falls into a closed loop of triangle shape on the stress and strain graph, and the loop proceeds rotationally with an increasing strain.

When the number of cycles increases, the status become semi-stationary, and the value of $\Delta \epsilon$ becomes a saturated final one. The saturated value of $\Delta \epsilon$ can be calculated also by analytical method under assumptions.

It is very interesting that the strain grows continuously without a stop in the C-type thermal cycle case.

If the stress strain characteristic of the material assumed to be an ideal elastoplastic one with $\gamma=0$, the final saturated value of $\Delta \epsilon$ is calculated as follows with various temperature distributions, of the temperature waves.

(a) Case of triangular shape wave:
If the proceeding temperature distribution is triangular shape, the value of $\Delta \epsilon$ becomes,

$$\Delta \epsilon = 3(3 - \sqrt{1 + 8\beta}) \alpha \cdot \frac{T_0}{2} \quad (18)$$

where

$$\beta = \frac{2(\sigma_r - \sigma_m)}{\alpha E T_0}$$

(b) Case of rectangular shape wave:
If the proceeding temperature distribution is rectangular shape, $\Delta \epsilon$ becomes,

$$\Delta \epsilon = 4(1 - \beta) \alpha \cdot \frac{T_0}{2} \quad (19)$$

(c) Case of sinusoidal waves:
If the temperature distribution is sinusoidal wave shape, $\Delta \epsilon$ is expressed approximately by the results of computer in the next from with the help of equation (18),

$$\Delta \epsilon = 6.15(3 - \sqrt{1 + 8\beta}) \alpha \cdot \frac{T_0}{2} \quad (20)$$

From the above three equations (18) to (20), the limiting condition for the generation of plastic flow is

$$1 \leq \beta = \frac{\sigma_r - \sigma_m}{\alpha E T_0 / 2}$$

which is simplified to

$$m + t \leq 1 \quad (21)$$

Including the other results already reported by the authors, all limiting conditions for the generation of plastic flow with various heating processes are shown in Fig. 4, which defines the limiting regions on $m-t$ plane.

Fig. 4 Limiting regions for the generation of plastic flow with various heating types

4. Experiments for plastic deformation caused by thermal ratchet mechanism

In order to observe actual cyclic-growth of plastic deformation by thermal ratcheting, experiments of two different kinds mentioned hereafter were performed in this investigation.

4.1 Thermal ratcheting under B-type thermal cycle

As shown in Fig. 5, a two-element assembly composed of two stainless steel bars (SUS 304) is set on a creep testing machine for the purpose of applying tensile stress at all times. On the other hand we perform air heating on one of the elements and air cooling on another. Under alternate repeating of heating and cooling for each we observe ratchet strain increments. Of course, in such situations the temperature of the elements is not always uniform in the axial direction, but its difference is measured to be negligibly small in the testing sections.

Fig. 6 shows an approximate time history of temperature variation of the elements.

Here, referring to the investigation⁽⁴⁾ on a three-bar assembly under B-type thermal cycle, the total tensile strain ϵ_w after N thermal cycles is given by

Fig. 5 Experimental set for two bar assembly

Fig. 6 Cyclic temperature variations applied to the two bar elements

$$\frac{\epsilon_N}{\epsilon_Y} - 1 = \left(\frac{\epsilon_\infty}{\epsilon_Y} - 1 \right) (1 - e^{-kN}) \quad (22)$$

and the quantity $\left(\frac{\epsilon_\infty}{\epsilon_Y} - 1 \right)$ is already defined by Eq. (14). For the parameter, we have

$$K = \frac{\epsilon_\infty - \epsilon_Y}{\epsilon_\infty - \epsilon_1} \quad (23)$$

Now, in order to get the value of various physical properties used in theoretical calculations, we performed other experiments using other specimens of the same kind. From these experiments it is found that the value of E_p is slightly influenced by temperature-degree, but in this study the value obtained at 40°C is adopted as typical. Furthermore, tensile yield stress and strain were determined by extending a straight line on the stress-strain diagram, namely, by extrapolation, since a clear yield point is unobtainable.

In Fig. 7 the experimental results for accumulated strain are compared with the theoretical results calculated by Eq. (22) which are indicated by the solid lines. As is evident from the figure, the experimental values are, as a whole, in considerably satisfactory agreement with the theoretical curves.

