Formation and Interaction of Two Parallel Vortex Streets

By Kyoji Kamemoto

The instability of two pairs of infinitely long parallel vortex sheets for initial disturbances is investigated by a linear analysis in an inviscid incompressible fluid, and it is shown paradoxically that two pairs of parallel vortex sheets cannot exist at a gap distance between two pairs less than the distance between vortex sheets of each pair. By calculating the non-linear growth in time of periodic disturbances in the vortex sheets, the essential features of the formation and interaction of two parallel vortex streets are analysed, and it is explained that the initially antisymmetric disturbance is more apt to cause the interaction between two vortex streets in their developing stage than the symmetric one. These theoretical results are compared with the experimental ones which were observed in a flow past two circular cylinders spaced in the direction perpendicular to a uniform flow.

1. Introduction

It is well known that interaction between wakes of multiple two-dimensional bluff bodies in a uniform flow is detected at a certain spacing of bodies. As the result of the interaction, the individual wakes differ from that behind a single body, well known as Karman vortex street. Nowadays, in relation to such problems as the local severe wind due to a group of skyscrapers and the vibration of tubes in a heat exchanger, it is necessary make clear the mechanism of the interaction.

Carpenter(1), Kawagut(2) and Dalton(3) investigated the potential flow around two or a group of circular cylinders. Spilvack(4) observed the effects of the interaction on the vortex shedding frequency with a hot-wire at higher Reynolds number, varying the gap distance of two cylinders in the direction perpendicular to the flow. Thomas(5)and Zdravkovich(6) also observed the formation and interaction of vortex streets visualizing a flow past two or three cylinders at lower Reynolds number. However, there are few theoretical investigations concerned with the mechanism of the interaction of vortex streets shed from multiple bluff bodies.

The main purpose of this paper is to explain the process of formation and interaction of two parallel vortex streets shed from two bluff bodies spaced in the direction perpendicular to the flow, using a rather simple flow model for theoretical treatment and observing the wakes behind two cylinders.

Nomenclature

- $a$: wave-length of disturbance,
- $c$: complex wave velocity,
- $c_r$: imaginary part of $c$,
- $c_i$: real part of $c$,
- $\delta$: gap distance between two pairs of vortex sheets,
- $h$: distance between vortex sheets in a pair,
- $m$: interference factor,
- $n$: number of vortex elements per wavelength,
- $p$: pressure,
- $t$: elapsed time from an original perturbation,
- $\Delta t$: time step in numerical integration,
- $u$: x-component of velocity,
- $v$: y-component of velocity,
- $x$: co-ordinate parallel to steady flow,
- $y$: co-ordinate normal to steady flow,
- $A$: complex constant of fluctuations,
- $B$: complex constant of fluctuations,
- $D$: diameter of circular cylinder,
- $C$: gap distance between two cylinders,
- $U$: velocity of steady flow,
- $Re$: Reynolds number ($UD/\nu$),
- $St$: Strouhal number ($fD/U$), where $f$ is vortex shedding frequency,
- $a$: wave number ($2\pi/a$),
- $\beta$: initial growth rate of disturbance ($ac_i$),
- $\xi$: x-component of displacement in vortex sheets,
- $\eta$: y-component of displacement in vortex sheets,

Superscripts

- $(\cdot)$: fluctuation
- $(\cdot)'$: complex representation of fluctuation,

Subscripts

- $i$: the number of vortex sheet,
- $j$: the number of vortex element,
- $N$: the number of flow region.

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2. Theoretical analysis

2.1 Flow model and linear analysis

It is desirable but complicated to analyse mathematically the time history of perturbations in the steady flow of a viscous incompressible fluid past two two-dimensional bluff bodies. Therefore, major simplifications of the flow model are needed.

There are such experimental evidences concerned with a vortex street shed from a single bluff body as the constancy of the Strouhal number over a considerable range of higher Reynolds numbers and the existence of an essentially universal Strouhal number for any shaped bluff body, examined by Fage and Roehl (13). Considering these evidences, Abernathy long paralleled that the essential process of the transition from vortex layers to street could be simulated by the growth in time of periodic disturbances in two infinitely long parallel vortex sheets without regard to the viscosity of fluid and the existence of a body. Meanwhile, in a flow past a pair of parallel circular cylinders, Spivack observed that the features of the interaction of vortex streets did not change at least in the range of Reynolds numbers from 5,000 to 93,000, and at a gap distance more than one diameter the Strouhal number of each vortex street is constant (0.2), the same as with a single cylinder. This experimental result suggests that viscosity is important only for the formation of vortex layers emanating from two parallel bluff bodies, but does not contribute significantly to the formation and interaction of two parallel vortex streets over a considerable range of Reynolds numbers.

