An Experimental Investigation of Flow in Blade Passage of Radial Inflow Turbine*

By Akiya SUGIMOTO**, Kozo NAKAMURA*** and Ken MATSUMOTO****

In this paper, the flow patterns in the blade passage of a radial inflow turbine are presented in three dimensions. The technique of tracer particles is developed in order to make three-dimensional analysis of a photograph of particles taken by means of the conventional double-flash illumination. The development is attained by the methods in which the diameters of the circles of diffusion of the picture of particles taken out of focus are measured, because the perspective distance can be estimated from them.

The availability of this three-dimensional method is verified by the experiments; therefore, in three dimensions, the particle trajectories can be compared with the flow patterns predicted by the compressible inviscid theory. The results show in a radial portion that the trajectories are in fairly good agreement with the flow patterns under the condition of shock free entry, and that the secondary flow and the flow passing through the axial clearance are appreciably feeble. Further, the theory is inapplicable to the run of reduced speed of rotation.

1. Introduction

Many papers have been published on the performance and the design of radial inflow turbines⁽¹⁾⁻⁽⁴⁾. The influence of splitter vanes⁽⁵⁾, the performance under peculiar conditions⁽⁶⁾ and the performance of high speed small turbines and turbo-expanders⁽⁷⁾⁻⁽¹⁰⁾ were investigated. In order to survey the flow in the passage of blades, the pressure measured on the blade surface^{(11),(12)} and the flow pattern measured by a yaw-meter⁽¹³⁾ were reported and the results were compared with the theoretically calculated values. These results have been very useful for the design of a radial inflow turbine.

The manufacture of a radial inflow turbine is easier than that of an axial flow turbine, but the radial inflow turbine has high efficiency only as a prime mover of low power. One of the demerits of the radial inflow turbine is low efficiency of blades as compared with that of the axial flow turbine. On this problem the papers mentioned above^{(1),(5),(11)-(13)} were available. These papers, however, were not enough to show the flow in the blade passage. The progress of the radial inflow turbine must be obstructed by the low efficiency of the blade. In the present experiments, the flow in the blade passage is investigated by means of the flow visualization using a

tracer. Since the flow is a three-dimensional one in the blade, it is preferable to investigate the flow also three-dimensionally. There are a few methods to investigate the third dimension of the flow, e. g., the use of a five-hole yaw-meter and the flow visualization from two directions ought to be suitable. However, these methods can not be applied structurally to conventional radial inflow turbines, for these turbines are too small. In case of the use of tracers, the general photograph can be taken easier than the stereoscopic photograph. The third dimension of the position and the velocity of a tracer can be obtained by diameter measurement of the circle of diffusion of the image of it. The results of experiments are compared with the prediction calculated theoretically on the assumption of inviscid flow.

2. Experimental equipment and method

Many techniques have been developed for the flow visualization. Recently, the quantitative measurements for flow patterns have been widely investigated. The visualization of a flow in the turbine which is running at very high speed is not easy even if any one of these methods is used. Insufficient as the use of tracers is for the quantitative measurement, it is suitable for visualizing a high speed gas flow; therefore, it is used here. The difference between a streamline and a corresponding path line is roughly estimated and it is discussed later.

A double-flash method⁽¹⁵⁾ is used for the measurement of velocity of a tracer. The conventional double-flash method makes two-dimensional measurement of velocity possible. In this experiment, the following technique is devised for the three-dimensional measurement. If a photograph is taken under the

* Received 14th September, 1973.

** Assistant, Faculty of Engineering, University of Osaka Prefecture, Sakai.

*** Professor, Faculty of Engineering, University of Osaka Prefecture.

**** Engineer, Hitachi Shipbuilding and Engineering Co., Ltd.

out of focus condition, the image of a tracer will become a circle of diffusion. The greater the absolute value of the distance s from a focal plane to the tracer, the greater the diameter δ of the circle. An aluminium flake used as the tracer is very small (less than 5μ), and it reflects light only in part. In this experiment, therefore, an object of the photograph can be assumed as a point source of light. In a narrow range, the relation between the diameter δ and the distance s is expressed approximately as a quadratic equation. Then, s can be estimated from the datum δ , and the error of s is reckoned within ± 0.2 mm in this equipment.

