621-567

Study on Impact Damper

Having a Spring-supported Additional Mass*

By Michio KATO**, Masayoshi DAZAI** and Hirofumi TAKASE***

While a number of studies on impact dampers having a free additional mass have been published, impact dampers having a spring-supported additional mass have been dealt with by only a few investigators. In this study, we devised a model of an impact damper having a spring-supported additional mass and made theoretical and experimental analyses of it, thus obtaining data useful for practical planning applications.

The obtained results are as follows:

- (1) Generally, an impact damper of one-sided impact design is superior in damping effect to one of two-sided impact design.
- (2) If the static distance between the impact damper and the stopper is chosen properly and the ratio of the natural frequency of the impact damper to that of the damped mass is set at a value of 0.6 - 0.8, a damping effect is achieved over a wide range of excitation frequencies.
- (3) The calculated and experimental results are in fairly close agreement with each other.

1. Introduction

A number of studies on impact dampers for suppressing oscillations through the utilization of impact vibration have been published in Japan and abroad. In most of these studies, impact dampers of the type in which a free additional mass is allowed to collide with the main mass are dealt with and, moreover, only steady-state periodic collision is considered with the main purpose of making theoretical analyses.

On the other hand, impact dampers of the type in which a spring-supported additional mass is allowed to collide with the main mass have been studied by only a few investigators such as Nishioka et al. (1) and Peterka (2) (3).

Nishioka et al. applied impact damper with a free additional mass and one with a spring-supported additional mass to expressway lighting poles, reporting that the latter had a superior damping effect as compared with the former. However, their study does not involve any detailed analyses.

Peterka used an analogue computer to analyze an impact damper in the case where an excitation force was applied to the main mass. But, the data presented in his report are not enough for practical planning use.

In this paper, we devised a model of an impact damper having a spring-supported additional mass and made theoretical and experimental analyses of it in the case where the main mass system is subjected to forced displacement. Thus, we obtained

data useful for practical planning applications.

2. Theoretical analysis

2.1 Basic Equations

The impact vibration system discussed in this paper is as shown in Fig. 1. The main mass system is constituted of a mass

Fig. 1

m_1 and a spring k_1 , while the impact damper is constituted of a mass m_2 and a spring k_2 .

In our study, we shall deal with the response of the impact vibration system to an excitation by which the remote end of the spring of the main mass system is subjected to forced sinusoidal displacement. Peterka took up a system similar to that of Fig. 1, but considered the case in which a sinusoidal force was applied to the mass m_1 .

In our experiments, the impact damper was designed in two fashions: two-sided impact design in which the mass m_2 was allowed to collide with the upper and lower stoppers fixed to the mass m_1 ; and one-sided impact design in which the mass m_2 was allowed to collide with only one of the stoppers. From the experimental results which will be shown later, it can be seen that an impact damper of one-sided

* Received 7th May, 1973.

** Research Fellow,
Research and Development Division,
NHK Spring Co., Ltd., Yokohama

impact design is superior in damping effect to one of two-sided impact design. Therefore, the following theoretical analysis will relate solely to the one-sided impact design.

The symbols used in the theoretical analysis are defined as follows:

- m_1 = Mass of the main mass system
- m_2 = Mass of the impact damper
- k_1 = Spring constant of the spring of the main mass system
- k_2 = Spring constant of the spring of the impact damper
- q = Amplitude of forced displacement subjected to the main mass system
- ω = Circular frequency of the excitation force
- t = Time
- d = Static distance between the stopper and the impact damper
- x_1 = Absolute displacement of the main mass
- x_2 = Absolute displacement of the mass of the impact damper
- $\bar{x}_1 = x_1/d$
- $\bar{x}_2 = x_2/d$
- ω_{11} = Natural circular frequency of the main mass system $= (k_1/m_1)^{1/2}$
- ω_{22} = Natural circular frequency of the impact damper $= (k_2/m_2)^{1/2}$
- ω_i = Natural circular frequency of the i -th mode of vibration of a two-degree-of-freedom system without stopper ($i = 1, 2$)
- $\mu = \omega_{22}/\omega_{11}$
- $r = m_2/m_1$
- $\bar{q} = q/d$
- $\nu = \omega/\omega_{11}$
- $\nu_i = \omega_i/\omega_{11}$ ($i = 1, 2$)

