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Flow Uniformalizing Effect of

Damping Screen in Pipe Flow

(The Case of S-Shape Velocity Distribution)*

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As part of a serial study of the effect of a damping screen on the velocity distribution of pipe flow, the case of a flow with S-shape velocity distribution is analysed theoretically. The analyses are performed for the case of a single isotropic damping screen and for three other combinations of (1) an isotropic damping screen and an isotropic lattice, (2) the screen and an annular lattice and (3) the screen and a radial lattice.

Theoretical results show that, for a single isotropic screen, the final damping ratio is the same as in the case of an axi-symmetric axial velocity distribution, but the flow uniformalizing process is more gradual than the axi-symmetric case. It is also made clear, for cases of combined damping devices, that the damping process vanishes downstream of an isotropic lattice, and a secondary flow is generated downstream of an annular or radial lattice.

1. Introduction

On the flow uniformalizing problems in pipe flow by damping screens, the case of an axi-symmetric flow is analysed theoretically in the previous paper⁽¹⁾. In the real pipe flow, however, being affected by non-symmetric boundary conditions such as a ground effect at the pipe intake or a bend in the pipe system, the perturbation velocity distribution often becomes non-symmetric besides axi-symmetric.

So the case of a pipe flow with S-shape distribution of axial velocity is treated in this paper. For damping devices are used combinations of an isotropic screen with an isotropic, annular or radial lattice. The purpose of this paper is to analyse the flow uniformalizing mechanisms of the above-mentioned systems and to assess the effectiveness of these devices.

2. Nomenclature

U_0 : axial component of main-flow velocity
 v'_z : axial component of perturbation velocity, $v'_z \equiv v'_z/U_0$
 v'_r : radial component of perturbation velocity, $v'_r \equiv v'_r/U_0$
 v'_ϕ : circumferential component of perturbation velocity, $v'_\phi \equiv v'_\phi/U_0$
 p' : perturbation static pressure, $p \equiv p'/\rho U_0$
 z' : axial coordinate, $z \equiv z'/a$
 r' : radial coordinate, $r \equiv r'/a$
 ϕ : circumferential angle

a : radius of pipe (D : diameter of pipe)
 K : static pressure loss coefficient of a screen of a perforated plate
 ρ : density of fluid
 J_n : Bessel function of first kind of order n
 Y_n : Bessel function of second kind of order n
 $f(r)$: function showing radial distribution of axial perturbation velocity far upstream (known function)
 $S(r)$: function showing radial distribution of axial perturbation velocity far downstream (unknown function)
 $|A|$: local damping ratio of axial perturbation velocity
 $|A_\infty|$: final damping ratio of axial perturbation velocity
 Subscripts (examples: 10^{Vzu} , 1^{Vrd} , $1^{V\phi d}$, see Fig.1)
 I : order number of damping screen or perforated plate (counted along flow direction)
 o : position at the inlet of damping screen
 l : position at the outlet of damping screen
 u : region upstream of damping screen (region 1)
 d : region downstream of damping screen (region 2)
 z : axial component
 r : radial component
 ϕ : circumferential component

3. Theory

following the previous paper⁽¹⁾, it is assumed that the flow in pipe is steady, incompressible and inviscid and perturbation velocities are small comparing with the

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axial component of main flow velocity U_0 . The slip at the wall is allowed in axial and circumferential directions. In the analysis the actuator disc method is applied.

3.1 Fundamental equations

A cylindrical coordinate is used. The main flow is uniform everywhere in a pipe and has the axial velocity U_0 , and perturbations of the flow are assumed to be small comparing with the main flow. In Euler's equation and the equation of continuity, if the second order small terms are neglected and the terms concerning the main flow are subtracted, the following equations concerning the first order small terms are obtained in a non-dimensional form.

$$\left. \begin{aligned} \frac{\partial v_r}{\partial z} &= -\frac{\partial p}{\partial r}, & \frac{\partial v_\phi}{\partial z} &= -\frac{1}{r} \frac{\partial p}{\partial \phi}, \\ \frac{\partial v_z}{\partial z} &= -\frac{\partial p}{\partial z} \end{aligned} \right\} \dots\dots\dots (1)$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0 \dots\dots\dots (2)$$

The boundary conditions are given as follows:

$$v_r = 0, \quad v_\phi = 0 \quad \text{at } z = -\infty \dots\dots\dots (3)$$

$$v_z = f(r) \cos \phi \quad \text{at } z = -\infty \dots\dots\dots (4)$$

$$v_r = 0 \quad \text{at } r = 1 \dots\dots\dots (5)$$

where, the boundary conditions at $z = -\infty$ of Eqs.(3) and (4) show the shape of velocity distribution far upstream, i.e. it has no radial or circumferential component but an axial component which has a shape of $f(r)$ in the radial direction and one of $\cos \phi$ in the circumferential direction. In this paper is treated only the case of a pipe flow with the S-shape distribution of the axial velocity, so the distribution function $f(r)$ in Eq. (4) is a known odd function.