Therefore, the flow model presented by Abernathy can be applied to the wakes of two bluff bodies spaced in the direction perpendicular to the flow, on the assumption that both bodies independently sheds the vortex layers at any gap distance. Then, the essential features of the formation and interaction of two parallel vortex streets can be analysed by calculating the growth in time of periodic disturbances in four infinitely long parallel vortex sheets. Also, it will be shown paradoxically that these two pairs of vortex sheets can not exist at an arbitrary gap distance but at a distance more than a certain value.

As shown in Fig. 1, two pairs of infinitely long parallel vortex sheets divide the inviscid incompressible flow field into five flow regions. In order to fix the four vortex sheets on the system of the co-ordinates, one may take the velocities of steady flow to be $U$ and $-U$ in the region I, III or V and in the region II or IV respectively. Small but arbitrary initial disturbances cause the horizontal and vertical displacements $\xi$ and $\eta$ (i=1, 2, 3, 4) of each element of the sheets, originally located at $x$ along the lines $y = z(t + \frac{h}{2})$ and $y = z(t - \frac{h}{2})$. Provided that each Fourier or Fourier series expansion of the initial disturbance exists, the subsequent linearized history of the system following an arbitrary disturbance can be determined from the response to a disturbance of a particular wave-length.

The velocity components and pressure in each flow region can be represented in the following forms, divided into steady terms and fluctuating terms:

$$u_0 = U + u(x, y, t), \quad v_0 = v(x, y, t), \quad p_0 = p(x, y, t),$$

$$\begin{align*}
\bar{u}_0 &= u_0(y)e^{i(\omega t - kx)}, & \bar{v}_0 &= v_0(y)e^{i(\omega t - kx)}, \\
\bar{p}_0 &= p_0(y)e^{i(\omega t - kx)}. & \end{align*}$$

where $\omega$ and $k$ are a wave number and a complex wave velocity respectively. The functions of $x$ only as $\bar{u}_0, \bar{v}_0$ and $\bar{p}_0$ in Eqs. (2) will be determined with two arbitrary complex constants in each region after integration of the fluctuating motion's equations; for instance, in the region I ($y > h + \frac{h}{2}$),

$$\begin{align*}
\bar{u}_1 &= \frac{1}{(h + \frac{h}{2})} \left[ A_1 e^{i\left(y - (h + \frac{h}{2})\right)} + B_1 e^{i\left(y - (h + \frac{h}{2})\right)} \right] \quad (3) \\
\bar{v}_1 &= -\frac{1}{(h + \frac{h}{2})} \left[ -A_2 e^{i\left(y - (h + \frac{h}{2})\right)} - B_2 e^{i\left(y - (h + \frac{h}{2})\right)} \right]
\end{align*}$$

in the region II ($h + \frac{h}{2} > y > \frac{h}{2}$),

$$\begin{align*}
\bar{u}_2 &= \frac{1}{h} \left[ A_3 e^{i\left(y - (h + \frac{h}{2})\right)} + B_3 e^{i\left(y - (h + \frac{h}{2})\right)} \right] \quad (4) \\
\bar{v}_2 &= \frac{1}{h} \left[ A_4 e^{i\left(y - (h + \frac{h}{2})\right)} + B_4 e^{i\left(y - (h + \frac{h}{2})\right)} \right]
\end{align*}$$

where the complex constant $A_1$ must be zero because the fluctuations die out away from the vortex sheet in the region I.

At first, the solutions of pressure fluctuations must satisfy the boundary condition of continuity of pressure across the vortex sheet. Thus, the following relation is obtained at $y = h + \frac{h}{2}$,

$$B_1 = A_1 e^{kh} + B_2 e^{kh} \quad (5)$$

Meanwhile, the displacements for each vortex sheet are related with the velocity fluctuation close to the sheet. Forcusing attention on the sheet 1, one can obtain the relations as follows:

