539.319 : 539.3.082.77 : 539.26

X-Ray Measurement of Residual Stresses in a Multi-Layered Sphere*

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Many theories of residual stress measurement have already been proposed for plate and cylinder, but only two methods such as the mechanical method by Kawaguchi⁽¹⁾ and the specific volume method by Yonetani⁽²⁾ are published for a single-layered sphere.

In this paper, the authors propose a general formula to calculate residual stresses in a multi-layered sphere by a new method using X-ray, and the formula of industrial use for a single sphere is shown to be reduced to a special case of general one.

As the linear expansion coefficient of each spherical layer is different in a general multi-layered sphere, residual stress distributions are affected by thermal stress due to temperature change, so that the authors show also a stress compensating formula for temperature.

Finally the authors give two examples of residual stress measurements by applying their new methods to chromium deposited steel spheres (SUJ-2) which have been quenched and tempered in different conditions.

1. Introduction

The residual stresses are determined by measuring the deformation (e.g. strain or curvature) accompanied with a stress relief caused by the thin layer removal from the surface of specimen and furthermore by applying the X-ray Method, which is fundamentally a nondestructive measuring method of a surface stress, to the destructive method (Thin Layer Removal Method), the residual stress distribution inside of a specimen can be obtained.

Many reports on the theory of residual stress measurement have been presented for plates and cylinders, but for spheres there are only few methods^{(1),(2)} known available to a single-layered sphere.

In this report the authors propose general formulas of residual stress measurement by X-ray Method for a multi-layered sphere and show the formulas of a single-layered sphere derived as a special case of the general formulas, and measure the residual stress distribution of a two-layered sphere for example.

In a multi-layered sphere, residual stresses are affected by temperature similar to a multi-layered plate or cylinder because of difference in linear expansion coefficient of each layer and therefore the base temperature must be expressed specifically. If the measuring temperature were different from the base one, a proper compensation for a temperature difference would be necessary.

Assumptions for the analysis are as

follows; (1) Residual stresses are a function of radius r . (2) Spherical layers are all isotropic and their elastic constants are different from each other. (3) Radial displacements are equal at the boundary of each layer.

2. Analytical formulas

In the theory of the authors, residual stresses are calculated from the calculating equations by using the relation curve between the surface stress measured by X-ray and the removed radius.

Generally, when the internal and external pressure P_1, P_2 act on a thick walled sphere with inside and outside radii r_1, r_2 , the radial and tangential stresses σ_r, σ_t at an arbitrary radius r of the sphere are

$$\left. \begin{aligned} \sigma_r(r) &= \frac{p_1 r_1^3 - p_2 r_2^3}{r^3 - r_1^3} - \frac{p_1 - p_2}{r^3 - r_1^3} \left(\frac{r_1 r_2}{r} \right)^3 \\ \sigma_t(r) &= \frac{p_1 r_1^3 - p_2 r_2^3}{r^3 - r_1^3} + \frac{p_1 - p_2}{2(r^3 - r_1^3)} \left(\frac{r_1 r_2}{r} \right)^3 \end{aligned} \right\} \dots\dots\dots (1)$$

The relations between the radial and tangential strains ϵ_r, ϵ_t and the radial displacement u are

$$\left. \begin{aligned} \epsilon_r(r) &= \frac{du}{dr} = \frac{1-2\nu}{E} \frac{p_1 r_1^3 - p_2 r_2^3}{r^3 - r_1^3} - \frac{1+\nu}{E} \frac{p_1 - p_2}{r^3 - r_1^3} \left(\frac{r_1 r_2}{r} \right)^3 \\ \epsilon_t(r) &= \frac{u}{r} = \frac{1-2\nu}{E} \frac{p_1 r_1^3 - p_2 r_2^3}{r^3 - r_1^3} + \frac{1+\nu}{E} \frac{p_1 - p_2}{2(r^3 - r_1^3)} \left(\frac{r_1 r_2}{r} \right)^3 \end{aligned} \right\} \dots\dots\dots (2)$$

E : Young's modulus, ν : Poisson's ratio

In i th layer of a sphere which consists of M layers (Fig.1), where r_{i+1}, r_i : the external and internal radii, E_i, ν_i : Young's modulus and Poisson's ratio, σ_{ir}, σ_{it} : radial and tangential residual stresses, $\sigma_{ir}', \sigma_{it}'$: the changes of them accompanied with thin layer removal, σ_{iT} : measured stress by X-ray, the following equations are obtained in removing thin layers from outer surface to radius r ($r_m \leq r \leq r_{m+1}$) in the m th layer.

$$\sigma_{ar}(r) = \sigma_{ir}(r) + \sigma_{ir}'(r) \dots\dots\dots (3)$$

* Received 18th April, 1974.