From Eq.(1) the following relation is obtained with the aid of Eq. (2):

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\partial^2 p}{\partial z^2} = 0 \dots\dots\dots (6)$$

Then using the terms corresponding to the given S-shape distribution of the axial velocity the following equations are obtained:

$$\begin{aligned} p &= \sum_n [(E_n J_1(k_n r) + F_n Y_1(k_n r)) \cos \phi \\ &\quad + (\tilde{E}_n J_1(k_n r) + \tilde{F}_n Y_1(k_n r)) \sin \phi] \\ &\quad (C e^{k_n z} + D e^{-k_n z}) \dots\dots\dots (7) \end{aligned}$$

where, J_1 and Y_1 are the first and the second kind Bessel functions, respectively. In Eq.(7), the perturbations should be finite even at points far upstream and far downstream, so an arbitrary constant D should be zero in the region upstream of a screen, and C should be zero in the region downstream of the screen. And from the fact that perturbations should be finite everywhere in a pipe including the point where $r=0$, F_n and $\tilde{F}_n$, the coefficients of $Y_1(k_n r)$ in Eq.(7), are determined to be zero.

3.2 The flow upstream of a screen (Region 1)

As mentioned above F_n , $\tilde{F}_n$ and D in Eq.(7) are put equal to zero in this case. Substituting Eq.(7) into the third equation of Eq.(1), the next equation is gained:

$$\begin{aligned} v_{zn} &= -\sum_n [E_{zn} J_1(k_n r) \cos \phi \\ &\quad + \tilde{E}_{zn} J_1(k_n r) \sin \phi] e^{k_n z} \\ &\quad + L_n(r) \cos \phi + \tilde{L}_n(r) \sin \phi \dots\dots\dots (8) \end{aligned}$$

Using the boundary condition given in Eq. (3) the arbitrary functions $L_n(r)$ and $\tilde{L}_n(r)$ in Eq.(8) are determined as $L_n(r) = f(r)$ and $\tilde{L}_n(r) = 0$. Then the perturbations p_u and v_{zu} upstream of a screen are obtained as Eqs.(9) and (10). Then using the second and third relations of Eq.(1) with Eqs.(9) and (3), the other components of the perturbation velocity upstream of a screen are obtained as Eqs.(11) and (12) after some reductions:

$$p_u = \sum_n (E_{zn} \cos \phi + \tilde{E}_{zn} \sin \phi) J_1(k_n r) e^{k_n z} \dots\dots\dots (9)$$

$$\begin{aligned} v_{zn} &= -\sum_n (E_{zn} \cos \phi \\ &\quad + \tilde{E}_{zn} \sin \phi) J_1(k_n r) e^{k_n z} + f(r) \cos \phi \dots\dots\dots (10) \end{aligned}$$

$$\begin{aligned} v_{rn} &= -\sum_n (E_{zn} \cos \phi \\ &\quad + \tilde{E}_{zn} \sin \phi) \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} e^{k_n z} \dots\dots\dots (11) \end{aligned}$$

$$\begin{aligned} v_{\phi n} &= \sum_n (E_{zn} \sin \phi - \tilde{E}_{zn} \cos \phi) \frac{J_1(k_n r)}{k_n r} e^{k_n z} \\ &\dots\dots\dots (12) \end{aligned}$$

Applying Eq.(4) to Eq.(11), the following relation is deduced, because the boundary condition given in Eq.(4) should be satisfied independently of ϕ and z :

$$J_0(k_n) - \frac{J_1(k_n)}{k_n} = 0 \dots\dots\dots (13)$$

From Eq.(13) the values of k_n used in Eqs. (9)~(12) are determined as shown in Table (1).

Putting z as zero in Eqs.(9)~(12) the perturbations just upstream of a screen are obtained as follows: (refer to figure (1), about subscripts)

$$10p_u = \sum_n (E_{zn} \cos \phi + \tilde{E}_{zn} \sin \phi) J_1(k_n r) \dots\dots\dots (9)'$$

$$\begin{aligned} 10v_{zn} &= -\sum_n (E_{zn} \cos \phi \\ &\quad + \tilde{E}_{zn} \sin \phi) J_1(k_n r) + f(r) \cos \phi \dots\dots\dots (10)' \end{aligned}$$

$$\begin{aligned} 10v_{rn} &= -\sum_n (E_{zn} \cos \phi \\ &\quad + \tilde{E}_{zn} \sin \phi) \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} \dots\dots\dots (11)' \end{aligned}$$

Fig. 1: Explanation of flow regions and notations

$$10v_{\phi z} = \sum_n (E_{zn} \sin \phi - \tilde{E}_{zn} \cos \phi) \frac{J_1(k_n r)}{k_n r} \dots\dots\dots(12)'$$

