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Effects of Inclination of Contact Line on Friction and Wear of Worm Gears*

By Taku UENO**, Takasuke YAMADA*** and Toshihiro ISHIZU****

The performance of worm gears is affected by the inclination of the contact line. In case of worm gears, the problem is that the contact lines of worm gears are nearly parallel to the sliding directions of the worm and the contact conditions are severe. But it has not been clear how much the inclination of the contact line affects the characteristics of friction. The authors, therefore, investigated quantitatively the relation between the performance and the contact line, and studied a method of designing and manufacturing worm gears with higher performance.

First, the authors analyzed the relative motion of tooth surfaces of worm and wheel and made a test apparatus on which the contact conditions could be simulated considerably well. Next, experiments were conducted by using the apparatus which indicated pure effects of this factor. Further, the authors investigated into the performance of side worm gears, and tried to clarify the relation between the performance and the contact line.

1. Introduction

The performance of worm gears is much affected by the inclination of the contact line to the sliding direction of the worm, as well as by the combination of materials, the lubricating oils, and the accuracy of the manufacture and assembling. In case of worm gears, the problem is that as the contact lines are nearly parallel to the sliding directions of the worm, and as the tooth contact pressures are high, the contact conditions are very severe. Accordingly, attempts have been made to solve the problem of the contact lines and to obtain high performance by using profile-shifted worm gears having a long addendum worm, the cone worm gears, the cavex worm gears, and so on.

But it has not been clear how much inclination of the contact line affects the characteristics of friction. The authors, therefore, investigated quantitatively the relation between the inclination of the contact line and the performance, to obtain a method of designing and manufacturing worm gears with higher performance. Conducting the performance tests by using actual worm gears is a practical method, but it is not easy to clarify the pure effects of the inclination of contact lines on the performance. One of the authors had been engaged in this study⁽¹⁾ of various inclination of the contact lines between a phosphor

bronze roller and a steel disc by means of four balls testing machine. But with respect to the simulation of the actual worm gears contact, this test was insufficient, because the roller could not be rotated and the contact load was restricted by the wear of the roller.

The authors made a new test apparatus which could give the test pieces the rolling and sliding to simulate the contact condition of the actual worm gears, and investigated into the effects of the inclination of the contact lines. Here, the relative motion of the tooth surfaces of the worm and wheel to the contact lines was analyzed, and according to this analysis, the effects of contact lines on the characteristics of worm gears were tested by means of the new test apparatus.

Further, the authors investigated into the performance of side worm gears and analyzed the relation between the performance and the contact line.

2. Apparatus and Simulation of worm gears

2.1 Apparatus and theory of simulation

Fig.1 shows the essential part of the manufactured apparatus and Fig.2 shows a scheme of the whole apparatus. The cylindrical surface of a roller is brought into contact with a disc plate, and both of the test pieces are rotated. The roller can be set at the proper position of the disc plate which is indicated by θ in Fig.1. So the angle between the contact line and the sliding direction is variable. Loads can be applied on the disc and the roller with the forces enlarged by lever action, and proper weights are suspended from the end of the lever. The roller can be put in the proper position on the disc plate by the

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** Professor, Faculty of Engineering, Kyushu University, Fukuoka

*** Assistant, Faculty of Engineering, Kyushu University, Fukuoka

**** Technical Official, Faculty of Engineering, Kyushu University, Fukuoka

change of the collar length l and the shaft height h .

But the velocity of the roller is always mechanically perpendicular to the contact line. So, the relative movement of the tooth surfaces of worm gears is analyzed and the contact condition can be simulated on this apparatus.