It is very important to choose the material for a tracer. The character required of the tracer is that a shot is easy, and that the entrainment coefficient K defined as in the following equation⁽¹⁶⁾ is large,

$$\frac{dv}{dt} = K(U - v) \dots \dots \dots (1)$$

Here, U is the velocity of air, v the velocity of the tracer and t the time. The coefficient K is affected by the specific weight, the shape, the size of the tracer, the relative velocity $U - v$ and the properties and the state of ambient fluid. Aluminium flakes are adopted as tracers in this experiment because of the merit of the high reflection for illumination, that is, because of the high facility for photographing among the conventional tracers. However, as the specific weight is fairly great, the disadvantage of the aluminium flake is that the entrainment coefficient K is low. The maximum value of K was approximately equal to 33000 s^{-1} in reference (16). In the preliminary experiments⁽¹⁷⁾, it was reported that the aluminium flakes available on the market contained many flakes of which the coefficient was larger than 10000 s^{-1} . The entrainment coefficients of a sphere, a disk and so on are regarded in steady flow as constants⁽¹⁸⁾, when the Reynolds number R_e is less than 2. If this relation holds with the coefficient of aluminium flakes, it is advisable that the velocity difference between the flake and the ambient air be set less than several m/s. In such case, the streamline can be easily estimated from the path lines. It is a rough assumption in practice that the coefficient K is constant. Because the flake has an indeterminate figure, rotating in the air, and because K is affected by the acceleration⁽¹⁹⁾ due to an unsteady flow, which is turbulent. As a result of rough calculation of the trajectory estimated in the same way as that of a centrifugal separator⁽²⁰⁾, it is found that the velocity difference can be kept to within several m/s even for the high speed flow in the present turbine, provided that K is greater than 20000 s^{-1} . Thus the flakes of which K is very small are excluded by means of a wind tunnel. The principle of the device for selection is the same as given in reference (21).

Since the absolute velocity of flow in the turbine is very high under the general condition, the shot of tracers is difficult even by using the technique of double-flash illumination. Therefore, the peripheral component of the velocity of blades is cancelled on the picture by means of a rotating mirror (though the peripheral velocity is in proportion to the radius, the velocity reduced by the mirror is constant on the picture; one should note that the position and the velocity of the tracer are not obtained unless compensated)

Fig. 1 Schematic diagram

1. control box	10. xenon lamp
2. flake scattering device	11. turbine
3. camera body	12. sensor for rotation
4. rotating mirror	13. pulse shaper
5. fixed mirror	14. oscilloscope
6. gate circuit	15. oscilloscope
7. time delay circuit	16. booster
8. light source circuit	17. sensor for mirror
9. double-flash	18. lens of camera

Figure 1 shows the schematic arrangement used to obtain photographs. The inlet temperature and the inlet and outlet pressures are kept at constant values by using an automatic control device and the rotating velocity is regulated by means of an eddy current torque-meter. A photograph can be taken only by the operation of pushing the switch of a control box 1. After this operation, a camera 3 and a flake scattering device 2 act with the predetermined time interval between them. A synchronizing signal of the camera opens the gate of a circuit 6. During these actions, a photoelectric sensor 12 is putting out a signal for every rotation, and the signal passes through a pulse shaper 13. The frequency of the signal is divided into $1/2$ and the signal is amplified by a booster 16 up to the driving power of the synchronous motor of a rotating mirror 4. Since the motor has four poles, the rotating velocity of the mirror is a quarter of that of the turbine. When the mirror comes to the position as shown in Fig. 1, that is, when the ray from flakes (by passing through a lens and being reflected twice by mirrors) just arrives at the surface of a film, a signal is generated at a photo-electric sensor 17 for the rotating mirror. This signal reaches the gate circuit 6, after passing through the pulse shaper 13. If the shutter of the camera is opened by the operation mentioned above, the signal passes through the circuit 6 and it enters a time delay circuit 7. Two pulses, of which time interval between them is predetermined ($20 \sim 200 \mu\text{s}$), are generated at the time delay circuit 7 and they trigger light sources 8 and 10. Thus, the photographing is accomplished. A lamp 9 is a double-flash one for illuminating the trac-