Fig. 7 Experimental results for accumulated strain

Fig. 8 Experimental apparatus for moving temperature variation together with vertical tensile stress

4.2 Thermal ratcheting under C-type thermal cycle

To realize in slab a movable horizontal-temperature-variation together with vertical tensile stress induced by external load, a testing device roughly shown in Fig. 8 was designed. The device can at the same apply not only a mechanical load to a specimen in the axial direction but also a uniformly moving temperature field in the circumferential direction. The thin wall tubular specimen is made of stainless steel tube (0.006%C, 9.5%Ni, 18.4%Cr) of which the dimensions are 14mm outside diameter and 1mm wall thickness.

Here, by applying electric current in the hollow cylindrical heater, the specimen tensioned axially by the creep testing machine is heated from the outside surface mainly by thermal radiation. On the other hand, as the cooling air is circulated through the air ducts which encircle the specimen at a constant rate, a moving temperature field is established on the surface of the specimen. Under the above steady conditions the behaviour of axial deformation of the specimen is observed by dial gauges at an appropriate time interval. The temperatures of the specimen and the heater are measured by thermocouples and a radiation pyrometer respectively.

Figure 9 (a) shows a circumferential temperature variation recorded actually for the specimen when the revolution of cooling device is $\frac{\pi}{27}$ (rad/sec). The temperature difference ΔT indicated in the figure, for instance, does not necessarily increase even if the revolutions are lowered. As for the pattern of the temperature field, it may be quite all right to specimen is shown in Fig. 9 (b). From this, we can recognize that a flat temperature region corresponds approximately to the heating region.

Fig. 9 Circumferential (a) and vertical (b) temperature variation

Now, to get data on the relationship between stress σ and strain ε under various temperature conditions we performed not a few material testings using specimens of the same kind under various conditions carefully controlled. From the above, the relationship is experimentally recognized to be fit for approximation by next expression (24) in the temperature region 0~400°C, though there exists a slight difference depending on the temperature degree.

$$\sigma = 4.77 (\varepsilon + 0.0015)^{0.22} \text{ kg/mm}^2 \quad (24)$$

and the modulus of elasticity in tension and compression, E , is

$$E = 1.65 \times 10^4 \text{ kg/mm}^2 \quad (25)$$

at the same temperature degree.

For the value of the coefficient of linear thermal expansion α , we adopt the next value,

$$\alpha = 1.6 \times 10^{-5} \text{ } ^\circ\text{C}^{-1} \quad (26)$$

Figure 10 shows the relation between an accumulated strain and the number of thermal cycles obtained by using the testing device shown in the former figure. As is easily seen from Fig. 10, ratchet strain increment per one thermal cycle has a tendency to decrease gradually with an increase of the cycling number. As the cause to a decreasing increment of plastic strain, quantitative rise of σ_y affected by strain-hardening may be considered. Of course, this is proved by the fact that the elastic upper limit of a specimen once used is higher about two times than that of one unused. Hence, in application of the parameter β defined by Eq. (20), it is necessary to correct

Fig. 10 Experimental results of accumulated strain with increasing thermal cycles

the value of β gradually in the process of numerical calculations, since β is obtained under the assumption that strain-hardening is absent. Accordingly, for the calculation of the theoretical curves shown by solid lines in the figure the following procedures were adopted.

(1) To begin with, picking up strain beyond the elastic limit, we calculate the corresponding stress to the strain by using Eq. (24).

(2) Regarding the computed value as a new tensile yield stress of the material under strain-hardening, we compute the value of β .

(3) Increment of strain $\Delta\varepsilon$ is obtainable by substituting the new value of β into Eq. (20).

(4) The accumulated strain ($=\sum \Delta\varepsilon$) is obtained by summing up the increment $\Delta\varepsilon$ computed for each thermal cycle. Then, the relation between the accumulated strain and the number of thermal cycles becomes clear.

As is easily seen from the figure, the results of calculation by the previous procedures agree fairly well with the experimental results, showing the propriety of the calculation. For the increasing disagreement in the region of large number of thermal cycles, however, the following causes may be considered:

- (1) An error due to assuming the actual shape of circumferential temperature field as sinusoidal wave.
- (2) Inaccuracy of representing stress-strain relation by Eq. (24).