Let us define $z = \omega t$ (which will be hereinafter referred to as dimensionless time). Using the above symbols and expressing in matrix representation, we can write the equation of motion for the system of Fig. 1 having no stopper as

$$M\ddot{X} + KX = F(z + \alpha) \quad (1)$$

where

$$M = \begin{bmatrix} 1 & 0 \\ 0 & r \end{bmatrix}, \quad X = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \quad (2)$$

$$K = \frac{1}{\nu^2} \begin{bmatrix} 1 + \mu^2 r & -\mu^2 r \\ -\mu^2 r & \mu^2 r \end{bmatrix}$$

$$F(z + \alpha) = \frac{\bar{q}}{\nu^2} \begin{bmatrix} \sin(z + \alpha) \\ 0 \end{bmatrix}$$

α is the phase angle of forced displacement. If $F(z + \alpha) = 0$ is assumed in Eq. (1), the modal matrix Φ and the eigenvalues ν_1, ν_2 are

$$\Phi = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \quad (3)$$

$$\left. \begin{aligned} \varphi_{1i} &= \frac{\mu^2 - (\nu\nu_i)^2}{[\mu^4 r + (\mu^2 - (\nu\nu_i)^2)^2]^{1/2}} \\ \varphi_{2i} &= \frac{\mu^2}{[\mu^4 r + (\mu^2 - (\nu\nu_i)^2)^2]^{1/2}} \end{aligned} \right\} \quad (4)$$

$$\nu_i^2 = [1 + \mu^2(1+r) \mp \sqrt{(1 + \mu^2(1+r))^2 - 4\mu^2}] / 2\nu^2 \quad (5)$$

Using the modal matrix (4), we can solve Eq. (1) under initial conditions $X(0), \dot{X}(0)$ as

$$X(z) = \Phi[\cos \nu z \Phi^T M \dot{X}(0) + G(z, \alpha)] \quad (6)$$

where

$$\left. \begin{aligned} \cos \nu z &= \begin{bmatrix} \cos \nu_1 z & 0 \\ 0 & \cos \nu_2 z \end{bmatrix} \\ \sin \nu z &= \begin{bmatrix} \sin \nu_1 z & 0 \\ 0 & \sin \nu_2 z \end{bmatrix}, \quad \nu = \begin{bmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{bmatrix} \end{aligned} \right\} \quad (7)$$

$$G(z, \alpha) = \int_0^z \sin \nu(z-\tau) \nu^{-1} \Phi^T F(\tau + \alpha) d\tau = \bar{q} \{ \sin(z + \alpha) - \cos \alpha \sin \nu z \nu^{-1} - \sin \alpha \cos \nu z \} Q \quad (8)$$

$$Q = \begin{bmatrix} \varphi_{11}/(\nu_1^2 \nu^2 - \nu^2) \\ \varphi_{12}/(\nu_2^2 \nu^2 - \nu^2) \end{bmatrix} \quad (9)$$

2.2 Calculating equations for impact vibration

On the basis of Eq. (6), let us find the calculating equations for impact vibration. The period of impact will not always agree with that of forced displacement. Therefore, we consider an instance of impact in which a collision occurs whenever the forced displacement repeats itself n times, which will be hereinafter referred to as the simple repeated impact with one collision for every n cycles of displacement.

In the following discussion, we shall find the calculating equations for the simple repeated impact with one collision for every n cycles of displacement. Other instances of impact vibration will be analyzed through simulation by means of a digital computer.