3.3 Flow downstream of a screen (Region 2)

In this case the arbitrary constant C , in Eq.(7) becomes zero as mentioned above. Following the previous section the perturbation pressure and three perturbation velocity components are obtained as follows:

$$p_d = \sum_n (E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi) J_1(k_n r) e^{-k_n z} \dots\dots\dots(14)$$

$$v_{zd} = - \sum_n (E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi) J_1(k_n r) e^{-k_n z} + S(r) \cos \phi + \tilde{S}(r) \sin \phi \dots\dots\dots(15)$$

$$v_{rd} = \sum_n \{ E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi \} \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} e^{-k_n z} + N_d(r) \cos \phi + \tilde{N}_d(r) \sin \phi \dots\dots\dots(16)$$

$$v_{\phi d} = - \sum_n (E_{dn} \sin \phi - \tilde{E}_{dn} \cos \phi) \frac{J_1(k_n r)}{k_n r} e^{-k_n z} + M_d(r) \cos \phi + \tilde{M}_d(r) \sin \phi \dots\dots\dots(17)$$

In Eqs.(14)~(17), the first terms of the right hand side satisfy the relation of continuity in the same way as the case of previous section. Then, in order that these equations may satisfy the condition of continuity, the arbitrary functions $N_d(r)$, $\tilde{N}_d(r)$, $M_d(r)$, $\tilde{M}_d(r)$ in Eqs.(14)~(17) have to satisfy the following relations:

$$\frac{dN_d(r)}{dr} + \frac{N_d(r)}{r} + \frac{\tilde{M}_d(r)}{r} = 0 \dots\dots\dots(18)$$

$$\frac{d\tilde{N}_d(r)}{dr} + \frac{\tilde{N}_d(r)}{r} - \frac{M_d(r)}{r} = 0 \dots\dots\dots(19)$$

Applying the boundary condition at the wall of Eq.(4) to Eq.(16), the following relations are obtained, because the boundary condition of Eq.(4) should be satisfied everywhere independently of ϕ and z as before:

$$J_0(k_n) - \frac{J_1(k_n)}{k_n} = 0 \dots\dots\dots(20)$$

$$N_d(1) = 0 \dots\dots\dots(21)$$

$$\tilde{N}_d(1) = 0 \dots\dots\dots(22)$$

Now, the relation of Eq.(20) is entirely the same as Eq.(13) which is deduced from the flow upstream of a screen, so the values of k_n which have been determined from the flow upstream of a screen can be applied to the flow downstream of the screen by themselves. Then Eqs.(18), (19), (21) and (22) become the new conditions which should be satisfied downstream of a screen.

Putting z as zero in Eqs.(14)~(17), the perturbations just downstream of a screen are determined as follows:

$$11p_d = \sum_n \{ E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi \} J_1(k_n r) \dots\dots\dots(14)'$$

$$11v_{zd} = - \sum_n \{ E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi \} J_1(k_n r) + S(r) \cos \phi + \tilde{S}(r) \sin \phi \dots\dots\dots(15)'$$

$$11v_{rd} = \sum_n \{ E_{dn} \cos \phi + \tilde{E}_{dn} \sin \phi \} \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} + N_d(r) \cos \phi + \tilde{N}_d(r) \sin \phi \dots\dots\dots(16)'$$

$$11v_{\phi d} = - \sum_n \{ E_{dn} \sin \phi - \tilde{E}_{dn} \cos \phi \} \frac{J_1(k_n r)}{k_n r} + M_d(r) \cos \phi + \tilde{M}_d(r) \sin \phi \dots\dots\dots(17)'$$

3.4 Relations connecting the flow just upstream of a screen with the flow just downstream of the screen

As all perturbations of the flow just upstream and downstream of a screen are obtained in Eqs.(9)~(12) and Eqs.(14)~(17)', respectively, they are connected using the characteristics of a screen and the relation of continuity as in the previous paper. The characteristics of the screen are described with the static pressure loss coefficient K and the outlet flow angle coefficient α .* About the coefficient α , the following empirical relations presented by Schubauer et al.(3) are also used:

$$\alpha = 1.1 \times (1+K)^{-1/2} \text{ for } K > 0.7 \dots\dots\dots(23)$$

The relation of Eq.(23) should be modified a little depending on the kinds of screen, but as it doesn't affect the key point of the theoretical analysis, Eq.(23) is used in this paper, too. If the individual values of α and K are obtained experimentally for each screen, more precise results will be obtained.

The characteristics of a screen is considered as isotropic, but the screen will be combined not only with an isotropic lattice but also with an annular or radial lattice. In these cases, at least for latter lattices, the outlet flow angle coefficient α will have different values in radial and circumferential directions, so in the following analysis depending on the flow directions, α will be used as α_R in the radial direction and α_ϕ in the circumferential direction, respectively. In the case of an isotropic screen being combined with an isotropic lattice, it is enough to treat as $\alpha_R = \alpha_\phi$. Here the analysis is proceeded considering the above-mentioned anisotropic characteristics. Then the equations connecting the flows just upstream and downstream of a damping device, which consists of an isotropic screen and an anisotropic lattice set closely, are obtained as follows:

$$11p_d = 10p_u - K 10v_{zu} \dots\dots\dots(24)$$

$$11v_{zd} = 10v_{zu} \dots\dots\dots(25)$$

$$11v_{rd} = \alpha_R 10v_{ru} \dots\dots\dots(26)$$

$$11v_{\phi d} = \alpha_\phi 10v_{\phi u} \dots\dots\dots(27)$$

* From Simmons & Cowdrey's experiment⁽²⁾, the following has been recognized. Putting the inlet flow angle to a screen as θ and the outlet flow angle out of the screen as ϕ , the relation between θ and ϕ is shown as $\phi = \alpha \theta$ using the coefficient α .