ers. The light source 10 is made of a xenon lamp for illuminating the blades; further, having long duration, it makes a thin line after an image obtained by the first flash. One oscilloscope 14 is used to measure the time interval between the flashes, and another 15 is used to ensure the synchronization of the turbine and the rotating mirror. Both the rotating mirror 4 and the fixed mirror 5 are installed on a sliding holder; therefore, provided that the position of the holder is adjusted by sliding in the axial direction of the camera lens, at a point on the picture (given at the center of the picture in the present case), the peripheral velocity can be kept apparently equal to 0 without a change of focusing. The turning angles of photo-electric sensors can be adjusted within the range of $\pm 20^\circ$. The angle of the sensor 17 to the mirror must be decided such that the ray produced at the light sources can be caught by the camera. The position of the turbine blade can be obtained arbitrarily on the picture by adjustment of the angle of the sensor 12.

The light source 10 for illuminating blades consists of xenon storobo light (radiated under 1 kV and $0.5 \sim 8.0 \mu\text{F}$). The light source 8 for illuminating the tracers consists of the sparks (in the air) of two condensers charged up to 10 kV. One of them is $1.75 \mu\text{F}$ in capacity and by the combination of a coil the discharging time is approximately made equal to $10 \mu\text{s}$. The shape of the image produced by this spark is an ellipse as in Fig.2. Moreover, as the image has a long tail due to the xenon lamp (the tail can not be seen in Fig.2, but it does on the film because of better contrast), the pair of the images of a tracer can be easily discovered. Since the other condenser is $0.4 \sim 1.0 \mu\text{F}$ in capacity, the shape of the image in this case is a circle. There is a blade at the right of Fig.2; the images of tracers concentrated on a line are those sticking to the blade. The ratio of the radius at the top of Fig.2 to that of the blade inlet is equal to 0.99. The light source 9 which makes the beam of ray illumi-

Fig.2 A typical photograph of tracers ($u/c_i = 0.519$, time interval between flashes $= 100 \mu\text{s}$)

nates the field apart from the focal plane of the object space of the camera lens (see Fig.3). Therefore, the larger the image, the deeper the position of the flake from the turbine casing. Obscure images happen to

appear in the case that the flakes drift outside of the beam. A counterpart of the pair is not always discovered (or it becomes extremely obscure). The fact is that owing to the rotation of the tracer during the illumination of flashes the intensity of reflection changes widely. Of course, the tracer which is apparently at rest on the picture has no counterpart. At the positions close to each other there are tracers which are moving individually in specific directions. Because, the directions are dependent on the turbulent motion generated in the flow. This behavior of the tracer is independent of the conditions of operation. However, the behavior was hardly observed in the preliminary experiments⁽¹⁷⁾ concerning the flow of low turbulence.

The dimension of the major parts of the turbine are shown in Fig.3. The light sources and the camera are movable in the directions indicated by arrows, respectively. Each focal point of the camera and the light source of flashes is illustrated by thin lines as an example. Two focal points are adjusted apart from each other. It is difficult to know by the relation mentioned above whether the position where the tracer is drifting is deeper than the focal plane or not. Therefore, by the illumination of only one side the disorder is prevented. The image of the circle of diffusion in one side as in Fig.3 is clearer than that of the other side.

Fig.3 Cross-section of major part of turbine (Arrows indicate direction of movement)

The turbine rotor has 16 blades but the nozzle has no vane. The passage of the scroll is divided into 4 parts for the sake of the equalization of the nozzle inlet pressure. The pressure ratio of the inlet to the outlet of the turbine is kept at 1.05 and the inlet temperature is at 40°C . The magnification of the image on a film is 3; the film is analyzed as projected on a screen under the total magnification of 100 times.

3. Results and discussion

The performance curves obtained in a preliminary test are illustrated in Fig.4. The non-dimensional peripheral velocity of the axis of abscissa is the ratio of the peripheral velocity u of the rotor at inlet to the theoretically calculated spouting velocity c_i based on the isentropic heat drop between the nozzle inlet and the blade

outlet. The loss of a journal is comparatively high for generating power and the Reynolds number is low⁽²²⁾; therefore, the efficiency is considerably low.