We assume that, as shown in Fig. 2,

Fig. 2

the simple repeated impact with one collision for every n cycles of displacement is taking place between two masses with a period z_0 (hence $z_0 = 2n\pi$).

Let $X_j = [\bar{x}_{1,j}, \bar{x}_{2,j}]^T$ be the position at which the j -th collision occurs, and

$\dot{X}_j^- = [\dot{\bar{x}}_{1,j}^-, \dot{\bar{x}}_{2,j}^-]^T$ and $\dot{X}_j^+ = [\dot{\bar{x}}_{1,j}^+, \dot{\bar{x}}_{2,j}^+]^T$ be the velocities immediately before and after the j -th collision, respectively.

From the law of conservation of momentum and the definition of the restitution coefficient e , the following relation will hold between $\dot{X}_j^-$ and $\dot{X}_j^+$:

$$\dot{X}_j^+ = e \dot{X}_j^- \quad (10)$$

$$e = \begin{bmatrix} \frac{1-e}{1+r} & \frac{r(1+e)}{1+r} \\ \frac{1+e}{1+r} & \frac{r-e}{1+r} \end{bmatrix}$$

Using Eqs. (6) and (10), we can express the position at which the $(j+1)$ -th collision occurs, X_{j+1} , and the velocity immediately before the $(j+1)$ -th collision, $\dot{X}_{j+1}^-$, in terms of X_j and $\dot{X}_j^-$ as

$$\left. \begin{aligned} X_{j+1} &= A_{11}(z_0) X_j + A_{12}(z_0) \dot{X}_j^- + \Phi G(z_0, \alpha) \\ \dot{X}_{j+1}^- &= A_{21}(z_0) X_j + A_{22}(z_0) \dot{X}_j^- + \Phi G_z(z_0, \alpha) \end{aligned} \right\} \quad (11)$$

where

$$\left. \begin{aligned} A_{11}(z_0) &= \Phi \cos \nu z_0 \Phi^T M \\ A_{12}(z_0) &= \Phi \sin \nu z_0 \nu^{-1} \Phi^T M \sigma \\ A_{21}(z_0) &= -\Phi \nu \sin \nu z_0 \Phi^T M \\ A_{22}(z_0) &= \Phi \cos \nu z_0 \Phi^T M \sigma \end{aligned} \right\} \dots\dots\dots (12)$$

$$G_z(z_0, \alpha) = \frac{\partial G}{\partial z}(z_0, \alpha) = \bar{q}(\cos(z_0 + \alpha) - \cos \alpha \cos \nu z_0 + \sin \alpha \nu \sin \nu z_0) Q \dots\dots\dots (13)$$

Since the simple repeated impact with one collision for every n cycles of displacement is under consideration, the relations $X_{j+1} = X_j$ and $\dot{X}_{j+1} = \dot{X}_j$ should hold. Substituting these relations in Eq. (11) and rearranging, we obtain

$$A \begin{bmatrix} X_j \\ \dot{X}_j \end{bmatrix} = C \dots\dots\dots (14)$$

or

$$\begin{bmatrix} X_j \\ \dot{X}_j \end{bmatrix} = A^{-1} C \dots\dots\dots (15)$$

$$A = \begin{bmatrix} A_{11} - E & A_{12} \\ A_{21} & A_{22} - E \end{bmatrix}, \quad C = \begin{bmatrix} -\Phi G(z_0, \alpha) \\ -\Phi G_z(z_0, \alpha) \end{bmatrix} \dots\dots\dots (16)$$

where E is the unit matrix of order 2. On the other hand, at the moment of the j -th collision, the following relation will hold between the displacements, x_{1j} and x_{2j} , of the two masses:

$$\bar{x}_{2,j} - \bar{x}_{1,j} = 1 \dots\dots\dots (17)$$

Therefore, if z_0 is given, x_j , $\dot{x}_j$ and α can be determined from Eqs. (15) and (17).

