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On Spring Constant of Connected Parts in Bolted Joints*

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In order to find the spring constant of connected parts in bolted joints, many experimental or theoretical researches have been done. But the theoretical and experimental results show a disagreement. As one of the reasons for this, is considered the deformation of a bolt-head which was included in the value regarded as the deformation of the connected parts. And the traditional theoretical researches have been done under the assumption that a uniformly distributed pressure is applied over the bearing surface. In this paper, using the point matching method coupled with the finite element method, a pressure distribution on the bearing surface is made clear as a contact problem of two elastic bodies, and then the deformations of the bolt-head and the connected parts under such a pressure distribution are analyzed and the spring constant are calculated. In addition, the spring constant is measured by experiment. According to the result, the value of the spring constant that was computed by taking account of the pressure distribution of the bearing surface, is not so different from the result under the assumption of a uniformly distributed pressure and Shibahara and other's theoretical value. When the deformation of the bolt-head is taken into account, the theoretical and experimental results show a fairly good agreement.

1. Introduction

Bolted joints have been used regularly in the machine or plant industry for a long time, and recently high tension bolts are used for the connection of structural parts in the building industry. Therefore, bolted joints are very important. One of important factors for the rational design of bolted joints is a spring constant of connected parts. In order to determine the spring constant, many researches^{(2)~(5)} starting from Röttscher's have been done. Most of these researches were experimental approaches and the proposed formulas were experimental ones, but recently a few researches^{(6)~(9)} based on the theory of elasticity were published. Table 1 shows the previously published methods to calculate the spring constant.

However, these theoretical and experimental results show a disagreement with each other. As one of the reasons for this, is considered the deformation of a bolt head which was included in the value

regarded as the deformation of the connected parts. Though, in the traditional theoretical researches, only the deformation of the connected parts has been treated under the assumption that a uniformly distributed pressure is applied on the bearing surface, also the deformation of the bolt head should be taken into consideration for strict treatment. In this paper, using the point matching method coupled with the finite element method,⁽¹⁰⁾⁽¹¹⁾ which has been employed for such a problem as an interference fit of a shaft and a hole, a pressure distribution on the bearing surface is made clear as a contact problem of two elastic bodies, and then the deformation of the bolt head and the connected parts under such a pressure distribution is analyzed, and the spring constants are calculated. In addition, the spring constants are measured using the experimental equipment which was newly developed, and they are compared with the theoretical values.

2. Analysis by the finite element method

Firstly, the pressure distribution over the contact surface between the bolt and the connected parts, namely a bearing surface, is computed by the finite element method combined with the point matching method which is an approximate analytical method for an elastic contact problem. Secondly, the deformation of the connected parts under this pressure distribution is computed and then the spring constant is calculated on it. The method to solve an elastic problem of a contact between two

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solid bodies by the finite element method combined with the point matching method has been reported in the literature (10), (11) and so on, therefore omitting the detailed explanation of this method, the form of the stress analysis model, contact condition and so on are described in this report.

2.1 Model of stress analysis

Supposing the case that a plate or a cylinder is fastened by a bolt and a nut with a screw thread of M24 as shown in Fig. 1, a three-dimensional axially symmetrical analysis by the finite element method is applied. It is supposed that the pressure resulting from the clamping force which acts on two pressure pieces corresponding to bolt head, acts on the annular ring around the hollow of the plate whose form is cylindrical. In order to simply compare with the solution of Shibahara and others,⁽⁸⁾ the computation is carried out for the plate of the following dimension: $D_i=25\text{mm}$, $B=40\text{mm}$, $l_f=50\text{mm}$ with three steps of D_o , that is 40mm, 50mm and 100mm. In addition, taking a symmetrical form into consideration, an analysis is carried out with a hatched section shown in Fig.1.