Using Eqs. (18), (19), (21), (22), (24), (26) and (27) the unknown coefficients E_{un} , $\tilde{E}_{un}$, E_{dn} and $\tilde{E}_{dn}$ and unknown functions $S(r)$, $\tilde{S}(r)$, $N_d(r)$, $\tilde{N}_d(r)$, $M_d(r)$ and $\tilde{M}_d(r)$ are all determined.

3.5 Determination of unknown coefficients

As Eq. (13) satisfies the condition for Dini's expansion in Bessel function, an arbitrary function $F(r)$ can be expanded using $J_1(k_n r)$ in the region of $0 < r < 1$. So the functions $f(r)$, $S(r)$ and $\tilde{S}(r)$ which are included in Eqs. (10), (15) and others are expanded as follows:

$$f(r) = \sum_n f_n J_1(k_n r) \quad (28)$$

$$S(r) = \sum_n S_n J_1(k_n r) \quad (29)$$

$$\tilde{S}(r) = \sum_n \tilde{S}_n J_1(k_n r) \quad (30)$$

Substituting Eqs. (28)~(30) into Eqs. (24) and (25) the following relations are obtained with the aid of orthogonality of Bessel function:

$$E_{un} = ((K-1)f_n + S_n)/K \quad (31)$$

$$\tilde{E}_{un} = \tilde{S}_n/K \quad (32)$$

$$E_{dn} = (-f_n + (1+K)S_n)/K \quad (33)$$

$$\tilde{E}_{dn} = (1+K)\tilde{S}_n/K \quad (34)$$

In these equations f_n is a known coefficient which determines the radial distribution of axial velocity far upstream. On the other hand S_n and $\tilde{S}_n$ are unknown coefficients which determine the radial distribution of axial velocity far downstream and assess the flow uniformizing function of a damping device.

In the expansion of the unknown functions $dN_d(r)/dr$ and $d\tilde{N}_d(r)/dr$ corresponding to the radial component of perturbation velocity v_{rd} , it should be considered that they may have some finite values at $r = 0$ when an anisotropic screen or lattice is used. So, in order to quicken the convergence of the series, these functions are expanded as follows including some constants:

$$\frac{dN_d(r)}{dr} = \sum_{n=1}^{n-2} N_{dn} J_1(k_n r) + N_1 \quad (35)$$

$$\frac{d\tilde{N}_d(r)}{dr} = \sum_{n=1}^{n-2} \tilde{N}_{dn} J_1(k_n r) + \tilde{N}_1 \quad (36)$$

Integrating Eqs. (35) and (36) about r , the functions $N_d(r)$ and $\tilde{N}_d(r)$ are obtained as follows:

$$N_d(r) = -\sum_{n=1}^{n-2} N_{dn} \frac{J_0(k_n r)}{k_n} + N_1 r + N_0 \quad (37)$$

$$\tilde{N}_d(r) = -\sum_{n=1}^{n-2} \tilde{N}_{dn} \frac{J_0(k_n r)}{k_n} + \tilde{N}_1 r + \tilde{N}_0 \quad (38)$$

Substituting Eqs. (18) and (19) into Eqs. (35)~(38) the following equations are obtained:

$$M_d(r) = \sum_{n=1}^{n-2} \tilde{N}_{dn} \left\{ J_1(k_n r) - \frac{J_0(k_n r)}{k_n r} \right\} r + 2\tilde{N}_1 r + \tilde{N}_0 \quad (39)$$

$$\tilde{M}_d(r) = -\sum_{n=1}^{n-2} N_{dn} \left\{ J_1(k_n r) - \frac{J_0(k_n r)}{k_n r} \right\} r - 2N_1 r - N_0 \quad (40)$$

Now substituting Eqs. (11)', (12)', (16)' and (17)' into Eqs. (26) and (27), and using relations of Eqs. (31)~(34) and Eqs. (39) and (40) which were obtained just above, the following relations are gained with the aid of the orthogonality of trigonometric functions:

$$\begin{aligned} & \frac{1+K+\alpha_R}{K} \sum_n S_n \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} - \sum_{n=1}^{n-2} N_{dn} \frac{J_0(k_n r)}{k_n} + N_1 r + N_0 \\ &= \frac{1+\alpha_R-\alpha_R K}{K} \sum_n f_n \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} \end{aligned} \quad (41)$$

$$\begin{aligned} & \frac{1+K+\alpha_\phi}{K} \sum_n S_n \frac{J_1(k_n r)}{k_n r} + \sum_{n=1}^{n-2} N_{dn} \left\{ J_1(k_n r) - \frac{J_0(k_n r)}{k_n r} \right\} r + 2N_1 r + N_0 \\ &= \frac{1+\alpha_\phi-\alpha_\phi K}{K} \sum_n f_n \frac{J_1(k_n r)}{k_n r} \end{aligned} \quad (42)$$

$$\begin{aligned} & \frac{1+K+\alpha_R}{K} \sum_n \tilde{S}_n \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} - \sum_{n=1}^{n-2} \tilde{N}_{dn} \frac{J_0(k_n r)}{k_n} + \tilde{N}_1 r + \tilde{N}_0 = 0 \end{aligned} \quad (43)$$

$$\begin{aligned} & \frac{1+K+\alpha_\phi}{K} \sum_n \tilde{S}_n \frac{J_1(k_n r)}{k_n r} + \sum_{n=1}^{n-2} \tilde{N}_{dn} \left\{ J_1(k_n r) - \frac{J_0(k_n r)}{k_n r} \right\} r + 2\tilde{N}_1 r + \tilde{N}_0 = 0 \end{aligned} \quad (44)$$