Fig.4 Effective efficiency and reaction degree

For the facility of comparison between experiments and predictions, at first the experiments are made under the condition of shock free entry ($u/c_t \approx 0.650$), which has been surveyed with the tracking of the tracers. The non-dimensional velocity is shifted from that of the maximum efficiency; but the difference is slight. The other non-dimensional velocities are chosen as 0.519 and 0.345 in the following experiments. Another one is equal to 0.173; the run is however given up, because the images of tracers become streaked at the rotor inlet and the data are scattered in the blade passage, as though vortices were generated.

Fig.5 Path line and streamline from ϕ -direction

Figure 5 shows the typical examples. The theoretically calculated stream-surfaces are illustrated by thin lines (see appendix 1). Arrows indicate the direction of path

lines, and the sizes of them are in proportion to the velocities of tracers. Fan shape figures show the positions of data in the axial direction. The moving directions of tracers are almost the same as those of the predicted lines. In the case of low u/c_t , the flow is complicated in the vicinity of the blade, that is, the flow deviates to the rotor disc on the suction side and it deviates slightly to the casing on the pressure side. In Fig.6 the behaviour of tracers is shown more distinctly. It seems that the tracers move toward the casing; they must however move on lines parallel to the casing, provided that they are projected along the arrow described by a broken line.

Fig.6 Path line from r -direction

Figures 7 ~ 9 are described from the axial direction according to the same rules as in Fig.5 except the projected plane. Broken lines show the average path lines (which are also the time average lines, because the number of shots is greater than several tens). The method for calculating these lines is as follows: the velocity components of the tracers are fitted to the polynomial equations by means of the least square,

$$v_r = f(r, \phi), \quad v_\phi = g(r, \phi), \dots \dots (2)$$

(here, v is the velocity, r , ϕ and z the coordinates and the subscripts indicate the corresponding components), then, the lines can be calculated by the Runge-Kutta method. The number of terms is restricted to 20 to save the calculation time; therefore, this calculation has a demerit in that the local flow pattern can not be given in detail. In this way, we can estimate approximately the maximum difference between the path line and the streamline as follows: substituting velocities and accelerations obtained by Eq. 2 into Eq.4 in appendix 2, we can calculate the values of u_r and u_ϕ at an arbitrary position for a presumed entrainment coefficient (the equation of the z -component is omitted). Then, a pseudo streamline can be estimated for the presumed entrainment coefficient K . Chain lines show the pseudo streamlines in Figs.7 ~ 9. The time t is set at 0 at the blade inlet, and the time lines are drawn at intervals of every 300 μ s.

The difference between the path line and the pseudo streamline (of which $K=20000 \text{ s}^{-1}$, the minimum value on the run) is insignificant; therefore, the quantitative interpretation can apparently hold on the streamline. However, the discrepancy of velocity for the corresponding curves must be noticed, which yields as a result of centrifugal force. It makes tracers move outward. Consequently, the following conclusions are obtained from

Fig.7 Path line and streamline from z -direction Fig.8 Path line and streamline from z -direction Fig.9 Path line and streamline from z -direction

Fig.7: (i) The theoretically calculated streamlines are expressed as radial straight lines in the range of the figure. (ii) Although the practical streamlines are approximately radial and straight, they converge to the center of the blade passage as they approach the outlet of it. (iii) The secondary flow is weak; more specifically, the flow deviates in the counter rotating direction, that is, the suction side in the vicinity of the casing. (iv) The flow seen

near the blade is parallel to it; a tracer passing through the axial clearance between the blade and the casing is hardly seen.

Although the runs shown in Fig.7 were expected to make the shock free entry as mentioned before, the rotating velocity is somewhat higher than expected. This difference results in indistinctness continued until the end of the runs.