2.2 Element division and loading condition

Figure 2 shows an example of the element division of the pressure piece and the plate, and the type of element division is triangular ring element. In the pressure piece, the number of nodes is 319 and the number of elements is 539. In the plate, the numbers of nodes are 182, 255 and 333 and the numbers of elements are 309, 444 and 594 for the diameters $D_o=40$, 50 and 100mm, respectively. The axial force which acts on the shaft end of the pressure piece is given such that the stress at the bearing surface may be equal to 1 kgf/mm^2 if the pressure distribution is uniform. That is, the area S of the bearing surface is given as follows:

Fig.1 Model for Stress Analysis (tension)

$$S = \frac{\pi(B^2 - D_i^2)}{4} = \frac{3.14 \times (40^2 - 25^2)}{4} = 765.4 \text{ mm}^2 \quad \dots\dots\dots(1)$$

Therefore, the computation is carried out, supposing that the axial force of 765.4 kgf distributes uniformly over the shaft end of the pressure piece.

2.3 Contact condition at the bearing surface

Disregarding the friction at the bearing surface, a pressure is applied perpendicularly on the bearing surface. In Fig. 3, expressing the component of displacement of the node i at the bearing surface of the pressure piece in the axial direction by δ_i and the component of the displacement of the node i' at the bearing surface of the plate corresponding to that of the pressure piece by $\delta_{i'}$, the following expression must be satisfied for all the nodes at the bearing surface in order that the nodes may be closely in contact with each other after the deformation:

$$\delta_i + \delta_{i'} = C \quad (i=1, 2, \dots, m) \quad \dots\dots\dots(2)$$

(a) Pressure piece

(b) Plate (outer diameter: 100mm)

Fig.2 Element division

Table 1 Previously published computing methods of a spring constant

Experimental formula		Theoretical analysis	
Hollow infinite plate	Hollow cylinder	Hollow infinite plate	Hollow cylinder
Rötscher's formula ⁽¹⁾	Yamamoto's formula ⁽⁵⁾	Mitsunaga's solution ⁽⁶⁾	Shibahara and Oda's solution ⁽⁸⁾
Weiss-Waller's formula ⁽²⁾		Fernlund's solution ⁽⁷⁾	
Fritsche's formula ⁽³⁾		Hashimoto's solution ⁽⁹⁾	

where C is a constant and m is the number of nodes on the bearing surface.

Then, expressing the nodal forces applied on the nodes of the bearing surface by F_i and F'_i , respectively and the influence coefficient by c_{ij} and c'_{ij} , respectively, δ_i and δ'_i are expressed as follows:

$$\begin{aligned} \delta_i &= c_{i1}F_1 + c_{i2}F_2 + \dots + c_{im}F_m + \dots \\ &+ c_{im}F_m = \sum_{j=1}^m c_{ij}F_j \quad \dots\dots\dots(3) \end{aligned}$$

$$\delta'_i = \sum_{j=1}^m c'_{ij}F'_j \quad \dots\dots\dots(4)$$

In addition, when computing the influence coefficient c'_{ij} of the bolt, supposing that the node on the axis at the shaft end of the bolt is fixed and that the stress distribution at the shaft end is uniform, the load which is equal to unit load is distributed among the nodes of the shaft end. When computing the influence coefficient c'_{ij} of the plate, the displacement at the central surface of the plate in the direction of axis is fixed as shown in Fig. 2(b).

The magnitude of the nodal force F_i applied on the pressure piece is equal to that of F'_i applied on the plate. Therefore:

$$F_i = F'_i \quad \dots\dots\dots(5)$$

The sum of the nodal forces F_i is equal to the axial force P of the pressure piece. Therefore the following expression is obtained:

$$P = \sum_{i=1}^m F_i \quad \dots\dots\dots(6)$$

If the axial force P is given, F_i is obtained from Eqs. (2)~(6).

Then, the procedures to obtain the value of F_i from these equations, shall be shown. Substituting Eqs. (3)~(5) into Eq. (2), we get:

$$\sum_{j=1}^m (c_{ij} + c'_{ij})F_j = C \quad (i=1, 2, \dots, m) \quad \dots\dots\dots(7)$$

Fig. 3 Contact condition at the bearing surface

In order to eliminate C in Eq. (7), subtracting the equation for $i=1$ from both sides of those for $i=2, 3, \dots, m$, respectively, we get:

$$\sum_{j=1}^m ((c_{ij} + c'_{ij}) - (c_{1j} + c'_{1j}))F_j = 0 \quad (i=2, \dots, m) \quad \dots\dots\dots(8)$$

where

$$(c_{ij} + c'_{ij}) - (c_{1j} + c'_{1j}) = k_{ij} \quad (i=2, \dots, m) \quad \dots\dots\dots(9)$$