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The vanishing of radial residual stress $\sigma_{mr}(r)$ at radius r means action of external pressure $\sigma_{mr}(r)$ on the remaining sphere, and then the contact pressure p changes also at individual boundary. The strains caused in each spherical element are

$$\left. \begin{aligned} \varepsilon_{ir}(\rho_i, r) &= \frac{1-2\nu_i}{E_i} \frac{p_{i-1}r_i^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} - \frac{1+\nu_i}{E_i} \frac{p_{i-1} - p_i}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \\ \varepsilon_{it}(\rho_i, r) &= \frac{1-2\nu_i}{E_i} \frac{p_{i-1}r_i^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} + \frac{1+\nu_i}{E_i} \frac{p_{i-1} - p_i}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \end{aligned} \right\} \quad (i=1, \dots, m) \quad (4)$$

where $r_i \leq \rho_i \leq r_{i+1}$, $p_m = \sigma_{mr}$, $p_0 = 0$, $r_{m+1} = r$, $\varepsilon_{ir}(\rho_i, r)$,

$\varepsilon_{it}(\rho_i, r)$: the strains at an arbitrary radius ρ_i of i th layer due to thin layer removal from outer surface to radius r ($r_m \leq r \leq r_{m+1}$).

The change of tangential stress $\sigma_{mt}'(r)$ in Eq.(3) is

$$\sigma_{mt}'(r) = \frac{\sigma_{mr}(r)}{2(r^3 - r_m^3)} \{3r_m^3 - \frac{p_{m-1}}{\sigma_{mr}(r)} - (2r^3 + r_m^3)\} \quad (5)$$

Considering Assumption (3) and the second equation of Eq.(4), radial stress $\sigma_{mr}(r)$ of m th layer is

$$\sigma_{mr}(r) = \frac{[E_{m-1}(r_m^3 - r_{m-1}^3)[2(1-2\nu_m)r_m^3 + (1+\nu_m)r^3] + E_m(r^3 - r_m^3)[2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3]}{3E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)r^3} p_{m-1} - \frac{E_m(1-\nu_{m-1})}{E_{m-1}(1-\nu_m)} \frac{(r^3 - r_m^3)r_{m-1}^3}{(r_m^3 - r_{m-1}^3)r^3} p_{m-2} \quad (6)$$

The symbol p_i , the change of contact pressure at the boundary between i th layer and $(i+1)$ th one, is described as follows;

$$p_i(r) = k_{i-1}p_{i-1}(r) - l_{i-1}p_{i-2}(r) \quad (i=2, \dots, m-1) \quad (7)$$

$$\left. \begin{aligned} k_i &= \frac{[E_i(r_{i+1}^3 - r_i^3)[2(1-2\nu_{i+1})r_{i+1}^3 + (1+\nu_{i+1})r_{i+2}^3] + E_{i+1}(r_{i+2}^3 - r_{i+1}^3)[2(1-2\nu_i)r_{i+1}^3 + (1+\nu_i)r_i^3]}{3E_i(1-\nu_{i+1})(r_{i+1}^3 - r_i^3)r_{i+2}^3} \\ l_i &= \frac{E_{i+1}(1-\nu_i)}{E_i(1-\nu_{i+1})} \frac{(r_{i+2}^3 - r_{i+1}^3)r_i^3}{(r_{i+1}^3 - r_i^3)r_{i+2}^3}, \quad p_0 = 0 \end{aligned} \right\}$$