where

$$\left. \begin{aligned} B &= -\Phi \nu^2 \cos \nu z_0 \Phi^T M X_j + A_{21} \sigma \dot{X}_j + \Phi G_{zz}(z_0, \alpha) \\ D_1 &= \Phi G_\alpha(z_0, \alpha) - \dot{X}_{j+1}, \quad D_2 = \Phi G_z(z_0, \alpha) - B_1 \\ G_{zz}(z_0, \alpha) &= \frac{\partial^2 G}{\partial z^2}(z_0, \alpha) = \bar{q}[-\sin(z_0 + \alpha) + \cos \alpha \nu \cos \nu z_0 + \sin \alpha \nu^2 \cos \nu z_0] Q \\ G_\alpha(z_0, \alpha) &= \frac{\partial G}{\partial \alpha}(z_0, \alpha) = \bar{q}[\cos(z_0 + \alpha) + \sin \alpha \sin \nu z_0 \nu^{-1} - \cos \alpha \cos \nu z_0] Q \\ G_{z\alpha}(z_0, \alpha) &= \frac{\partial^2 G}{\partial z \partial \alpha}(z_0, \alpha) = \bar{q}[-\sin(z_0 + \alpha) + \sin \alpha \cos \nu z_0 + \cos \alpha \nu \sin \nu z_0] Q \end{aligned} \right\} \dots\dots\dots (20)$$

On the other hand, we obtain from Eq. (17)

$$\partial \bar{x}_{1,j} = \partial \bar{x}_{2,j}, \quad \partial \bar{x}_{1,j+1} = \partial \bar{x}_{2,j+1} \dots\dots\dots (21)$$

Let us define

$$A(z_0) = \begin{bmatrix} A_{11}(z_0) & A_{12}(z_0) \\ A_{21}(z_0) & A_{22}(z_0) \end{bmatrix} = (a_{ij}), \quad D = [D_1, D_2] = (d_{ij}), \quad B = (b_i) \\ R_j = \begin{bmatrix} \partial \bar{x}_j \\ \partial \dot{X}_j \end{bmatrix}, \quad \partial \bar{x}_j = \partial \bar{x}_{1,j} = \partial \bar{x}_{2,j}$$

Then, performing certain calculations by use of Eq. (21) and rewriting Eqs. (18) and (19), we obtain

$$R_{j+1} = TR_j \dots\dots\dots (22)$$

$$T = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 \\ a_{31} + a_{32} + e_1 b_1 & a_{33} + e_2 b_1 & a_{34} + e_3 b_1 & d_{12} + e_4 b_1 \\ a_{41} + a_{42} + e_1 b_2 & a_{43} + e_2 b_2 & a_{44} + e_3 b_2 & d_{22} + e_4 b_2 \\ e_1 & e_2 & e_3 & e_4 \end{bmatrix} \dots\dots\dots (23)$$

where

$$\begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} = \begin{bmatrix} a_{11} + a_{12} - a_{21} - a_{22} & a_{13} - a_{23} \\ a_{14} - a_{24} & d_{11} - d_{21} \end{bmatrix} \frac{1}{\bar{x}_{2,j} - \bar{x}_{1,j}} \dots\dots\dots (24)$$

$$\begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix} = \begin{bmatrix} e_1 \bar{x}_{1,j} + a_{11} + a_{12} & a_{13} + e_2 \bar{x}_{1,j} \\ a_{14} + e_3 \bar{x}_{1,j} & d_{11} + e_4 \bar{x}_{1,j} \end{bmatrix} \dots\dots\dots (25)$$

Therefore, in order to secure the stability of the simple repeated impact with one collision for every n displacements, it is necessary that $R_j \rightarrow 0$ when $j \rightarrow \infty$. This will be satisfied only when $|\lambda| < 1$ where λ is the eigenvalue of the matrix T . That is, the root λ of the following equation must satisfy the condition $|\lambda| < 1$ (5):

$$|\lambda E - T| = 0 \dots\dots\dots (26)$$

where E is the unit matrix of order 4.