These four equations in which r is a non-dimensional parameter showing a radial position contains $2n$ unknown coefficients of S_n and $\tilde{S}_n$, $2(n-2)$ ones of N_{dn} and $\tilde{N}_{dn}$ and 4 ones of N_1 , $\tilde{N}_1$, N_0 and $\tilde{N}_0$. Then if these four relations are applied at arbitrary n radial positions of r , $4n$ relations containing the above $4n$ unknown coefficients are obtained and these unknown coefficients are determined. These relations satisfy the conditions of Eqs. (21) and (22) at the point where $r=1$, so it is enough to solve these $4n$ relations including the point where $r=1$. Dini's expansion used in Eqs. (35)~(38), however, are only applicable to the region where $0 < r < 1$, so the calculation is performed at the limit where $r \rightarrow 1$ instead of at the point where $r=1$.

From Eqs. (41)~(44), it is clear that Eqs. (41) and (42) only contains $2n$ unknown coefficients of S_n , $\tilde{S}_n$, N_1 and $\tilde{N}_1$, while Eqs. (43) and (44) only contains the other $2n$ unknown coefficients of $\tilde{S}_n$, $\tilde{N}_{dn}$, $\tilde{N}_1$ and $\tilde{N}_0$, where f_n is a known coefficient giving the radial distribution of axial perturbation velocity far upstream. In order to satisfy the relations of Eqs. (43) and (44) independently of r , each coefficient in these equations must be zero and the following equations are obtained:

$$\tilde{S}_n = 0, \quad \tilde{N}_{dn} = 0, \quad \tilde{N}_1 = 0, \quad \tilde{N}_0 = 0 \quad (45)$$

Then from the relations of Eqs.(32), (34) and (39), the following equations are obtained:

$$\tilde{E}_{un}=0, \tilde{E}_{dn}=0 \text{ and } M_d(r)=0 \dots\dots(46)$$

As the above, the 2n unknown coefficients of 4n ones in Eqs.(41)~(44) are determined as Eq.(45), then another 2n unknown coefficients of S_n , N_{dn} , N_1 and N_0 are determined from Eqs.(41) and (42). After that, using the relations of Eqs.(31), (33) and (40), another unknown coefficients of E_{un} , E_{dn} and an unknown function $\tilde{M}_d(r)$ are all determined.

4. Analytical results and discussions

Now, as unknown coefficients are all determined, the decaying aspect of perturbation pressure and velocities of a flow through a damping screen or a perforated plate are made clear from Eqs.(9)~(12) and Eqs.(14)~(17). Then the local damping ratio of axial perturbation velocity is defined as Eqs.(47) and (48) for the flow upstream and downstream of a screen, respectively.

$$|A_u| = \left[\frac{\overline{v_{su}}}{(\overline{v_s})_{-\infty}} \right] = \left[\frac{\int_0^1 \left\{ \sum_n (f_n - E_{un} e^{-k_n r}) J_1(k_n r) \right\}^2 r dr}{\int_0^1 \left\{ \sum_n f_n J_1(k_n r) \right\}^2 r dr} \right]^{1/2} \dots\dots\dots(47)$$

$$|A_d| = \left[\frac{\overline{v_{sd}}}{(\overline{v_s})_{-\infty}} \right] = \left[\frac{\int_0^1 \left\{ \sum_n (S_n - E_{dn} e^{-k_n r}) J_1(k_n r) \right\}^2 r dr}{\int_0^1 \left\{ \sum_n f_n J_1(k_n r) \right\}^2 r dr} \right]^{1/2} \dots\dots\dots(48)$$

The final damping ratio $|A_\infty|$ is obtained as follows:

$$|A_\infty| = \left[\frac{(\overline{v_s})_{+\infty}}{(\overline{v_s})_{-\infty}} \right] = \left[\frac{\int_0^1 \left\{ \sum_n S_n J_1(k_n r) \right\}^2 r dr}{\int_0^1 \left\{ \sum_n f_n J_1(k_n r) \right\}^2 r dr} \right]^{1/2} \dots\dots\dots(49)$$

As a lattice in pipe flow, can be conceived anisotropic lattices like an annular lattice, a radial lattice and so on besides the isotropic lattice mentioned above. When these anisotropic lattices are combined with an isotropic screen, a special anisotropic damping device is assembled. The following discussions are developed including these cases.