Rearranging Eq.(7),

$$\left. \begin{aligned} \frac{p_2}{p_1} &= k_1 \\ \frac{p_3}{p_1} - k_2 \frac{p_2}{p_1} &= -l_2 \\ &\vdots \\ \frac{p_i}{p_1} - k_{i-1} \frac{p_{i-1}}{p_1} + l_{i-1} \frac{p_{i-2}}{p_1} &= 0 \\ &\vdots \\ \frac{p_{m-1}}{p_1} - k_{m-2} \frac{p_{m-2}}{p_1} + l_{m-2} \frac{p_{m-3}}{p_1} &= 0 \end{aligned} \right\} \quad (8)$$

The coefficient matrix of the simultaneous equation of $(m-2)$ unknown in Eq.(8) is

$$\begin{vmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & -k_2 \\ & \ddots & & \vdots \\ 0 & \dots & 1 & -k_{m-2} \\ 1 & -k_{m-2} & l_{m-2} & 0 \end{vmatrix} = j^{(m+1)(m-2)} \quad (9)$$

where $j = \sqrt{-1}$

Put $D_i^{(m-2)}$ as the determinant with the $(m-2)$ rows and columns made by substituting the column vector of the right side of Eq.(8) in the i th column of Eq.(9), then p_i is shown in the following.

$$p_i = D_{m-i}^{(m-2)} j^{(m+1)(m-2)} p_1 \quad (i=2, \dots, m-1) \quad (10)$$

Namely, $p_i \sim p_{m-2}$ may be represented by p_1 only. Putting $K(r)$, $L(r)$ in Eq.(6) as follows and considering the relation of Eq.(10),

Fig.1 Multi-layered sphere of m layers

$$\begin{aligned} K(r) &= \frac{[E_{m-1}(r_m^3 - r_{m-1}^3)[2(1-2\nu_m)r_m^3 + (1+\nu_m)r^3] + E_m(r^3 - r_m^3)[2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3]}{3E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)r^3} \\ L(r) &= \frac{E_m(1-\nu_{m-1})}{E_{m-1}(1-\nu_m)} \frac{(r^3 - r_m^3)r_{m-1}^3}{(r_m^3 - r_{m-1}^3)r^3} \end{aligned}$$

$$\sigma_{mr}(r) = p_{m-1} \left\{ K(r) - \frac{p_{m-2}}{p_{m-1}} L(r) \right\} = p_{m-1} \left\{ K(r) - \frac{D_2^{(m-2)}}{D_1^{(m-2)}} L(r) \right\} \quad (11)$$

Putting $\left\{ K(r) - \frac{D_2^{(m-2)}}{D_1^{(m-2)}} L(r) \right\} = \frac{3}{F(r)}$, equation(12) is obtained.

$$p_{m-1}/\sigma_{mr}(r) = F(r)/3 \quad (12)$$

Substituting Eq.(12) into Eq.(5) yields,

$$\sigma_{mt}'(r) = \frac{\sigma_{mr}(r)}{2(r^3 - r_m^3)} \{F(r)r_m^3 - (2r^3 + r_m^3)\} = \frac{H_m(r)}{2} \sigma_{mr}(r) \quad (13)$$

where $H_m(r) = [r_m^3 \{F(r) - 1\} - 2r^3]/(r^3 - r_m^3)$

From Eqs.(3), (13),

$$\sigma_{mr}(r) = \frac{2}{H_m(r)} \{\sigma_{mr}(r) - \sigma_{mt}(r)\} \quad (14)$$

By substituting the equation $\sigma_{mr}(r) = \frac{2}{r^2}$

$\times \int_{r_{m+1}}^r \rho \sigma_{mt}(\rho) d\rho$ into Eq.(14),

$$\frac{1}{r^2} \int_{r_{m+1}}^r \rho \sigma_{mt}(\rho) d\rho = \frac{1}{H_m(r)} \{\sigma_{mr}(r) - \sigma_{mt}(r)\} \quad (15)$$

Letting $\phi(r)$ as follows, $\int_{r_{m+1}}^r \rho \sigma_{mt}(\rho) d\rho = \phi(r)$.