Equation (8) is expressed as follows:

$$\sum_{j=1}^m k_{ij}F_j = 0 \quad (i=2, 3, \dots, m) \quad \dots\dots\dots(10)$$

Then, expressing Eqs. (6) and (10) by matrix, we get:

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ k_{21} & k_{22} & \dots & k_{2m} \\ k_{31} & k_{32} & \dots & k_{3m} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1} & k_{m2} & \dots & k_{mm} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_m \end{bmatrix} = \begin{bmatrix} P \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \dots\dots\dots(11)$$

Solving Eq. (11) for F_i , values of F_i are obtained as follows:

$$\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ k_{21} & k_{22} & \dots & k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1} & k_{m2} & \dots & k_{mm} \end{bmatrix}^{-1} \begin{bmatrix} P \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \dots\dots\dots(12)$$

Substituting these values of F_i into Eq. (7), value of C can be obtained.

Fig. 4 Pressure distribution (tension) (influence of the outer diameter of the plate)

2.4 Calculation of the pressure distribution at the bearing surface

Expressing the radius of the node i on the bearing surface as shown in Fig. 2 by r_i , value of P_i is calculated by using the values of F_i obtained in section 2.3, as follows:

$$p_i = \frac{F_i}{\pi \left\{ \left(\frac{r_i + r_{i+1}}{2} \right)^2 - \left(\frac{r_i + r_{i-1}}{2} \right)^2 \right\}} \quad (13)$$

where $r_0 = r_1$ and $r_{m+1} = r_m$. As the value which is equal to the area of the bearing surface is given for the axial force P , previously, the ratio to the uniform pressure distribution is obtained by Eq. (13).

2.5 Calculation of the spring constant

In order to calculate the spring constant K_{FT} of the plate, using the mean displacement $\bar{\delta}$ of the bearing surface defined by Mitsunaga, (6) the spring constant is given as follows:

$$K_{FT} = -\frac{P}{2\bar{\delta}} \quad (14)$$

where

$$\bar{\delta} = \frac{1}{S} \sum_{i=1}^m \pi \delta_i \left\{ \left(\frac{r_i + r_{i+1}}{2} \right)^2 - \left(\frac{r_i + r_{i-1}}{2} \right)^2 \right\} \quad (15)$$

In addition, in computing, modulus of elasticity $E=21000 \text{ kgf/mm}^2$ and Poisson's ratio $\nu=0.3$ are used. Referring to the program for the two-dimensional problem which is described in the literature (12), a new program for an axially symmetrical three-dimensional problem has been developed by one of the authors.

2.6 Results of computation

2.6.1 Pressure distribution at the bearing surface

Figure 4 shows the pressure distribution at the bearing surface which has been obtained by the above mentioned method. As expected, the pressure is not uniformly distributed and rises suddenly near the bolt hole of the plate. In the case that

Fig. 5 Displacements at the top surface of the plate and at the section under head of the pressure piece (tension)

the outside diameter of the plate is larger than that of the bearing surface (when $D_0=50$ or 100mm), the pressure rises also near the outside circumference of the bearing surface. But, there it drops in the case that the outside diameter of the plate is equal to that of the bearing surface of the pressure piece (when $D_0=40\text{mm}$). As the reason for this, is considered the rigidity of the plate which is small near the outside circumference of the bearing surface. In addition, as a reason why the pressure near the inside circumference of the bearing surface is the highest, is considered the rigidity of the bearing surface of the pressure piece becoming large to a greater extent than that of the plate becomes small.

2.6.2 Deformation of the bearing surface

Figure 5 shows the displacement at the surface of the plate and at the portion under the head of the pressure piece. Naturally, both the displacement curves show a good agreement over the bearing surface. Table 2 shows the spring constants of the plates which are calculated on the mean displacement at the bearing surface. In this table, the spring constants which are computed under the assumption of a uniform pressure distribution at the bearing surface, are entered for comparison. Their differences become a little larger with an increase of the outside diameter of the plate, but generally they show good agreement.