2.2 Relative movement of tooth surfaces of worm gears and simulation on apparatus

Fig.3 shows that the tooth surfaces of a worm and a wheel contact at the point Q_0 . Now, the authors tried to simulate the contact condition at Q_0 . The co-ordinate XYZ at the point O and the co-ordinate xyz at the point Q_0 which is parallel to the co-ordinate XYZ are defined as shown in the figure. Fig.4 is an enlarged view at or near the point Q_0 . Now, the authors refer to the situation that both the tooth surfaces which contact at the point Q_0 con-

Fig.1 Principle of Apparatus

Fig.2 Mechanism of Apparatus

Fig.3

tact at the point Q after a unit time. During the time the point Q_0 on the worm moves to D at a velocity of v in the xz -plane and the same point on the wheel moves to E at a velocity of V in the yz -plane. ϕ is the angle of the worm which indicates the position Q_0 on the worm surface and the direction of the velocity v , and ψ is the angle of the wheel. $BCAD$ -plane is the common tangent plane of the worm and the wheel at the contact point Q . The normal components of the velocity v and V to both surfaces are equal. However, the worm and the wheel slide relatively at the contact point.

Fig.4 Enlarged view about point Q

Fig.5
Contact plane viewed from OO'
(Axis ξ is parallel to line AB)

Fig.6 State simulated on Apparatus

Fig.5 is a view from the perpendicular direction to BCAD-plane of Fig.4. QQ' indicates the contact line. When its inclination δ to X axis viewed from the direction of Y axis is defined, the inclination to AB-line on ABC-plane becomes the angle δ' induced by δ as shown in the figure. As the worm moves at the relative velocity of v_s in the direction of the angle $(\theta_w - \delta)$ to the contact line and the wheel moves at the relative velocity V_s in the direction $(\theta_w + \delta')$ to the contact line, the same movement must be carried out on the apparatus.

Fig.6 shows that the disc corresponds to the worm and the roller corresponds to the wheel. On the apparatus the contact line of the disc and the roller agrees with the line QQ' of the actual worm gears, so the velocity u_1 , the velocity u_2 , and the angle θ must be selected to compensate the velocity V_c which is caused by the difference in the directions of the roller velocity u_2 and the wheel sliding velocity V_s . The calculation is as follows.

- γ ; Lead angle of worm at point Q.
- ϕ ; Axial pressure angle of worm at point Q.
- δ' ; Angle of inclination of contact line on tooth surfaces
- v_s ; Sliding velocity of worm at point Q.
- V_s ; Sliding velocity of wheel at point Q.
- u_1 ; Velocity of disc
- u_2 ; Velocity of roller
- θ ; Angle indicating position of roller (angle between contact line and u_1)

First, the sliding velocities v_s and V_s are given by

$$v_s = \sqrt{v^2 - v_n^2}, \quad V_s = \sqrt{V^2 - V_n^2} \quad (1)$$

Here, the velocity v_n or V_n of the normal direction to the tooth surfaces is determined in Fig.4 as follows:

$$\begin{aligned} v_n &= V_n = \overline{Q_0F} \cos \angle FQ_0Q = \overline{Q_0F} \cos \angle FCQ_0 \\ &= \overline{Q_0A} \sin \gamma \sqrt{\frac{1}{1 + \tan^2 \angle FCQ_0}} \\ &= \overline{Q_0B} \cos \gamma \sqrt{\frac{1}{1 + \tan^2 \angle FCQ_0}} \end{aligned}$$

Fig.7

where $\tan \angle FCQ_0 = \tan \alpha \cos \gamma$

$$\overline{Q_0A} = \overline{Q_0G} - \overline{AG} = v \cos \phi - v \sin \phi \frac{\tan \alpha}{\tan \gamma}$$

$$\begin{aligned} \overline{Q_0B} &= \overline{Q_0C} \tan \alpha = V \frac{\sin \angle Q_0EC}{\sin \alpha} \tan \alpha \\ &= V \frac{\cos(\alpha - \phi)}{\cos \alpha} \end{aligned}$$