Equation(15) is reduced to the following differential equation.

$$\frac{d\phi(r)}{dr} + \frac{H_m(r)}{r} \phi(r) = r \sigma_{mr}(r) \quad (16)$$

where

$$\begin{aligned} H_m(r) &= \frac{[2E_{m-1}(r_m^3 - r_{m-1}^3)[(1-2\nu_m)r_m^3 - (1+\nu_m)r^3] - E_m(2r^3 + r_m^3)[2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3]}{r^3} \\ &\quad \times \left\{ \left(1 - \frac{3D_2^{(m-2)}}{D_1^{(m-2)}} \right) + \nu_{m-1} \left(1 + \frac{3D_2^{(m-2)}}{D_1^{(m-2)}} \right) \right\} r_{m-1}^3 \\ &= \frac{1}{r} - \frac{3p^2 \left\{ E_{m-1}(1+\nu_m)(r_m^3 - r_{m-1}^3) + E_m[2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3] \right\}}{r^3} \\ &\quad \times \left\{ \left(1 - \frac{3D_2^{(m-2)}}{D_1^{(m-2)}} \right) + \nu_{m-1} \left(1 + \frac{3D_2^{(m-2)}}{D_1^{(m-2)}} \right) \right\} r_{m-1}^3 \end{aligned} \quad (17)$$

Putting the denominator of the second term of the right side in Eq.(17) as $Y(r)$, the relation of Eq.(17) is finally given by

$$\frac{H_m(r)}{r} = \frac{1}{r} - \frac{Y'(r)}{Y(r)} \quad (18)$$

Using the relation $\int_r^{\xi} \frac{H_m(\xi)}{\xi} d\xi = U_m(r) - U_m(\rho)$ and considering the initial conditions at the external surface, the solution of Eq.(16) is

$$\phi(r) = \frac{Y(r)}{r} \int_{r_{m+1}}^r \frac{\rho^2}{Y(\rho)} \sigma_{mT}(\rho) d\rho \quad (19)$$

The radial and tangential residual stresses of m th layer are given by the following equations.

$$\sigma_{mr}(r) = \frac{2}{r^2} \phi(r) = \frac{2}{r^2} Y(r) \int_{r_{m+1}}^r \frac{\rho^2}{Y(\rho)} \sigma_{mT}(\rho) d\rho \quad (20)$$

$$\sigma_{mt}(r) = \frac{1}{r} \frac{d\phi(r)}{dr} = \sigma_{mT}(r) - \frac{H_m(r)}{r^2} Y(r) \int_{r_{m+1}}^r \frac{\rho^2}{Y(\rho)} \sigma_{mT}(\rho) d\rho \quad (21)$$

In order to obtain the radial, tangential stresses σ_{ir} , σ_{it} of an arbitrary i th layer, the stress relieves σ_{ir}' , σ_{it}' in i th layer during removal of $m \sim (i+1)$ th layers must be considered.

$$\left. \begin{aligned} \sigma_{ir}''(r) &= - \left[\frac{r_i^3 \sum_{k=i+1}^m \{p_{i-1}(r_k)\} - r_{i+1}^3 \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \right. \\ &\quad \left. - \frac{\sum_{k=i+1}^m \{p_{i-1}(r_k)\} - \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{r} \right)^3 \right] \\ \sigma_{it}''(r) &= - \left[\frac{r_i^3 \sum_{k=i+1}^m \{p_{i-1}(r_k)\} - r_{i+1}^3 \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \right. \\ &\quad \left. + \frac{\sum_{k=i+1}^m \{p_{i-1}(r_k)\} - \sum_{k=i+1}^m \{p_i(r_k)\}}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{r} \right)^3 \right] \end{aligned} \right\} \quad (22)$$

The true residual stresses at radius r ($r_i \leq r \leq \rho \leq r_{i+1}$) are as follows;