For the theoretical estimation in Fig.8, the boundary conditions at the blade inlet are not exactly decided. Before the estimation, the tracers were examined above the blade inlet in order to substitute the experimental values into the boundary conditions. According to the result of this examination, the flow was fairly uniform at $0.8 \sim 1.0$ mm above the blade inlet. Thereupon, for the convenience of the estimation, w_ϕ (the peripheral component of relative velocity) is assumed to be constant at $r_0/40$ (1.32 mm) above the blade inlet (r_0 is a radius of the rotor). Since the results of calculation are greatly different from those of experiments, the values of the boundary condition used here are not appropriate. Although the results of the estimation are insufficient, the problem of the boundary condition is not the chief aim in this paper; therefore, the result are left untouched.

Since the path lines rather deviate from the pseudo streamlines (of which $K=10000 \text{ s}^{-1}$) as shown in Fig.8, the practical streamlines are considerably difficult to restore. Moreover, the flow pattern in the

vicinity of the blade inlet can be hardly seen in detail, because the abrupt change of flow is averaged in the computation of the least square. Hence, the data obtained in the vicinity of the blade inlet are illustrated in Fig.10. There are tracers on the pressure side which are moving as follows: (i) moving inward, (ii) moving outward, (iii) passing through the clearance and (iv) colliding into the blade. A stagnation point can be seen on the blade

in Fig.10, which is also seen on the line of prediction in Fig.8. Considering the temporal fluctuation and the difference between the path line and the streamline, we can not precisely show the position of the stagnation point, though the empirical position is nearer to the blade inlet than to that of the prediction. On the suction side the tracers are inhaled.

Fig.10 Path line in the vicinity of inlet ($u/c_t=0.519$)

The conclusions from Fig.8 are as follows ($u/c_t=0.519$): (i) The theoretically calculated streamlines are considerably different from those obtained in experiments. The streamlines calculated in disregard of the turbulence and the viscosity in the absence of shock free entry are in poor agreement with those obtained in experiments. However, the theoretical estimation leaves room for improvement on the way of taking the boundary condition. (ii) Since a few tracers move outward on the suction side, there must exist a vortex or a separation. However, a vortex does not appear in Fig.8, because the position of the vortex varies with the lapse of time, and it disappears owing to the curve-fitting. (iii) A tracer which passes through the clearance is not found in case of $u/c_t=0.650$ but found in this case. (iv) The secondary flow is delicately appreciable from Figs.6 and 8. A leak of flow through the clearance and an influence of friction produced by the casing cancel each other. Further, the flow is considerably uniform above the blade inlet. The secondary flow accordingly can not grow abruptly along the blade passage.

Figure 9 shows the case of $u/c_t=0.345$ and the prediction is omitted. This figure puts emphasis upon the tendency shown in Fig.8. For example, the movements of the tracers are complicated and more tracers pass through the clearance. According to the observation of photographs, there are two positions where the vortices happen to be seen on the suction side, and $r/r_0 \neq 0.95$ and 0.70 .

The trajectories of tracers found very near to the casing and the blade are different from those observed in the main stream of the blade passage; therefore, the data obtained in the vicinity of them are not included in the calculation of the least square. The velocity of the tracer measured near the rotor is somewhat slow. This reduction corresponds to that of the prediction. In particular, disagreement between theory and practice is revealed about the lines drawn for every $300 \mu s$. This is caused by the influence of the viscosity, the vortex

and the separation of flow.

4. Advisability of the use of tracers

The flow in the turbomachine (such as a compressor or a turbine) of which the working fluid is gaseous is a turbulent high speed flow; therefore, the flow visualization methods applicable in practice are restricted mainly to the uses of a wool tuft, electric sparks, a pitot tube and a laser Doppler-meter. The advisability of these methods is reported already in another paper, and here is explained the advisability concerning the use of aluminium flakes as tracers demonstrated in this experiment as follows: (i) A photograph taken from one direction may permit a three-dimensional analysis of tracers. (ii) This technique can be applied to an experimental object which is too small for a probe to be attached thereto (e. g., a pitot tube, a yaw-meter, etc.). (iii) The flow can be observed qualitatively (and also instantaneously) on the photograph. Provided that both the blade and the streamline are radial and straight, the deviation of a path line from the corresponding streamline is small and this is advantageous in the experiments. (iv) This technique is insufficient for analyzing tracers quantitatively. The aspect of turbulence can not be observed in detail due to the poor fidelity for turbulence. (v) The measuring in portion of the exducer is difficult. However, there is room for improvement. (vi) As tracers adhere to the inner surface of the turbine, the disassembling and the cleaning of the turbine are often necessary. (vii) The flow must be changed, when a number of tracers are injected for a shot. Further, there is a space which the tracers can hardly reach. Hence, many photographs must be taken to obtain the data required for the analysis. (viii) The velocity of rotation is limited to that of the present turbine more or less in the use of aluminium flakes obtainable from the market, because a very high entrainment coefficient can not be realized.