4.1 The case of isotropic damping device ($\alpha_R = \alpha_\phi = \alpha$)

In this case the coefficients of terms of S_n and f_n in Eqs.(41) and (42) are identical with each other, so Eqs.(41) and (42) are rearranged as follows:

$$\sum_n \left(\frac{1+K+\alpha}{K} S_n - \frac{1+\alpha-\alpha K}{K} f_n \right) \left\{ J_0(k_n r) - \frac{J_1(k_n r)}{k_n r} \right\} - \sum_{n=1}^{n-2} N_{dn} \frac{J_0(k_n r)}{k_n} + N_1 r + N_0 = 0 \dots\dots\dots(50)$$

$$\sum_n \left(\frac{1+K+\alpha}{K} S_n - \frac{1+\alpha-\alpha K}{K} f_n \right) \frac{J_1(k_n r)}{k_n r} + \sum_{n=1}^{n-2} N_{dn} \left\{ J_1(k_n r) - \frac{J_0(k_n r)}{k_n r} \right\} r + 2N_1 r + N_0 = 0 \dots\dots\dots(51)$$

For these relations to hold independently of r, all the coefficients of function including r are to be zero, and the unknown coefficients are determined as follows:

$$N_{dn}=0, N_1=0, N_0=0 \dots\dots\dots(52)$$

$$S_n = [(1+\alpha-\alpha K)/(1+K+\alpha)] f_n \dots\dots\dots(53)$$

Then E_{un} and E_{dn} are obtained from Eqs.(31) and (33), and $\tilde{M}_d(r)$ is also obtained from Eqs.(40), respectively.

$$E_{un} = [K/(1+K+\alpha)] f_n \dots\dots\dots(54)$$

$$E_{dn} = -[\alpha K/(1+K+\alpha)] f_n \dots\dots\dots(55)$$

$$\tilde{M}_d(r) = 0 \dots\dots\dots(56)$$

Using these coefficients the local damping ratios $|A_u|$ and $|A_d|$ are determined by Eqs.(47) and (48) for the flow upstream and downstream of a screen, respectively, and the final damping ratio $|A_\infty|$ is gained as follows from Eq.(49):

$$|A_\infty| = (1+\alpha-\alpha K)/(1+K+\alpha) \dots\dots\dots(57)$$

This value is entirely the same as that of the two-dimensional case of single screen treated by Taylor et al.⁽⁴⁾ and also that of the axi-symmetric case treated by the author⁽¹⁾. In this case, the functions $\tilde{M}_d(r)$, $\tilde{M}_d(r)$, $N_d(r)$ and $\tilde{N}_d(r)$ all become zero. This fact means, from Eqs.(16) and

Table 1: The values of K_n satisfying Eq. (13)

k_1	1.84	k_6	18.02
k_2	5.33	k_7	21.16
k_3	8.54	k_8	24.31
k_4	11.71	k_9	27.46
k_5	14.86	k_{10}	30.60

Fig. 2: Radial distribution of axial perturbation velocities far upstream

(17), that the radial and circumferential components of perturbation velocity damp out finally flowing through an isotropic damping device. Now, it is considered that this is the reason why the final damping ratio of this case coincides with that of the case of an axi-symmetric velocity perturbation.

4.2 The case of $\alpha_R = \alpha_\phi = 0$

This is the case where an isotropic damping screen is set close to an isotropic lattice. In this case from Eqs.(53), (54) and (55) the following equations are obtained:

$$S_n = [1/(1+K)]f_n \quad (58)$$

$$E_{zn} = [K/(1+K)]f_n \quad (59)$$

$$E_{\phi n} = 0 \quad (60)$$

The final damping ratio $|A_\infty|$ is gained from Eq.(57) as follows:

$$|A_\infty| = 1/(1+K) \quad (61)$$

The facts of E_{dn} and $\tilde{E}_{dn}$ being equal to zero as shown in Eq.(60) and Eq.(46) respectively, mean that the damping process of flow perturbations entirely vanishes downstream of this damping device as realized in Eqs.(14)~(17). In other words this means that the damping process of flow perturbations only exists upstream of the damping device. In this case, as mentioned above, an isotropic damping screen is set close to an isotropic lattice. The characteristics of an isotropic lattice shown as $\alpha_R = \alpha_\phi = 0$ is to restrict the flow to the axial direction of flow through the lattice. As the result, radial and circumferential components of the flow velocity become zero in the flow downstream of the damping device, and the static pressure is balanced in any cross section, so the flow uniformizing function is lost downstream of the damping device.

Some examples of computational result are shown in Figs.3~5. In Figs.4 and 5 are also shown the results of the cases of anisotropic damping devices which will be explained in the next section. Fig.2 shows the shape of radial distribution of the axial perturbation velocities. In the figure the abscissa shows radial positions in the non-dimensional form as divided by a quarter wave length of the perturbation for the case of a sinusoidal distribution used in the two-dimensional analysis and by a radius a for the cases of a pipe flow. In the range of r'/a or $y/(\lambda/4)$ used in Fig.2 (i.e. $-1.0 \sim 1.0$), the shape of $\sin 2\pi y/\lambda$ is almost the same as that of $J_1(k_1 r)$ where $k_1 = 1.84$, so in Fig.2 the curves for these two cases are represented by one curve.