$$\left. \begin{aligned} \sigma_{ir}(r) &= \frac{2}{r^2} \left[\frac{E_{i-1}(r_i^3 - r_{i-1}^3) \{2(1-2\nu_i)r_i^3 + (1+\nu_i)r^3\}}{+ E_i(r^3 - r_i^3) \{2(1-2\nu_{i-1})r_i^3\}} \right. \\ &\quad \left. + \left\{ \left(1 - \frac{3D_2^{(i-2)}}{D_1^{(i-2)}} \right) + \nu_{i-1} \left(1 + \frac{3D_2^{(i-2)}}{D_1^{(i-2)}} \right) \right\} r_{i-1}^3 \right] \\ &\quad \times \int_{r_{i+1}}^r \frac{\rho^2 \sigma_{iT}(\rho)}{\left[\frac{E_{i-1}(r_i^3 - r_{i-1}^3) \{2(1-2\nu_i)r_i^3 + (1+\nu_i)\rho^3\}}{+ E_i(\rho^3 - r_i^3) \{2(1-2\nu_{i-1})r_i^3\}} \right.} d\rho \\ &\quad \left. - \left[\frac{r_i^3 \sum_{k=i+1}^m \{p_{i-1}(r_k)\} - r_{i+1}^3 \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \right. \right. \\ &\quad \left. \left. - \frac{\sum_{k=i+1}^m \{p_{i-1}(r_k)\} - \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{r} \right)^3 \right] \right. \\ \sigma_{it}(r) &= \sigma_{ir}(r) - \frac{2}{r^2} \left[\frac{2E_{i-1}(r_i^3 - r_{i-1}^3) \{ (1-2\nu_i)r_i^3 - (1+\nu_i)r^3 \}}{- E_i(2r^3 + r_i^3) \{2(1-2\nu_{i-1})r_i^3\}} \right. \\ &\quad \left. + \left\{ \left(1 - \frac{3D_2^{(i-2)}}{D_1^{(i-2)}} \right) + \nu_{i-1} \left(1 + \frac{3D_2^{(i-2)}}{D_1^{(i-2)}} \right) \right\} r_{i-1}^3 \right] \\ &\quad \times \int_{r_{i+1}}^r \frac{\rho^2 \sigma_{iT}(\rho)}{\left[\frac{E_{i-1}(r_i^3 - r_{i-1}^3) \{2(1-2\nu_i)r_i^3 + (1+\nu_i)\rho^3\}}{+ E_i(\rho^3 - r_i^3) \{2(1-2\nu_{i-1})r_i^3\}} \right.} d\rho \\ &\quad \left. - \left[\frac{r_i^3 \sum_{k=i+1}^m \{p_{i-1}(r_k)\} - r_{i+1}^3 \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \right. \right. \\ &\quad \left. \left. - \frac{\sum_{k=i+1}^m \{p_{i-1}(r_k)\} - \sum_{k=i+1}^m \{p_i(r_k)\}}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{r} \right)^3 \right] \right] \end{aligned} \right\}$$

$$\left. + \frac{\sum_{k=i+1}^m \{p_{i-1}(r_k)\} - \sum_{k=i+1}^m \{p_i(r_k)\}}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{r} \right)^3 \right] \quad (i=1, \dots, m) \quad (23)$$

The calculating equations of residual stresses in a two-layered sphere can be derived in the following form.

$$\left. \begin{aligned} \sigma_{2r}(r) &= - \frac{2}{r^2} \left[\frac{E_1(r_2^3 - r_1^3) \{2(1-2\nu_2)r_2^3 + (1+\nu_2)r^3\}}{+ E_2(r^3 - r_2^3) \{2(1-2\nu_1)r_2^3 + (1+\nu_1)r_1^3\}} \right. \\ &\quad \times \int_{r_2}^r \frac{\rho^2 \sigma_{1T}(\rho)}{\left[\frac{E_1(r_2^3 - r_1^3) \{2(1-2\nu_2)r_2^3 + (1+\nu_2)\rho^3\}}{+ E_2(\rho^3 - r_2^3) \{2(1-2\nu_1)r_2^3 + (1+\nu_1)r_1^3\}} \right.} d\rho \\ \sigma_{2t}(r) &= \sigma_{2r}(r) - \frac{2}{r^2} \left[\frac{2E_1(r_2^3 - r_1^3) \{ (1-2\nu_2)r_2^3 - (1+\nu_2)r^3 \}}{- E_2(2r^3 + r_2^3) \{2(1-2\nu_1)r_2^3 + (1+\nu_1)r_1^3\}} \right. \\ &\quad \times \int_{r_2}^r \frac{\rho^2 \sigma_{1T}(\rho)}{\left[\frac{E_1(r_2^3 - r_1^3) \{2(1-2\nu_2)r_2^3 + (1+\nu_2)\rho^3\}}{+ E_2(\rho^3 - r_2^3) \{2(1-2\nu_1)r_2^3 + (1+\nu_1)r_1^3\}} \right.} d\rho \end{aligned} \right\} \quad (r_2 \leq r \leq \rho \leq r_1) \quad (24)$$