3. Experiment

It is desirable to obtain the spring constant by the measurement of the displacement at the bearing surface of the plate, which is clamped with a bolt and a nut, in an experiment, but it is not simple to measure the displacement at the bearing surface. Therefore, in the previously performed experiments, the following two methods have been used:

- (1) Measuring the elongation of the bolt when an external load is applied on the plate which is clamped with a bolt and a nut, the spring constant of the plate may be calculated from the load-elongation curve (Thum⁽¹³⁾ or Sekido⁽¹⁴⁾).
- (2) Measuring a change of distance between two pressure pieces when the plate is compressed by them, the spring constant

Table 2 Comparison of spring constants (tension)

Outer diameter of plate	Spring constant		Error %
	Non-uniform pressure, $K_{FT} \text{ kgf/}\mu\text{m}$	Uniform pressure kgf/mm	
40φ	322	322	0.0
50φ	505	515	+2.0
100φ	641	658	+2.7

may be calculated from it (Wiegand⁽⁴⁾).

In the method (1), it is considered that the spring constant may be different according to the portion on which the external load is applied; that is, this method is apt to be influenced by the load point. Therefore it seems that Wiegand employed the method (2). But he did not take the deformation of the pressure piece into consideration.

Therefore, in this paper, the experiment is carried out by the method (2) and then, the deformation of the pressure pieces is removed by using the result of the computation by the finite element method and finally the spring constant of the plate is obtained. But the experimental apparatus which was used by Wiegand,⁽⁴⁾ is improved such that an experiment can be carried out with stability. In the model of stress analysis described in Chapter 5, an axial force is applied on the pressure piece as well as on a practical bolt head. But the experiment can treat only the case that a compressive force is applied on the pressure piece. Therefore, in Chapter 4, this case is analyzed by the finite element method and the difference between both is made clear.

3.1 Experimental apparatus

Supposing the case that a plate was clamped by a bolt and a nut of M24, an apparatus for experiment was developed. Figure 6 shows the sketch of the experimental apparatus. The plate ② is shrunk by the compressive force given from the

(a) Sketch

(b) Photograph

Fig.6 Experimental equipment

pressure piece ① and the pressure stand ④. At that time, the displacement between the shaft end of the pressure piece and the upper surface of the pressure stand is detected by an electric micrometer which is included in ④. A load is applied on the pressure piece by a universal material testing machine and detected by a load cell ⑤ which is set under the pressure stand ④. They are recorded by an X-Y recorder. As the aim is to know the spring constant when the plate is compressed by the two pressure pieces, the spring constants in the case that a compressive ring ⑥ is put on ① and removed from it, are measured and then the effect of the pressure stand is compensated by them.

3.2 Load cell

In order to transform the load, which is applied on the plate, into an electric signal, the load cell as shown in Fig. 7 was newly developed. The load cell consists of a body, which has three projections in the upper- and under-surface, respectively, and a pressure plate. As the load is applied on the body through the projections, a bending stress is produced on the outside circumferential surface. This is detected by a wire strain-gauge and amplified in a dynamic strain meter; that is, the load is transformed into an electrical signal. Twelve strain-gauges are stuck in the vicinity of the upper- and lower sides at six positions where the projections exist on the top or the bottom, in order to detect a mean strain, for unbalanced loads may be applied on the six projections of the load cell body. As the signs of the bending strains which are produced in the vicinity of the top surface and the bottom are opposite, a temperature compensation is carried out. Figure 8 shows the relationship between the load and the strain, and the connection among the strain gauges. This figure shows that the linearity of the load cell is good and that the hysteresis is small.

3.3 Specimens

The dimensions of the plate are equal to those computed by the finite element method: the form is cylindrical, the di-

Fig.7 Load cell

iameter of hole D_i is 25mm, the diameter of bearing surface B is 40mm, the height l_f is 50mm, the outer diameters D_o are 40, 50 and 100mm, the materials are S25C and S45C (in the case that D_o is 40mm, only S45C), and the number of test pieces is five (Fig. 9).