Equations of v_n and V_n thus become:

$$\begin{aligned} v_n &= v \cos \phi \frac{\sin \gamma - \tan \phi \tan \alpha \cos \gamma}{\sqrt{1 + \tan^2 \alpha \cos^2 \gamma}} \\ V_n &= V \cos(\alpha - \phi) \frac{1}{\sqrt{1 + \tan^2 \gamma \cos^2 \alpha}} \end{aligned} \quad (2)$$

Next, the angles θ_w and θ_w shown in Fig.5 must be explained. The velocity components in AB-direction of v_s and V_s shown in Fig.4 are expressed by $v \cos \phi \cos \gamma$ and $V \cos \phi \sin \gamma$. Hence

$$\begin{aligned} \cos \theta_w &= v \cos \phi \cos \gamma / v_s \\ \cos \theta_w &= V \cos \phi \sin \gamma / V_s \end{aligned} \quad (3)$$

Lastly, the disc velocity u_1 , the roller velocity u_2 , and the setting angle θ conducted on the apparatus are decided as follows, as the compensating velocity V_c is $V_s \cos(\theta_w + \delta')$:

$$\begin{aligned} u_1 &= \{v_s^2 + V_s^2 \cos^2(\theta_w + \delta') \\ &\quad + 2v_s V_s \cos(\theta_w + \delta') \cos(\theta_w - \delta')\}^{1/2} \\ u_2 &= V_s \sin(\theta_w + \delta') \end{aligned} \quad (4)$$

$$\theta = \sin^{-1}(v_s \sin(\theta_w - \delta') / u_1) \quad (5)$$

Thus, if u_1, u_2 and θ are computed from Eq.(1) to (5), the state at the contact point of worm gears can be represented on this apparatus.

3. Results of the experiments with this apparatus

3.1 Test method and test conditions

3.1.1 Measurement of coefficient of friction

Table 1

	Worm	Wheel
Tooth form	Involute worm	
Nominal pitch	30.562 mm	
No. of thread, No. of teeth	2	25
Lead angle (direction)	18°26' (right)	
Axial pressure angle	22°	
Center distance	150.79 mm	
Face width of wheel	44.45 mm	
No. of revolution (Sliding velocity)	1800 rpm	144 rpm
	(5.7 m/s at pitch point)	
Allowable horse power (BSS)	14 H	
Lubricating oil	Diamond cylinder oil 680W (fatty additives 10 %)	
	(Viscosity 369cSt at 37.8°C)	
Materials	SCM4 (Hs 70)	PBB2 (Hs 20)
Roughness	1.5 μm (ground)	2 ~ 4 μm (shaved)
Efficiency (Coeff. of friction)	92.5 % (μ=0.023)	
Wear rate (wheel)	0.35 × 10 ⁻⁴ μm/rev	

When a load is applied with the force of P and the test pieces are rotated as shown in Fig.1, it is considered that the friction force of μP acts in the direction the relative sliding velocity u_s (Fig.7). The component F_1 in the direction to the disc velocity u_1 of μP (or the component F_2 in the direction to the roller velocity u_2) can be measured by using the torque meter. Accordingly, the coefficient of friction may be calculated from

$$\left. \begin{aligned} u_s &= \sqrt{u_1^2 + u_2^2 - 2u_1 u_2 \sin \theta} \\ F_1 &= \mu P(u_1 - u_2 \sin \theta)/u_s \\ F_2 &= \mu P(u_2 - u_1 \sin \theta)/u_s \end{aligned} \right\} \dots\dots\dots (6)$$

3.1.2 Measurement of wear

The reference surfaces for measuring the wear of the roller are made at both side parts of the cylindrical surfaces (cf. Fig.1). The diameters of the parts are smaller than the diameter of the contact surface, and the steps of both side parts are 0.04 mm. So the wear of the roller after the test is a decrease of the step

Fig. 8

Fig. 9

heights measured by a TALY-SURF roughness meter. As to the disc, a worn groove occurs on the contact part after the test. So the wear of the disc is the depth of the groove measured by the TALY-SURF meter.