$$\left. \begin{aligned} \sigma_{1r}(r) &= \frac{2(r^3 - r_1^3)}{r^2} \int_{r_2}^r \frac{\rho^2}{\rho^3 - r_1^3} \sigma_{1T}(\rho) d\rho + \frac{(r^3 - r_1^3)r_2^3}{(r_2^3 - r_1^3)r^2} \sigma_{2r}(r_2) \\ \sigma_{1t}(r) &= \sigma_{1r}(r) + \frac{2r^3 + r_1^3}{r^2} \int_{r_2}^r \frac{\rho^2}{\rho^3 - r_1^3} \sigma_{1T}(\rho) d\rho + \frac{(2r^3 + r_1^3)r_2^3}{2(r_2^3 - r_1^3)r^2} \sigma_{2r}(r_2) \end{aligned} \right\} \quad (r_1 \leq r \leq \rho \leq r_2) \quad (25)$$

For a single-layered sphere, the equations are obtained as follows;

$$\left. \begin{aligned} \text{[Hollow sphere]} \\ \sigma_{1r}(r) &= - \frac{2(r^3 - r_1^3)}{r^2} \int_r^{r_2} \frac{\rho^2}{\rho^3 - r_1^3} \sigma_{1T}(\rho) d\rho \\ \sigma_{1t}(r) &= \sigma_{1r}(r) - \frac{2r^3 + r_1^3}{r^2} \int_r^{r_2} \frac{\rho^2}{\rho^3 - r_1^3} \sigma_{1T}(\rho) d\rho \quad (r_1 \leq r \leq \rho \leq r_2) \\ \text{[Solid sphere]} \\ \sigma_{1r}(r) &= -2 \int_r^{r_2} \frac{\sigma_{1T}(\rho)}{\rho} d\rho \\ \sigma_{1t}(r) &= \sigma_{1r}(r) - 2 \int_r^{r_2} \frac{\sigma_{1T}(\rho)}{\rho} d\rho \quad (r_1=0, 0 \leq r \leq \rho \leq r_2) \end{aligned} \right\} \quad (26)$$

3. Stress compensation for temperature

In a multi-layered sphere composed of many spherical elements with different elastic constants and linear expansion coefficients, the thermal stress corresponding to temperature change affects residual stress distributions, so that the base temperature must be assigned and expressed clearly together with the measuring temperature for the description of residual stresses. Furthermore if both temperatures were different, a proper compensation would be needed.

The authors investigate the effect of the thermal stress for the temperature difference and show the compensating formulas of residual stresses in a two-layered sphere with different materials.

The relation between thermal stress $\Delta\sigma$ and strain $\Delta\varepsilon$ for a unit temperature rise is as follows, using spherical co-ordinates r, t, φ .

$$\Delta\sigma_{ij} = 2G_i \left(\Delta\varepsilon_{ij} + \frac{\nu_i}{1-2\nu_i} \Delta\varepsilon_i \right) - \frac{E_i \alpha_i T_i}{1-2\nu_i} \quad (i=1, 2, j=r, t, \varphi) \quad (28)$$

where $\Delta\varepsilon_i = \Delta\varepsilon_{ir} + \Delta\varepsilon_{it} + \Delta\varepsilon_{i\varphi}$, $\Delta\sigma_{ii} = \Delta\sigma_{ir}$

The symbols E, G, ν, α are Young's mod-

Fig.2 Compensation of X-ray measured stress

Fig.3 Schematic view of electrolytic apparatus

Fig.4 $2\theta - \sin^2\psi$ diagram for diameter d of sphere

Table 1 Chemical composition and condition of electropolishing

Chemical components % (vol)	H_3PO_4 50
	H_2SO_4 50
Temperature °C	30
Voltage V	DC 100
Current density A/dm ²	10
Time S	60

Fig.5 X-ray incident angle (ψ_0) - Diffraction intensity diagram for diameter d of sphere Fig.6 Half width - X-ray incident angle (ψ_0) diagram for diameter d of sphere