3.4 Virtual spring constant

A model for experiment corresponding to one for stress analysis as shown in Fig. 1, is shown in Fig. 10(a). The experiment is carried out in order to obtain the spring constant in the case that a plate is compressed by two pressure pieces. The equivalent system of spring in this case is shown in Fig. 10(b), namely, a spring is serially connected with two springs whose spring constants are equal to each other. Then, denoting the virtual spring constant by K_E , the value of K_E is expressed as follows:

$$K_E = \frac{P}{2\Delta_1 + \Delta_2} \quad (16)$$

where Δ_1 is the shrinkage of the spring corresponding to the pressure piece,

Δ_2 is the shrinkage of the spring corresponding to the plate, and

P is the load.

In order to obtain the value of K_E , experiments are carried out firstly for the case where the plate is compressed by the pressure piece and the pressure stand, and secondly for the case where the plate is compressed by the pressure piece, on which the compressive ring is placed as shown in

Fig.8 Calibration curve of the load cell

Fig.9 Photograph of the plates.

Fig. 12(a), and the pressure stand.

The equivalent system of spring in the case of Fig. 11(a) is shown in Fig. 11(b). Then, expressing the virtual spring constant which is recorded in the X-Y recorder by K_N , the value of K_N is expressed as follows:

$$K_N = \frac{P}{\Delta_1 + \Delta_2 + \Delta_3} \quad (17)$$

where Δ_3 is the shrinkage of the spring corresponding to the pressure stand.

In the case of Fig. 12(a), as the

Fig.10 Spring system when the plate is compressed by two pressure pieces (spring constant: K_E)

Fig.11 Spring system when the plate is compressed by a pressure piece and a pressure stand (spring constant: K_N)

Fig.12 Spring system when the plate is compressed by a compressive ring put on the pressure piece (spring constant: K_R)

compressive ring is placed on the pressure piece, the compressive force is not applied on the axis of the pressure piece, but on the contact surface of the compressive ring, with an almost uniform pressure distribution. On the other hand, as the pressure stand receives the compressive force through its bottom, it is considered that the vicinity of the contact surface with the plate is in the state of an almost uniform pressure distribution. Therefore, supposing that the state of deformation at the bearing surface of the pressure piece is equal to that of the pressure stand, the equivalent system of spring is obtained as shown in Fig. 12(b). Then, the virtual spring constant K_R is expressed as follows:

$$K_R = \frac{P}{\Delta_2 + 2\Delta_3} \quad (18)$$

From Eqs. (16) (18), the following equation is obtained.

$$K_E = \frac{K_N K_R}{2K_R - K_N} \quad (19)$$

From Eq. (19), the virtual spring constant K_E in the case where the plate is compressed by the two pressure pieces, is obtained. However, as the spring constant of the plate necessary for design is the value of P/Δ_2 , attention must be paid to the fact that this spring constant K_E includes the deformation $2\Delta_1$ of the pressure pieces.

4. Results of experiments and consideration

The relationship between the load and the displacement of the plate, which is recorded by the X-Y recorder, shows the same pattern as the experiment by Wiegang,⁽⁴⁾ as shown in Fig. 13. Namely, when the plate is compressed firstly, the slope is changed as shown along the curve $0 \rightarrow A$ by the plastic deformations due to the surface roughness or the waviness at the contact surface. But when the plate

Fig. 13 An example of records of load-displacement

is unloaded at the point of A, the curve follows $A \rightarrow B$ with a steeper slope, and then when the plate is loaded again, the curve goes to the point A following $B \rightarrow A$. When the load is increased more, the curve changes with the slope of the curve $0 \rightarrow A$ as if it were an extension to the curve $0 \rightarrow A$. In the case of the load cycle that the plate is unloaded from such a certain point as A and then loaded at such a certain point as B again, the slopes of the curve $A \rightarrow B$ and $B \rightarrow A$ are nearly unchanged. Therefore, the spring constant of the plate is not measured from the curve $0 \rightarrow A$ which includes local plastic deformations, but from the slope of the straight line in the curve $B \rightarrow A$ corresponding to the external load after tightening. The spring constants were measured for both cases with and without a compression ring on the pressure piece, and the results are shown in Table 3 at column a.