3.1.3 Material of test pieces

On this apparatus, the material of the disc corresponding to the worm is CrMo-steel (hardened, Hs70) and the material of the roller corresponding to the wheel is phosphor bronze (forged, Hs20) just as in the actual worm gears (Table 1). The surfaces of the discs are finished by grinding ($\sim 1.5 \mu\text{m Rmax}$) and the surfaces of the rollers are finished by cutting ($\sim 4 \mu\text{m Rmax}$).

3.1.4 Speed and loads

The revolution of the disc is constant 740rpm (5.66m/s) and the revolution of the roller is constant 182rpm (0.69m/s), so as to test at nearly the pitch point. Experiments are conducted under loads of 100kg (Hertzian stress 27kg/mm^2 , corresponding to 14HP) and 200kg (40kg/mm^2 , 28HP). Before tests, running-in is conducted for an hour under light loads of 25kg.

3.1.5 Lubricating oil

Cylinder oil containing fatty additives 10% is used, which is flooded at a rate of 2 l/min and kept at a temperature of 60°C . The viscosity is 369cSt at 37.8°C and 27.4cSt at 98.9°C .

3.1.6 Setting position of roller

The worm gears used in the tests are shown in Table 1 and their performance has been investigated⁽²⁻⁴⁾ sufficiently up to now. As shown in Fig.8, the thick lines are the contact lines of a standard wheel. The two dot chain lines in the figure show the ranges of the side worm gears examined in this paper. In Fig.8, it is presumed that the inclination angles of the contact lines to the velocities of the worm change in the range $+70^\circ \sim -70^\circ$; and the lubrication properties are widely variable at each point on a tooth surface. Therefore, as shown in Fig.9, the five points A~F on the

Fig. 10

pitch line of the actual worm are picked up to clarify the characteristics. These points correspond to each setting position of the same signal A~F on this apparatus.

3.2 Results

Before the experiments, running-in is done under a quarter of the allowable load. Fig.10 is one of the running examples. During the first hour of the running-in, the roughness is smoothened gradually, and the coefficient of friction continues to reduce. Thereafter, the coefficient of friction is almost constant and a small amount of wear continues in a steady state. However, when the roller is set near the po-

(a) Coeff. of friction and wear in Steady state

(b) Coeff. of friction and wear at Commencement of Running

Fig.11

sition of $\theta=0^\circ$, a longer time is taken for this. Fig.11(a) shows the test results of the effects of θ on the friction and the wear in the steady state.

3.2.1 Coefficient of friction and wear

As seen in Fig.11, the coefficient of friction and the wear rate increase twice in the range from 90° to -30° . This excellent performance at 90° is caused by easiness of supplying lubricating oil to the contact part, as the component of the disc velocity perpendicular to the contact line increases with approaching to 90° . Now, attention must be paid to the peak at $\theta=-6^\circ33'$. The reason why the peak appears at $\theta=-6^\circ33'$ is probably that at this point it is difficult to make an oil film by supplying the oil, because the components of the disc velocity perpendicular to the contact line and the roller velocity are equal in the reverse direction. Further, in comparison with the results of the actual worm gears, both the coefficient of friction and the wear rate on the worm gears agree with the values near the point 0°. This agreement must come from the contact condition near the center of the wheel surface which practically governs the performance of the actual worm gears corresponding to the condition in the range of $\theta=-15^\circ$ to 25° on the apparatus. However, it would be necessary to investigate more carefully whether any other factors affect the performance in reality or not.

3.2.2 Effects of roughness

Fig.11(b) shows the results at the commencement of the running with the initial roughness. Although the tendency of the experimental results is similar to the steady state, the performance is generally worse. Specially, lubrication properties are not good in the range of A~D which has a large sliding parallel to the contact line and severe condition on the formation of oil film.