2.3 Stability conditions

In the preceding section, we obtained a solution for the simple repeated impact with one collision for every n cycles of displacement. But, whether the motion represented by this solution will actually occur or not depends on the stability of the solution. Then, we shall examine its stability.

In the preceding section, we considered a section between one and the next collisions and assumed the moment of the former to be the origin of time. In this section, however, let us newly define an origin of time common to all sections.

Now, we assume that the system has undergone an infinitesimal disturbance and, as a result, α , z_j (the dimensionless time at which the j -th collision occurs), x_j , $\dot{x}_j$, x_{j+1} and $\dot{x}_{j+1}$ have changed to $\alpha + \delta\alpha$, $z_j + \delta z_j$, $z_{j+1} + \delta z_{j+1}$, $z_0 + \delta z_0$ (where $\delta z_0 = \delta z_{j+1} - \delta z_j$), $x_j + \delta x_j$, $\dot{x}_j + \delta \dot{x}_j$, $x_{j+1} + \delta x_{j+1}$ and $\dot{x}_{j+1} + \delta \dot{x}_{j+1}$ respectively.

Substituting $x_j + \delta x_j$, $\dot{x}_j + \delta \dot{x}_j$ etc. for x_j , $\dot{x}_j$ etc. in Eq. (11) and neglecting the terms of δ^2 , we obtain

$$\delta \dot{x}_{j+1} = A_{11}(z_0) \delta \dot{x}_j + A_{12}(z_0) \delta \dot{X}_j + \dot{X}_{j+1} - \delta z_{j+1} + D_1 \delta z_j \dots\dots\dots (18)$$

$$\delta \dot{x}_{j+1} = A_{21}(z_0) \delta \dot{x}_j + A_{22}(z_0) \delta \dot{X}_j + B \delta z_{j+1} + D_2 \delta z_j \dots\dots\dots (19)$$

Let us expand Eq. (26) and assume that

$$\lambda^4 + b_1 \lambda^3 + b_2 \lambda^2 + b_3 \lambda + b_4 = 0 \dots\dots\dots (27)$$

Then, in order to satisfy the condition $|\lambda| < 1$, the following relations must hold for $b_1 \sim b_4$ (6):

$$\left. \begin{aligned} 1 + \sum_{i=1}^4 b_i &> 0, \quad 1 + \sum_{i=1}^4 (-1)^i b_i > 0 \\ (1 - b_4)^2 (1 - b_2 + b_4) - (b_1 - b_3)(b_1 b_4 - b_3) &> 0 \\ -b_1 + b_3 - 2(1 - b_4) &< 0 \\ -b_1 + b_3 + 2(1 - b_4) &> 0 \end{aligned} \right\}$$

Furthermore, in order to allow the masses to collide with each other, the following relations must hold for their velocities and displacements:

$$\bar{x}_{2,j}^- > \bar{x}_{1,j}^- \quad \dots\dots\dots(29)$$

$$\bar{x}_{2,j} - \bar{x}_{1,j} < 1, \quad z_j < z < z_{j+1} \quad \dots\dots\dots(30)$$

After all, the simple repeated impact with one collision for every n cycles of displacement will be stable only when the conditions represented by Eqs. (28-through (30)) are satisfied.

2.4 Simulation by means of a digital computer

The simple repeated impact could be analyzed as described in the preceding section. For other complicated instances of impact, however, we made numerical analysis through simulation by means of a digital computer.

The equations used in the simulation are as follows:

$$\begin{aligned} \mathbf{X}(z) &= \mathbf{A}_{11}(z)\mathbf{X}(0) + \mathbf{A}_{12}(z)\dot{\mathbf{X}}^-(0) + \mathbf{G}(z, \alpha) \\ \dot{\mathbf{X}}^-(z) &= \mathbf{A}_{21}(z)\mathbf{X}(0) + \mathbf{A}_{22}(z)\dot{\mathbf{X}}^-(0) + \mathbf{G}_z(z, \alpha) \end{aligned} \quad \dots\dots\dots(31)$$

The flow chart for the simulation is shown in Fig. 3. Some examples of the numerical calculations according to the above equations will be set forth later.