However, the computation by the finite element method which was described in Chapter 2, is for the case where a tensile force is applied on the shaft end of the pressure piece, and the model for stress analysis is different from one for the experiment. Therefore, using the model for stress analysis, as shown in Fig. 14, corresponding to the above mentioned experiments, the pressure distributions and the displacements of the bearing surfaces were computed. They are as shown in Figs. 15 and 16, considerably different from those in the case where the shaft end is pulled. Comparing Fig. 4 with Fig. 15, the pres-

Fig. 14 Model for deformation analysis (compression)

Table 3 Experimental values of spring constant

constant				unit: kgf/μm		
Outer diameter of plate				40φ	50φ	100φ
a	K_E including the deformation of pressure pieces	Material	S25C	—	316	348
			S45C	243	312	367
b	K without the deformation of pressure pieces, referring to Fig.16	Material	S25C	—	447	521
			S45C	317	438	565

Each value is mean of every three experimental data.

sure at the inside bearing surface in Fig. 4 is higher than in Fig. 15, and the pressure at the outside bearing surface is lower than in Fig. 15. The pressure distribution in Fig. 15 is more uniform. Comparing Fig. 5 with Fig. 16, the deformations of the pressure pieces corresponding to a bolt head are considerably large in both cases, specially in Fig. 5 (tension). Therefore, they can not be disregarded in the calculation of the spring constant of the bolt. And the spring constant K_{FC} of the plate in this case was calculated and compared with that in the case of a uniform pressure distribution at the bearing surface. They show a fairly good agreement as shown in Table 4.

From Fig. 16, taking the deformations of the pressure piece into consideration and removing Δ_1 in Eq. (19), the values of $K=P/\Delta_2$ were calculated and shown in Table 3 at column b. Here the displacements at the shaft end of the pressure piece were also taken into consideration, corresponding to the experiments in Fig. 6. In order to compare the spring constants with each other, the experimental values in column b of Table 3, Röttscher's, Shibahara

Table 4 Comparison of spring constant (compression)

Outer diameter of plate	Spring constant		Error %
	Non-uniform pressure K_{FC} kgf/ μ m	Uniform pressure kgf/ μ m	
40 ϕ	322	322	0.0
50 ϕ	518	515	-0.6
100 ϕ	671	658	-1.9

Fig.15 Pressure distribution over the bearing surface (compression)

* Yamamoto's formula was derived by modifying Weiss and other's one⁽²⁾ in order to enable to apply a hollow cylinder, based on Wiegand's experiment.⁽⁴⁾

and Oda's, and Yamamoto's* among traditional formulas, and the results of computation in Table 2 by the finite element method are shown in Fig. 17. The numerical solutions by the finite element method show good agreement with Shibahara and others' values by the theory of elasticity. The experimental values show fairly good agreement with the numerical solutions by the finite element method and Shibahara and Oda's values, but the difference between the experimental and the theoretical values becomes a little larger with an increase of the outside diameter D_0 . It is considered that this is an effect of the surface roughness and so on at the bearing surface.

5. Conclusions

The results are summarized as follows:

- (1) The pressure distribution at the bearing surface in the bolted joints is made clear, using the finite element method combined with the point matching method.
- (2) According to the results, the pressure distribution is not uniform over the bearing surface, and the pressure is considerably higher at the inside bearing surface. In the case where the outside diameter of the plate is larger than that of the bearing surface, the

Fig.16 Displacements at the top surface of the plate and at the section under head of the pressure piece (compression)

Fig.17 Comparison between theoretical and experimental values

pressure becomes a little higher at the outside, and in the case where the outside diameter of the plate is equal to that of the bearing surface, the pressure becomes a little lower.

- (3) The pressure distribution at the bearing surface has little effect on the spring constant of the plate. Therefore the spring constant of the plate computed by assuming uniform distribution will not cause such a large error.
- (4) An experiment corresponding to the computation has been carried out. When the deformation of the bolt head is taken into consideration and compensation is made, the experimental values of the spring constants of the plates show a fairly good agreement with the numerical solutions by the finite element method.

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