5. Experiment on critical for the generation of thermal ratchet deformation.

To make clear the critical condition which decides whether thermal ratchet deformation occurs or not, an experiment on A-type thermal cycle frequently observed in industry was performed. For the present test, carbon steel (S35C) annealed (550°C, 6hrs) in a vacuum (10^{-5} mmHg) was used as the testing material. The test section of specimen is a hollow cylinder of which dimensions are 14mm outside diameter and 2mm wall thickness. Furthermore, the test section is divided axially by four slits running at equal interval.

Before going into main experiment, material testing of several specimens was performed so as to get stress-strain relation using dial gauges and strain gauges for the measurement of strain.

From this, as we obtained considerable coincidence of the experimental results, the following values are adopted for the physical property of the material.

$$E = 2.2 \times 10^4 \text{ kg/mm}^2$$

$$E_p = 1.6 \times 10^2 \text{ kg/mm}^2$$

$$\sigma_Y = 3.42 \text{ kg/mm}^2$$

$$\alpha = 1.4 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

Now, the specimen which is set on the creep testing machine (maximum tensile load 3 tons) is heated by a ceramic rod-heater running through center of the specimen. On the other hand, as is easily seen from a detail picture of A-A cross section in Fig. 11, any two partitions facing each other are forcibly cooled from outside surface by introducing the cooling air into air-ducts, while the remainder adjacent to them is not so much cooled by slits and casings. Hence, depending on presence or absence of air flow in the air-ducts thermal cycle can be given to the specimen. Temperature difference between the cooling partitions and the non-cooling ones is changeable by making allowance for air flow and electric heating density of the rod-heater. In this case, the difference is changed in the range of $30^\circ\text{C} \sim 60^\circ\text{C}$, which is measured by a radiational surface pyrometer.

By such procedures, with not only temperature difference T_0 but also sustaining mechanical load changed variously in the range of $80\text{Kg} \sim 150\text{Kg}$, we examined whether a permanent plastic-flow occurs or not.

Fig. 12 shows the degree of the plastic deformation in several specimens observed when the number of thermal cycles is 30. The value of abscissa is $m + 0.5(1-\gamma)t$, and a solid line shown in the figure means that the value is 1.

As is evident from Eq. (12), the left domain of the line is theoretically considered to be one where plastic flow doesn't occur.

From this figure, though experimental data are partly scattered and the number of the points plotted near the line is not abundant, we can deduce the existence of such a line, namely, the critical boundary.

In Fig. 13, the experimental data are plotted on $m \sim t$ plane. Judging from this figure, the points which indicate obvious occurrence of ratcheting are scattered in the domain defined by

$$m + \frac{t}{2} \geq 1 \quad (\gamma = 0)$$

The chief conceivable cause of the scattering is that the temperature distribution realized in the specimen is not always a complete-rectangular one obtained in the case of a two-bar assembly.

6. Conclusions

Fig. 11 Experimental apparatus for the determination of critical conditions for the generation of obvious thermal ratchet deformation

Fig. 12 Accumulated plastic deformation data after 30 cycle heating in the apparatus shown in Fig. 11

Fig. 13 Comparison of theoretical predictions and experimental data on the critical region for the generation of obvious thermal ratcheting

Taking up thermal ratcheting as an object of our investigation, we not only classified various ratcheting actually observed depending on the type of thermal load but also theoretically explained the thermal ratchet mechanism for each type above classified. Moreover, several experiments were performed in order to check whether our theoretical explanations were reasonable, and by comparing the results theoretically calculated with the experimental results the reasonableness of the former was proved.

Besides those, the existence of critical conditions for the plastic flow generation was also studied in this investigation. The experimental results of A-type thermal cycle proved the propriety of theories described in other papers⁽¹⁾⁻⁽³⁾ and cited in this paper. Now, considering not only many kinds of material and actual shape but also complexity of temperature field arising from complex thermal load, it is necessary to perform more investigations of thermal ratcheting besides the problems of creep and thermal shock.

Recently, as ratchet problems have become the center of wide interest,⁽⁷⁾⁽⁸⁾ the authors hope that this study can be of any use to further investigations.

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