Table 2 Chemical composition and condition of chromium electrodeposition and electrolyte for chromium and steel

	Electrodeposition bath	Electrolyte(Cr)	Electrolyte(Steel)
Chemical components	CrO ₃ 250g H ₂ SO ₄ 2.5g H ₂ O 1000g	NaOH 40g Na ₂ CO ₃ 50g H ₂ O 1000g	H ₃ PO ₄ 50%(vol) H ₂ SO ₄ 50%(vol)
Temperature °C	30	30	30
Current density A/dm ²	30(Specimen 1) 20(Specimen 2)	15(Specimen 1) 5(Specimen 2)	100~120

ulus, shear modulus, Poisson's ratio, linear expansion coefficient respectively. From Eq.(28), the equilibrium equation of radial and tangential stresses

$$\Delta\sigma_r = \Delta\sigma_t + \frac{r}{2} \frac{d(\Delta\sigma_r)}{dr}$$

and the relations between strains and displacement

$$\Delta\epsilon_r = d(\Delta u)/dr, \Delta\epsilon_t = \Delta u/r,$$

the displacement Δu_i is given as

$$\Delta u_i = A_i r + \frac{B_i}{r^2} + \frac{1+\nu_i}{1-\nu_i} \alpha_i \frac{1}{r^2} \int_{r_i}^r T_i \rho^2 d\rho \quad \dots\dots\dots(29)$$

The thermal stresses (i.e. stress compensations) are obtained from Eq.(29).

$$\left. \begin{aligned} \Delta\sigma_{ir} &= 2G_i \left\{ \frac{1+\nu_i}{1-2\nu_i} A_i - 2 \frac{B_i}{r^3} \right. \\ &\quad \left. - \frac{1+\nu_i}{1-\nu_i} \alpha_i \frac{2}{r^3} \int_{r_i}^r T_i \rho^2 d\rho \right\} \\ \Delta\sigma_{it} &= 2G_i \left\{ \frac{1+\nu_i}{1-2\nu_i} A_i + \frac{B_i}{r^3} \right. \\ &\quad \left. + \frac{1+\nu_i}{1-\nu_i} \alpha_i \left(\frac{1}{r^3} \int_{r_i}^r T_i \rho^2 d\rho - T_i \right) \right\} \end{aligned} \right\} \dots\dots\dots(30)$$

where the radius is designated r_1, r_2, r_3 from inside, and $r_1 \leq \rho \leq r_2$ for $i=1, r_2 \leq \rho \leq r_3$ for $i=2$.

The constants A_i, B_i in Eq.(30) can be determined from the following boundary conditions.

- (1) $\Delta\sigma_{ir}(r_1)=0$ at the internal surface $r = r_1$ of 1st layer (hollow sphere), and $B_1=0, r_1=0$ (solid sphere).
- (2) $\Delta\sigma_{ir}(r_2)=\Delta\sigma_{2r}(r_2)$ at the boundary $r = r_2$ of 1st and 2nd layers.
- (3) $\Delta u_1(r_2)=\Delta u_2(r_2)$ at the boundary $r = r_2$ of 1st and 2nd layers.
- (4) $\Delta\sigma_{2r}(r_3)=0$ at the external surface $r = r_3$ of 2nd layer.

As an example, the compensation for X-ray measured stress per unit temperature rise from the base temperature is shown in Fig.2 regarding the chromium deposited solid sphere.

4. Examples of application

As applications of this method, the authors measured residual stresses of both chromium layer and steel base in a two-layered solid sphere produced by electrodeposition of chromium.

The base steel sphere is a bearing ball of SUJ-2 with 1 inch diameter (JIS B

Table 3 Conditions of X-ray stress measurement

Apparatus	Parallel beam type(GM tube)
Measured part	SUJ-2, Cr layer
Characteristic X-ray	Cr-K α
Tube voltage	30kV
Tube current	8 mA
Count range	600, 400 c/s
Time constant	16 s
Slit divergent angle	0.35°
Irradiated area	2x7 mm ²
Measuring method	sin ² ψ method(0° - 45° method)
Determination of peak point	Half width method

Table 4 Elastic constant and linear expansion coefficient

		Steel(SUJ-2)	Cr(Deposited layer)
E	Kg/mm ²	2.1×10^4	1.85×10^4 (4)
ν		0.28	0.23 (5)
α	1/°C	1.20×10^{-5}	0.65×10^{-5} (6)

1501, G 4805). After initial bright annealing at 800°C for 90 minutes, the following heat treatments are done to each specimen.