Fig. 3

3. Experimental examination

The results of theoretical analysis of the impact vibration system were shown above. In the following, we shall try a comparison between calculated and experimental results and thereafter carry out a number of experiments so as to determine the optimum constructional conditions of the impact damper being discussed.

3.1 Experimental method and specifications

The model of an impact vibration system used in this study is shown in Fig. 4. The impact damper was constituted of

Table 1

r	Main mass system		Mass m_2 (kg)
	Spring constant (kg/mm)	Mass m_1 (kg)	
0.01	20.9	4.34×10^{-3}	48×10^{-3}
0.02	10.6	2.20 "	"
0.04	5.8	1.21 "	"

Fig. 4

Fig. 5

a leaf spring and a weight, while the main mass system was constituted of a coil spring and a rigid body (including stoppers and excluding the impact damper) attached to the upper end of a coil spring. All of these elements were made of steel except the leaf spring of the impact damper.

The distances between the impact damper and the main mass was controlled by moving the stoppers up and down. For the one-sided impact design, any one of the stoppers (the lower one in this study) was so adjusted that no collision would occur.

In carrying out the experiments, the lower end of the impact vibration system shown in Fig. 4 was connected to an electromagnetic exciter and subjected to forced displacement with a constant amplitude, during which the responsive displacement of the mass of the main mass system was measured.

The excitation frequency was continuously varied at a constant sweep rate (0.3 Hz/s) within a certain range (0.3 - 1.2 Hz). The experimental system drawing is shown in Fig. 5.

In these experiments, the excitation amplitude was set at a constant value of 0.05 mm while the mass ratio was selected from among three values of 0.01, 0.02 and 0.04. For each of these three values,

the frequency ratio μ and the distance d between stopper and impact damper were varied diversely.

The values employed for the mass and spring constant of the main mass system and the mass of the impact damper are specified in Table 1.

Each spring was so designed that it had the smallest possible mass (about 10%) as compared with the mass of the corresponding weight. For each mass value given in Table 1, however, the mass of the spring is taken into account by adding the equivalent mass of the spring (that is, $1/3$ of the mass for a coil spring or $33/140$ of the mass for a leaf spring) to the mass of the corresponding weight.

The natural frequency of the main mass system was kept at a constant value of 34.5 Hz throughout the experiments. The frequency ratio μ was varied by altering the spring constant of the impact damper. If the spring constant of the impact damper is altered, the equivalent mass of the spring will change more or less. Therefore, the material and specifications of the spring was so modified as to eliminate such a change.

3.2 Comparison between experimental and calculated results

With respect to the example in which the parameters are set as $\mu = 0.9$, $r = 0.04$ and $\bar{q} = 0.02$ ($q = 0.05$ mm, $d = 2.5$ mm), the experimental and calculated results are shown in Fig. 6.

Fig. 6

In this figure, the solid line shows a characteristic curve for m_1 based on the calculated results. In making the calculations, the restitution coefficient e was set at a constant value of 0.6 by reference to literature and previous data.

The broken line shows a characteristic curve for m_1 based on the experimental results. The black-cored circle ($\odot$) and triangle ($\triangle$) marks indicate the dynamic characteristics observed when the excitation frequency was increased and decreased, respectively.

The alternate long and short dash line and the alternate long and two short dashes line show resonance curves for m_1 and m_2 , respectively, based on the calculated results regarding the simple two-degree-of-freedom system without a stopper.

From geometrical examination of the alternate long and short dash line and the alternate long and two short dashes line, it will be too evident to require analysis that collisions occur if (absolute displacement of m_2) - (absolute displacement of m_1) $> d$.