5. Conclusions

The flow in the blade passage of the radial inflow turbine is visualized by use of aluminium flakes as tracers near the design point and below it. The conclusions are as follows:

- (1) Combined use of tracers, the double-flash method and the measuring method of a perspective distance estimated from the diameter of the circle of diffusion makes three-dimensional analysis of the path line possible. In this way, the experimental results can be compared with those of the theoretical estimation and the effective range of predictions is examined.
- (2) In case of the shock free entry, a streamline predicted by the inviscid flow theory is in fairly good agreement with one obtained from experiments and the flow converges to the center of the blade passage as it approaches the outlet of the passage. The tracers which are passing through the axial clearance between the casing and the blade are hardly seen, and the peripheral component

of the relative velocity, that is, the leak is likely to be feeble.

(3) The secondary flow is weak in the range of this experiment. The path line changes in every photographing at the position where the vortex is expected to generate, where the flow is to be unstable and it can not be investigated in detail by the calculation of a time and space average.

(4) The choice of the boundary condition for the theoretical estimation is not simple, and the flow is unstable and disturbed; therefore, the prediction is not proper for the low velocity of rotation so long as Eq.3 (see appendix 1) is used.

Appendix 1. Theoretical estimation

The methods of the theoretical estimation of flow in the radial inflow turbine were reported by Mori and others⁽¹¹⁾ and Watanabe and others⁽¹³⁾. Moreover, the flow in the radial inflow turbine can be calculated by the theoretical formula for the flow of the centrifugal compressor; therefore, many approaches can be made for the flow of the radial inflow turbine. The formulae mentioned above were based on the assumptions of inviscid flow and others, and furthermore great efforts have been made to save the computation time and secure high accuracy. As to the comparison between theory and experiments, it is preferable that the assumptions be made as few as possible. Thus, though much calculation time is necessary, the successive over relaxation method is adopted here using the following equation^(2,3).

$$\begin{aligned} & \left(1 - \frac{w_r^2}{a^2}\right) \frac{\partial^2 \Phi}{\partial r^2} + \left(1 - \frac{w_\phi^2}{a^2}\right) \frac{\partial^2 \Phi}{r^2 \partial \phi^2} \\ & + \left(1 - \frac{w_z^2}{a^2}\right) \frac{\partial^2 \Phi}{\partial z^2} - \frac{2}{a^2} \left(w_r w_\phi \frac{\partial^2 \Phi}{r \partial \phi \partial r} \right. \\ & + w_r w_z \frac{\partial^2 \Phi}{r \partial r \partial z} + w_\phi w_z \frac{\partial^2 \Phi}{r \partial \phi \partial z} \left. \right) \\ & + \frac{1}{r} \left(1 + \frac{r^2 \omega^2}{a^2} \right) \frac{\partial \Phi}{\partial r} = 0. \dots \dots \dots (3) \end{aligned}$$

Here, r, ϕ and z are the cylindrical coordinates, w_r, w_ϕ and w_z the components of relative velocity (directions are indicated with subscripts), a the sound velocity, Φ the potential and ω the angular velocity. By this equation, the flow can be estimated three-dimensionally. However, it is considerably difficult to determine the boundary conditions. The shock free entry is assumed at the rotor inlet and the angle of outflow is assumed to be equal to that of the exducer in Fig.7. Further, the thickness of the blade is neglected. The calculation is done by using a small electric computer with the aid of an auxiliary magnetic tape memory.

Appendix 2. Path line and streamline

Owing to the difference of specific weight between the tracer and the working fluid, the use of a tracer is not suitable for the quantitative measurement⁽¹⁴⁾. If the

deviation between the streamline and the path line can be estimated, the demerit mentioned above can be compensated for a little more.