3.2.3 Effects of loads

As shown in Fig.11(a), when the load is changed, the tendency in the effects of

Table 2

θ	Experimental conditions		Coefficient of friction	Wear rates
	u_1	u_2		
$-6^\circ33'$	5.66m/s	0.69m/s	0.075	$12.0 \times 10^{-4} \mu\text{m/rev}$
-90°	-0.69	0.69	0.095	4.1×10^{-4}
$+90^\circ$	5.66	0.69	0.014	0.8×10^{-4}
-90°	-2.8	2.8	0.054	1.5×10^{-4}

Fig.12

the angle θ on the characteristics is similar. But the wear rate is more than twice that when the load is increased from 100kg to 200kg, though the coefficient of friction is nearly unchanged. In comparison with the actual worm gears, the coefficients of friction and the wear rates under 100kg and 200kg agree considerably well with the results of 14HP and 20HP as seen in Fig.11(a).

3.2.4 Cause of peak at $-6^\circ 33'$

In order to clarify the cause of the peak, the lubrication properties are investigated at the position of $\theta = -90^\circ$ (see Fig.12). The experimental roller and disc are rotated at speed of 0.69 m/s in the reverse direction, the applied load is 100kg, and the surfaces of the roller and the disc are finished by grinding ($1.5\mu\text{m } R_{\text{max}}$ and $2\mu\text{m } R_{\text{max}}$). The test results are shown in Table 2. The lubrication conditions at the equal velocity in the reverse direction become very severe compared with the result of $\theta = -90^\circ$ and agree well with the value of $\theta = -6^\circ 33'$.

3.2.5 Effects of relative sliding velocity

From the experiments under constant sliding velocity conditions, it can be stated that a little variation (see Table 3 and Fig.13) of the relative sliding velocity does not affect the test results shown in Fig.11.

4. Performance of side worm gears

Worm gears are generally designed for the standard state shown in Fig.14A. However, considering the inclination of the

contact lines on the tooth surface (Fig.8), it seems that more efficient worm gears will be obtained using the so-called side worm gears made by shifting the wheel in the axial direction.

4.1 Performance tests of type B side worm gears

First, the authors investigated into the performance of the side worm gears shown in Fig.14B. Fig.15 shows the dimensions, the finishing methods and the allowable horse power of the worm gears designed normally (Fig.14A) are shown in Table 1. The testing machine used in the experiment is a power circulating type worm gears testing machine made in the authors laboratory at Kyushu University. The test worm is rotated at a speed of 1800rpm. A cylinder oil containing 10% of fatty additives in mineral oil is used (viscosity is 450cSt at 37.8°C and 32cSt at 98.9°C , VI=108). The unit gear box is bath-lubricated with the whole depth of the tooth of the worm dipped into oil. The wear of the wheel is measured in terms of the decrease of tooth thickness in the central plane, and the wear of the worm is measured in the neighbourhood of pitch point using the ZEISS Universal measuring microscope. The variations of the performance of the worm gears and tooth contact areas in operation are shown in Fig.16. Fig.17 indicates the test results of the standard worm gears. Table 4 and Table 5 are the summaries of the test results. Although the efficiency of the standard worm gears indicates 92% ($\mu = 0.025$) or more than 92%, the efficiency of the type B shows a low value of 91~91.5% ($\mu = 0.029 \sim 0.027$). To clarify the cause, the authors have analyzed the movement of the surfaces of the side worm gears.