Fig.3 shows the decaying aspect of the axial velocity v_z in terms of local damping ratio $|A|$. The result of two-dimensional analysis by Taylor et al.⁽⁴⁾ (dotted line), that of the previous paper⁽¹⁾ for the axi-symmetric distribution of axial perturbation velocities (chain line) and that of the present case of S-shape radial distribution of axial perturbation velocities

(solid line) are compared with each other for the case of an isotropic damping device. In the figure the abscissa shows the distance along the flow direction in the non-dimensional form as divided by a wave length λ for the case of a sinusoidal distribution in two-dimensional analysis and by a diameter D for the case of a pipe flow, respectively. $(v_z)_\infty$ in the figure shows the radial (or Y-direction) distribution of axial perturbation velocities far upstream (refer to Fig.2). The decaying aspects of the case of two-dimensional flow and that of pipe flow with axi-symmetric distribution of axial perturbation velocities resemble each other. The minor difference between these two cases depends on the minor difference in the exponents appearing, for example, in Eq.(10) or (15) which determines the damping rate in flow direction and is $\pi (=3.1416)$ for the case of a sinusoidal distribution and $k_1 (=3.832)$ for the case of distribution of $J_1(k_1 r)$. On the other hand, in the present case of an S-shape radial distribution of axial perturbation velocities, it is seen that the decaying aspects in the flow direction are rather gradual comparing with the former two cases. The reason for this fact also lies in the value of exponent k_1 being as small as 1.84 which is shown in Table 1 for the present case.

In the case of an S-shape radial distribution of axial perturbation velocities, the length similar to the wave length of flow perturbation is $2D$ and it corresponds to twice the pipe diameter. Then it is recognized that the value of k_1 for this case is nearly a half of that for the case of an axi-symmetric distribution of axial perturbation velocities and, as the result, the decaying process of S-shape case is more gradual than that of axi-symmetric case. In the figure, the length in flow direction is shown in the non-dimensional form as divided by the pipe diameter D even for the S-shape case. However, when it is compared with the axi-symmetric case or the two-dimensional case, considering that the wave length of the S-shape case corresponds to $2D$, if it is shown in the non-dimensional form as divided by $2D$ instead of D the decaying aspects for both cases will show similar tendencies.

Fig. 3: Comparison of decaying aspects of axial perturbation velocity in flow direction due to the shape of radial distribution of axial velocity far upstream (single screen)

Figure 4 shows the relation between the final damping ratio of axial perturbation velocity $|A_\infty|$ and the pressure loss coefficient of a damping screen K for the case of an S-shape radial distribution of axial perturbation velocities. The solid line in the figure corresponds to the case of a single isotropic damping screen and the dotted line corresponds to the case of a combined isotropic damping device which consists of an isotropic damping screen and an isotropic lattice set close to it. In the case of a single isotropic damping screen, as mentioned before, the final damping ratio $|A_\infty|$ coincides with that of a two-dimensional case and an axis-symmetric case, so $|A_\infty|$ of this case also becomes zero at the K value of 2.76. On the other hand, in the latter case (dotted line) it is clear that the general tendency of the change of $|A_\infty|$ with the change of K is more gradual than the former case (solid line) especially in the region of low K values smaller than 5.0, which is considered to be important from the practical point of view.

Fig. 5 shows the damping aspects in flow direction of axial perturbation velocity for the case of an S-shape radial distribution of axial perturbation velocities with a single damping screen having a K value of 3.0. The definitions of lines in the figure are the same as in Fig. 4. Comparing the solid line and the dotted line, the damping aspects upstream of the damping device resemble each other, but those downstream of it are completely different. As mentioned before when an isotropic damping screen is set close to an isotropic lattice (dotted line), the function of damping the flow perturbation vanishes completely downstream of the damping device. The tendency of the dotted line in the figure shows this fact clearly.

Fig. 4: Variation of final damping ratio $|A_\infty|$ for each damping device with the change of pressure loss coefficient K of a damping screen

Fig. 5: Comparison of decaying aspects of axial perturbation velocity in flow direction with the changes of characteristics of damping devices

4.3 The case of anisotropic damping device ($\alpha_R \neq \alpha_\phi$)

In this case Eqs. (50) and (51) can not be used to determine the unknown coefficients S_n , N_{dn} , N_l and N_o . Instead, these coefficients are determined numerically from Eqs. (41) and (42). If the values of S_n are determined numerically the values of E_{un} and E_{dn} are also determined from Eqs. (31) and (33); then using these values the local damping ratios $|A_u|$ (upstream of the damping device) and $|A_d|$ (downstream of it) and the final damping ratio $|A_\infty|$ are all determined.