Assuming that Eq.1 holds even in the turbine, we consider the equation of motion for a tracer as follows:

$$\left. \begin{aligned} \frac{dv_r}{dt} &= \frac{v_\phi^2}{r} \left(1 - \frac{U_\phi^2}{v_\phi^2} \frac{\rho_a}{\rho_t} \right) + K(U_r - v_r), \\ \frac{dv_\phi}{dt} &= -\frac{v_r v_\phi}{r} + K(U_\phi - v_\phi), \\ \frac{dv_z}{dt} &= K(U_z - v_z). \end{aligned} \right\} \dots (4)$$

Here, U and v are the absolute velocities with respect to the air and the tracer, respectively (the subscripts indicate the corresponding components), ρ_a and ρ_t the densities of the air and the tracer, respectively. The entrainment coefficient K is not constant as mentioned before. Assuming that K is constant and equal to the minimum value of that used in this experiment, we can obtain the maximum deviation of the streamline from the path lines (the maximum deviation can not always be obtained). Of course, the fidelity of this calculation is not sufficient, when the turbulence and the acceleration are significant in the flow. In such case, we should note that the discussion can not be easily concluded.

References

- (1) Mizumachi, N., Science of Machine, (in Japanese), Vol.10, No.9 (1958), p.1003; Vol.10, No.10 (1958), p.1226; Vol.10, No.11 (1958), p.1353; Vol.10, No.12 (1958), p.1473.
- (2) Mori, Y., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.21, No.109 (1955), p.679.
- (3) Wallace, F. J., Proc. Inst. Mech. Engr., Vol.172, No.33 (1958), p.931.
- (4) Cox, H. R., Gas Turbine Principles and Practice, (1955), p.9-1, George Newnes.
- (5) Watanabe, I. and Ando, T., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.25, No.149 (1959), p.4.
- (6) Sawada, T. and Nishi, A., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.32, No.238 (1966), p.965; Vol.35, No.280 (1969), p.2401.
- (7) Kapitza, P., J. Phys., Vol.1, No.4 (1939), p.7.
- (8) Mori, Y., J. Sci. Res. Inst., Vol.50, No.1430 (1956), p.107.
- (9) Sawyer, J. W., Gas Turbine Engineering Handbook, (1966), p.281, Gas Turbine Publications.
- (10) Nakamura, K., et al., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.35, No.275 (1969), p.1539.
- (11) Mori, Y., et al., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.29, No.198 (1963), p.365.
- (12) Sawada, T. and Nishi, A., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.34, No.259 (1968), p.537.
- (13) Watanabe, I., et al., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.30, No.216 (1964), p.966.

- (14) Asanuma, T., Jour. Japan Soc. Mech. Engr. (in Japanese), Vol.72, No.609 (1969), p.1370.
 - (15) York, J. L. and Stubbs, H. E., Trans. ASME, Vol.74 (1952), p.1157.
 - (16) Ladenberg, R. W., et al., Physical Measurement in Gas Dynamics and Combustion, (1955), p.145, Geoffrey Cumberlege Oxford Univ.
 - (17) Sugimoto, A., et al., Bull. Univ. Osaka Pref., Vol.20, No.1 (1971), p.49.
 - (18) Japan Soc. Mech. Engr., Mech. Engr. Handbook, (in Japanese), (1968), p.8-24, Japan Soc. Mech. Engr.
 - (19) Hughes, R. R. and Gilliland, E. R., Chem. Engng. Progress, Vol.48, No.10 (1952), p.497.
 - (20) Iinoya, K., Jour. Japan Soc. Mech. Engr. (in Japanese), Vol.67, No.541 (1964), p.211.
 - (21) Tanasawa, Y. and Hiroyasu, H., Trans. Japan Soc. Mech. Engr. (in Japanese), Vol.26, No.162 (1960), p.224.
 - (22) Hiett, G. F. and Johnston, I. H., Instn. Mech. Engng., Paper 13 (1964), p.1.
 - (23) Wu, C., Trans. ASME, Vol.74 (1952), p.1365.
-