Experiment A ($\theta = -30^\circ$)

Fig.13

Fig.14

Fig.15

Table 3

Experiments	A	B	C	D	E	F	
Setting angle θ	-30°	$-6^\circ 33'$	0°	$6^\circ 50'$	30°	60°	90°
Sliding velocity u_s m/s	6.00	5.76	5.69	5.62	5.37	5.09	5.03
Angle between Contact line and u_s	$-35^\circ 12'$	$-12^\circ 46'$	$-6^\circ 22'$	26°	$24^\circ 11'$	$56^\circ 14'$	90°

Table 4 Performance of Type B side worm gears

	Efficiency	Maximum Temperature	Temperature rise	Wear rate of worm	Wear rate of wheel
14HP	91.5%	66°C	47°C	0.1μm/h	0.24μm/h
20	91.0	83	56	slight	0.33
28	seizure				

Table 5 Performance of Standard worm gears

	Efficiency	Maximum Temperature	Temperature rise	Wear rate of worm	Wear rate of wheel
14HP	92.0%	60°C	43°C	slight	slight
20	92.3	73	55	0.14μm/h	0.15μm/h
28	93.0	80	68	slight	0.15

Fig.16 Running performance of Type B side worm gears

Fig.17 Running performance of Standard worm gears

The calculations are based on Eq.(1) ~ (5) mentioned before, and the calculated disc velocity, roller velocity and setting position on the apparatus are shown in Fig.18. In the figure, though the contact lines and the moving directions on the type B side worm gears are not parallel (Fig.8), it seems that the tooth surface of the type B has an unfavorable zone of lubricating which corresponds to negative θ position on the apparatus. The zone is hatched in Fig.19.

Next, the authors examined the type C side worm gears which have a zone corresponding to the positive θ .

Disc velocity on apparatus u_1 (m/s) (roller velocity u_2) Setting angle on apparatus θ (degree)

Fig.18

Fig.19 Component of worm(wheel) velocity perpendicular to contact line (m/s)

Table 6 Performance of Type C side worm gears

	Efficiency	Maximum Temperature	Temperature rise	Wear rate of worm	Wear rate of wheel
14HP	93.5%	67°C	48°C	slight	0.15 $\mu\text{m}/\text{h}$
20	92.4	82	62	slight	0.16

Fig. 20

Fig. 21 Running performance of Type C side worm gears

Fig. 22 Equivalent radius (mm)

4.2 Performance tests of type C side worm gears

The performance experiments are conducted under the above-mentioned conditions of the type A and B. Prof. Wakuri has once proposed that when the pitch of the worm is extended, the worm gears get crowning on the tooth surface (Fig. 20). As the type C has a tendency to make a narrow tooth contact area, the pitch of this worm was extended such that the crowning on the type C may be as much as on the types A and B. Fig. 21 shows the variation of the performance and the tooth contact mark, and Table 6 is the summary of the results. The efficiency is 92.4~93.5 % ($\mu=0.023\sim 0.019$) and the wear rates are small. Thus, the performance of the type C is more excellent than the types A and B. These results agree with the facts expected from the simulating tests on the apparatus. However, in the case of type C, attention must be paid to the equivalent radius of the tooth surfaces tending to become smaller (see Fig. 22), because the decrease of the equivalent radius causes an increase of the contact pressure.

5. Conclusions

- (1) The authors made an apparatus

which was nearly similar to the contact condition of the actual worm gears by using a disc and a roller and in which the angle between the contact line and the sliding direction was variable. It was recognized that the results gained by using this apparatus agreed considerably well with the performance of the actual worm gears. Accordingly, this apparatus is found useful in developing new materials and lubricating oils for worm gears.

(2) The coefficient of friction and the wear rate are affected by the angle between the contact line and the sliding direction. They increase twice with approach from 90° to -30° . Then it seems proper to test in the range of $\theta=0^\circ\sim 15^\circ$ when the performance test is conducted on the apparatus.

(3) The coefficient of friction and the wear rate have a peak at $\theta=-6^\circ 33'$ on the apparatus where the roller velocity and the component of the disc velocity perpendicular to the contact line are equal and reverse. The drop of efficiency and the increase of wear seem due to difficulty of the oil film formation. These unfavorable points exist in the tooth surfaces of the actual worm gears, but they do not seem to damage the surface immediately. However, such a severe condition had better be avoided, if possible.

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