These collisions will take place along the section B-C and I-J of the characteristic curve shown in Fig. 6.

Referring to Fig. 6, we shall explain the state of motion of the main mass.

When the excitation frequency is gradually increased, the amplitude of the vibratory motion of m_1 first increases (A $\rightarrow$ B) along the characteristic curve for the two-degree-of-freedom system without a stopper. After m_1 and m_2 begin to collide with each other at the point B, the amplitude still increases (B $\rightarrow$ C). On passing through the point C, the geometrical condition of collision ceases to be satisfied. However, collisions continue to occur (C $\rightarrow$ E-F) if the frequency is further increased. On arriving at the point F, the amplitude decreases sharply to the point G. In the course from the point G to the point I, no collision occurs while the amplitude increases along the characteristic curve for the two-degree-of-freedom system. At the point I, collisions begin to occur again (I $\rightarrow$ J). On passing through the point J, the geometrical condition of collision ceases to be satisfied again, but collisions continue to occur (J $\rightarrow$ K). Beyond the point K, no collision occurs. Next, if the excitation frequency is decreased, the amplitude changes along a different path L $\rightarrow$ J $\rightarrow$ I $\rightarrow$ G $\rightarrow$ H $\rightarrow$ C $\rightarrow$ B $\rightarrow$ A. Thus, the dynamic characteristics of m_1 observed when the excitation frequency is decreased differ from those observed when the excitation frequency is increased, which is a hysteresis phenomenon peculiar to nonlinear vibratory motion.

In Fig. 6, one collision occurs for

(a) $\nu = 0.86$

(b) $\nu = 0.92$

(c) $\nu = 1.03$

$\mu = 0.9, r = 0.04, \bar{q} = 0.02$

Scale of m_2 is $1/2$ times one of m_1

Fig. 7

every two cycles of forced displacement along the section B'-D hence $z_0=4\pi$), while one collision occurs for every one cycle of forced displacement along the section E-F hence $z_0=2\pi$). On the other hand, transient aperiodic collisions are observed along the sections B-B' and D-E. at the transition point E where the mode of impact changes, the slope of the tangent to the characteristic curve undergoes a discontinuous change. This is recognized in the calculated characteristic curve as well as in the experimental one. Along the section J-K, not a simple repeated impact but a beat phenomenon is noted.

The modes of impact corresponding to various portions of the characteristic curve were observed by means of a non-contact type optical displacement vibrograph (sold by Zimmer AG; West Germany). Then, we shall explain the observed results.

Fig. 7 shows some photographs taken by connecting the output of the vibrograph to a synchroscope. In each photograph, the upper and lower curves indicate the wave forms of vibratory motion of the impact surfaces of m_1 and m_2 , respectively. Fig. 7(a) displays the simple repeated impact with one collision for every two cycles of displacement which was observed along the section B'-D; Fig. 7(b) the simple repeated impact with one collision for every one cycle of displacement which was observed along the section E-F and Fig. 7(c) the beat phenomenon which was observed along the section J-K.

The experimentally observed modes of impact were also confirmed by calculations. That is, the calculated and experimental results were in close agreement with each other.

With respect to the above-mentioned dynamic characteristics and modes of impact, similar tendencies are noted in the study of Peterka who made analysis by means of an analogue computer.

3.3 Discussion about experimental results

In order to examine the influence of the frequency ratio on the dynamic characteristics of the main mass, the frequency ratio μ was varied widely while keeping $r = 0.04$ and $\bar{q} = 0.02$ constant. The results are shown in Fig. 8. Each graph indicates the dynamic characteristics observed when the excitation frequency was increased.

The broken line shows the dynamic characteristics corresponding to the case of the linear dynamic damper without a stopper; the solid line the dynamic characteristics corresponding to the case of the impact damper of one-sided impact design; and the alternate long and short dash line the dynamic characteristics corresponding to the case of the impact damper of two-sided impact design. For the impact damper of two-sided impact design, the static distances between the damper mass and the two stoppers were so adjusted that they were equal to each other.