- (Specimen 1) Water quenching at 850°C for 8 minutes → Tempering at 400°C for 30 minutes.
- (Specimen 2) Water quenching at 850°C for 8 minutes → 1st tempering at 200°C for 60 minutes → 2nd tempering at 400°C for 30 minutes.

A hole of 2.5 mm ϕ diameter and about 6 mm depth is bored at a portion of the surface and plugged with a leading wire. Electrolytic polishing of the sphere base has been performed previously under the condition as shown in Table 1. The experimental conditions are shown in Table 2 and a schematic view of electrolytic apparatus in Fig.3.

X-ray stress is measured by Geiger Counter method with an apparatus of parallel beam type under the conditions shown in Table 3 on the equator of a sphere whose pole is a connected point of a leading wire until about 10 mm diameter. Diameter d of the sphere is measured by a micrometer with minimum scale 1/100 mm. The elastic constants and linear expansion coefficients for stress calculations are shown in Table 4.

The preliminary experiments shown in Figs.4~6 are carried out for different diameters ($d=38.04 \sim 6.32$ mm) of spheres (SUJ-2) whose inherent stresses have been

relieved by annealing for the possibility of measuring by X-ray up to 10 mm diameter because of a spherical surface.

A sufficient temperature control is necessary owing to occurrence of thermal stress. In the experiments, a double vessel and air stirring are adopted to minimize the bath temperature change and consequently the variation of temperature is kept at less than $\pm 0.5^\circ\text{C}$. Since the base temperature is set at 30°C and the measuring temperature of X-ray stress at 20°C , the stress compensations are done for temperature difference of 10°C from Eq.(30).

The X-ray measured stresses and the residual stress distributions calculated

from the authors' equations are shown in Figs.7 and 8. The residual stresses of chromium layer shown in Figs.7(b),8(b) have both similar tendencies to decrease in the exterior, but the tangential stress at low current density (Fig.8(b)) is higher. Regarding the residual stresses of steel base shown in Figs.7(a) and 8(a), the tangential stresses are compressive at exterior and tensile at interior in either case, but the stress of Specimen 2 turns compressive again at the smaller diameter. The radial residual stresses of chromium layer are compressive in both specimens, their absolute values being very small and both stresses of chromium layer and steel base

(a) Residual stress distributions in steel base(SUJ-2) (base temperature 30°C)

(a) Residual stress distributions in steel base(SUJ-2) (base temperature 30°C)

(b) Residual stress distributions in chromium layer (base temperature 30°C , depositing temperature 30°C , current density of electrodeposition 30A/dm^2)

(b) Residual stress distributions in chromium layer (base temperature 30°C , depositing temperature 30°C , current density of electrodeposition 20A/dm^2)

Fig.7 Residual stress distributions in chromium deposited two-layered sphere (Specimen 1)

Fig.8 Residual stress distributions in chromium deposited two-layered sphere (Specimen 2)

coincide at the boundary. In a steel base, radial stress of Specimen 1 is tensile in almost all regions, but one of Specimen 2 turns compressive inside.

5. Conclusions

(1) The authors propose a general theory for residual stress measurement of a multi-layered sphere composed of different materials by applying X-ray to the thin layer removal method from the external surface.

(2) The formulas of practical use for a single-layered sphere can be derived as a special case of the general equations of the authors.

(3) Each spherical element has a different elastic constant and a different linear expansion coefficient, so that the residual stresses of a multi-layered sphere are affected by the surrounding temperature. Therefore the authors emphasize the necessity to specify the base temperature in expressing the residual stress and show the compensating method of stresses for a

difference between the base temperature and the measuring one.

(4) As an application of this theory, the authors show two examples of residual stress measurement for chromium deposited steel spheres whose base spheres have been quenched and tempered under different conditions.

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