The following two cases are presented as numerical examples, i.e. one case where an isotropic damping screen and annular lattice are set close to each other ($\alpha_R = 0, \alpha_\phi = \alpha$) and the other case where an isotropic damping screen and a radial lattice are set close to each other ($\alpha_R = \alpha, \alpha_\phi = 0$) are shown in Figs. 4 and 5 comparing with the case of a single isotropic damping screen (solid line) and the case of an isotropic damping device where an isotropic damping screen and an isotropic lattice are set close to each other (dotted line). These numerical results concerning both the final damping ratio $|A_\infty|$ shown in Fig. 4 and the damping aspects in flow direction of axial perturbation velocity shown in Fig. 5 situate between the solid line and the dotted line which are explained in the previous section, and especially in Fig. 4 it is seen that, comparing these anisotropic damping devices, the one with an annular lattice is slightly superior to the one with a radial lattice in the final damping ratio.

4.4 The secondary flow in the field downstream of anisotropic damping devices

In cases of anisotropic damping devices the coefficients N_{dn} , N_l and N_o don't become zero but have some values. Then from Eq. (40) $\bar{M}_d(r)$ shows some radial distribution. This means, as recognized from Eqs. (16) and (17), that the radial and circumferential components of perturbation velocities v_r and v_ϕ remain far downstream of anisotropic damping devices and secondary flows occur. If a stream function ψ is defined in a cross section far downstream, ψ is obtained as follows with the aid of

Fig. 6: Secondary flow induced by an anisotropic damping device composed of an isotropic damping screen and an annular lattice set close to each other

Fig. 7: Secondary flow induced by an anisotropic damping device composed of an isotropic damping screen and a radial lattice set close to each other

equations showing $(v_r)_{+\infty}$ and $(v_\phi)_{+\infty}$ which are gained from Eqs.(16) and (17) putting $z \rightarrow +\infty$:

$$\begin{aligned} \psi = & \left[\sum_{n=1}^{n-1} \frac{\tilde{M}_{an}}{k_n} (J_0(k_n r) - J_0(k_n)) \right. \\ & - \sum_{n=1}^{n-2} \frac{N_{an} J_0(k_n r)}{k_n} r - (r-1) \tilde{M}_0 \\ & \left. + r^2 N_1 + r N_0 \right] \sin \phi \dots\dots\dots (62) \end{aligned}$$

In Eq.(62) the above mentioned function $\tilde{M}_a(r)$ is expanded as follows by Dini's expansion:

$$\tilde{M}_a(r) = \sum_{n=1}^{n-1} \tilde{M}_{an} J_1(k_n r) + \tilde{M}_0 \dots\dots\dots (63)$$

n coefficients $\tilde{M}_{an}$ and $\tilde{M}_0$ in Eq.(62) are all determined from Eq.(63) because in Eq. (63) the function $\tilde{M}_a(r)$ has been determined in Eq.(40). Using Eq.(62) stream lines in a cross section far downstream are gained as shown in Fig.6 for the case of an anisotropic damping device with an annular lattice ($\alpha_R = 0$, $\alpha_\phi = \alpha$). In the figure it is shown clearly that fluid particle flows along the wall surface from the part marked as $((v_z)_{-\infty} = +)$, where the axial velocity far upstream is higher than the other, to the part marked as $((v_z)_{-\infty} = -)$ where the axial velocity far upstream is lower than the other.

On the other hand Fig.7 shows the case of an anisotropic damping device with a radial lattice. From the figure it is recognized that in this case a secondary flow with inverse direction comparing with the case of Fig.6 occurs.

5. Concluding remarks

In order to compare with the case of an axi-symmetric distribution of axial velocity analysed in the previous paper⁽¹⁾,

the cases of an S-shape radial distribution of axial velocities are treated in this paper and the following conclusions are obtained:

(1) In the case of a single isotropic damping screen the final damping ratio $|A_\infty|$ coincides with that of the two-dimensional case and the axi-symmetric case in pipe flow.

(2) The damping process of axial perturbation velocity in flow direction, however, is more gradual than the above latter two cases, owing to the fact that the wave length of flow perturbation of S-shape radial distribution is twice as great as that of axi-symmetric distribution.

(3) In the case of an isotropic damping device, which is composed of an isotropic damping screen and isotropic lattice set close to each other, the damping characteristics becomes worse especially in the region of K values smaller than 5.0, which is important in practical use.

(4) In the case of an anisotropic damping device which is composed of an isotropic damping screen and a different kind of anisotropic lattice set close to each other, the final damping ratio $|A_\infty|$ comes in between the two cases mentioned above (1) and (3). Among the anisotropic damping devices the combination with an annular lattice is slightly superior in the final damping ratio to the combination with a radial lattice, especially in the region of K values smaller than 5.0.

(5) When the flow with S-Shape radial distribution of axial velocities passes through anisotropic damping devices, secondary flows shown in Figs.6 and 7 remain in the field far downstream of the damping device. The flow directions of these secondary flows are determined by the characteristics of each damping device, e.g. the one com-

bined with an annular lattice or a radial lattice. In the former case the flow in a cross section far downstream moves in circumferential direction along pipe surface from the part of high total pressure in a cross section of a pipe ($(v_z)_{\infty}=\oplus$ in Fig. 6) to the part of low total pressure ($(v_z)_{\infty}=\ominus$) and returns through the center part of a pipe (Fig. 6), while in the latter case the flow direction is oppsite (Fig. 7).

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