With respect to the impact dampers,

it can be seen from Fig. 8 that the secondary peak value is larger than the primary one if μ is less than 1, as where the secondary peak value is smaller than the primary one if μ approaches 1 (and even if μ exceeds 1, though not shown in Fig. 8). Therefore, an optimum damping effect will be achieved when the primary and secondary peak values are equal to each other. Moreover, it can be seen that, as the value of μ increases, the primary peak shifts to the right and approaches the secondary one.

Fig. 8

In Fig. 9, the maximum value $|x_1/q|_{\max}$ on the characteristic curve for m_1 corresponding to the impact damper of one-sided impact design is plotted against $1/\bar{q}$. It can be seen from this figure that the optimum value of $\bar{q}$ which minimizes the value of $|x_1/q|_{\max}$ lies within a range of about 0.02 - 0.05.

In Figs. 10 - 12, the maximum value on the characteristic curve for m_1 is plotted against the three parameters r , μ and $\bar{q}$. For each graph, the solid circle ($\bullet$) plotted on the axis of ordinates $|x_1/q|_{\max}$ shows the maximum value on the characteristic curve for m_1 corresponding to the case in which the impact damper was omitted from the system. From these figures, it can be concluded that:

(a) Generally, an impact damper of one-sided impact design is superior in damping effect to one of two-sided impact

Fig. 9

Fig. 10

Fig. 11

Fig. 12

design. This has also been pointed out by Peterka.

(b) If μ and q are kept constant, the impact damper of greater mass produces a better damping effect.

(c) If r and q are given, the best damping effect is achieved at a frequency

ratio in the range of about 0.6 - 0.8

From its nature, the damping effect of the linear dynamic damper cannot be relied upon when the excitation frequency varies widely. On the contrary, if the value of μ is set at about 0.7 and the value of q at 0.02 - 0.05, the damping effect of the impact damper may be expected over a wide range of excitation frequencies.

4. Conclusions

We conducted a study on a model of an impact damper having spring-supported additional mass. The obtained results are as follows:

(1) Generally, an impact damper of one-sided impact design is superior in damping effect to one of two-sided impact design.

(2) If the ratio of the amplitude of forced displacement to the static distance between stopper and impact damper is set at a value of about 0.02 - 0.05 and the ratio of the natural frequency of impact damper to that of main mass system at a value of 0.6 - 0.8, a damping effect is achieved over a wide range of excitation frequencies.

(3) The calculated and experimental results are in fairly close agreement with each other.

5. Postscript

Impact dampers of the type discussed in this paper have been applied to beams which may be subjected to the oscillation due to the pulsation of pipings or the oscillation and shock due to random inputs, their excellent damping effect being thus observed. Details will be reported at a later date.

Last but not least, we are deeply indebted to Dr. Setsuo Kobayashi, Chief of the Machinery Research Laboratory, NHK Spring Co., Ltd., for his guidance in conducting this study.

Also, we express our grateful thanks to Profs. Tamura and Yamada for their careful examination of the manuscript and helpful criticism on it.

References

- (1) Nishioka et al.: "Sumitomo Kinzoku", 16-1 (January, 1964), 44 (in Japanese).
- (2) Peterka, F.: *Strojnický Časopis*, Paper 1, No. 5 (1970); II, No. 6 (1970).
- (3) Peterka, F.: *Strojnický Časopis*, XXII, C.5 (1971).
- (4) Kawai: "Lectures on Technology of Structures Using a Computer", I-4-B (Baifukan) (in Japanese).
- (5) Masri, S.F., and Caughey, T.K.: *Trans. ASME, Ser. E*, 33-3 (September, 1966), 586.
- (6) Jury, E.I., and Gupta, S.C.: *Regelungstechnik*, 10 (1962), 494.