$$\begin{align*}
\frac{\partial \bar{u}_1}{\partial t} + U \frac{\partial \bar{u}_1}{\partial x} &= \nu \frac{\partial^2 \bar{u}_1}{\partial x^2} & y = (h + \frac{h}{2}) & + \rho_0 g \frac{\partial \bar{u}_1}{\partial x} & = 0 \quad (6) \\
\frac{\partial \bar{v}_1}{\partial t} - U \frac{\partial \bar{v}_1}{\partial x} &= \nu \frac{\partial^2 \bar{v}_1}{\partial x^2} & y = (h + \frac{h}{2}) & = 0 \quad (7) \\
\frac{\partial \bar{u}_2}{\partial t} &= (u_1') + (u_2') & = 0 \quad (8)
\end{align*}$$
Table 1. The parameters of the normal modes of solution and the interference factor.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( m )</th>
<th>( C_r )</th>
<th>( C_i )</th>
<th>( \tilde{\eta}_i, \tilde{\xi}_i )</th>
<th>( \tilde{\eta}_i/\tilde{\xi}_i )</th>
<th>( \tilde{\eta}_i/\tilde{\xi}_i )</th>
<th>( \tilde{\eta}_i/\tilde{\xi}_i )</th>
<th>( \tilde{\eta}_i/\tilde{\xi}_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{2}(e^{-e^{ak} - e^{-ak}}) )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>2</td>
<td>( -e^{ak} + e^{ak} )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{2}(e^{ak} - e^{-ak}) )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>4</td>
<td>( -e^{ak} + e^{ak} )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{1}{2}(e^{ak} - e^{-ak}) )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>6</td>
<td>( -e^{ak} + e^{ak} )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{1}{2}(e^{ak} - e^{-ak}) )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>8</td>
<td>( -e^{ak} + e^{ak} )</td>
<td>( U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( -U(1 - m^2 e^{ak}) \frac{1}{2} )</td>
<td>( (1 + m e^{ak})/(1 - m e^{ak}) )</td>
<td>( 1 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
</tbody>
</table>

Figure 2. A sketch of the eight independent periodic modes of a disturbance in four parallel vortex sheets. The displacements of individual elements of the sheets are shown by the arrows.

Also representing preliminary complex displacements as

\[ \xi_i = \tilde{\xi}_i e^{a(x+c-t)} \]

\[ \eta_i = \tilde{\eta}_i e^{a(x-c-t)} \]

... (9)

one may reduce Eqs. (6), (7) and (8) to

\[ \alpha(U - c) \tilde{\eta}_i = B_1 i(U - c) \] .......................... (10)

\[ -\alpha(U + c) \tilde{\eta}_i = (A_1 e^{ak} - B_2 e^{ak})/i(Uc) \] .......................... (11)

\[ -\alpha c \tilde{\xi}_i = [-B_2/(U + c) + (A_1 e^{ak} + B_2 e^{ak})/(Uc)]/2 \] .......................... (12)

The equation obtained by eliminating \( \tilde{\eta}_i \) from Eqs. (10) and (11) represents the condition of continuity of normal velocity component across the sheet.

In the same way, one can obtain the equations corresponding to Eqs. (5), (10), (11) and (12), concerned with the other vortex sheets. Consequently, these equations, eight independent solutions for fluctuations will be calculated, where each solution corresponds to a particular mode of displacement of the individual elements of the sheets. These eight independent modes (called normal modes) can be combined with arbitrary amplitude and phase to satisfy the initial disturbance functions.

2.2 Interference factor

It is convenient to define a factor which indicates quantitatively the rate of interaction between two pairs of vortex sheets. When the gap distance \( g \) is very large, no interaction will be detected and each pair may independently respond to disturbances. According to Abernathy, the normal modes in a single pair of vortex sheets are symmetrical and antisymmetrical. Therefore, at infinitely large gap distance, the normal modes in the flow region II must be symmetrical and antisymmetrical, and corresponding to these modes the complex constants in Eqs. (4) will be related to one another as \( A_1 = B_2 \) and \( A_2 = B_1 \). However, the gap distance is comparable with the distance between vortex sheets in each pair, and the normal modes in each pair are distorted to be quasi-symmetrical and quasi-antisymmetrical by the interaction between two pairs. Thus, one can represent the rate of interaction using the ratio of \( B_2 \) to \( A_1 \) as follows:

\[ m = -B_2/A_1 \] .......................... (13)

where \( m \) is named the interference factor and the departure of its absolute value from unity indicates the level of the interaction.

2.3 Results of linear analysis

The eight normal modes of displacements calculated above are shown in Table 1, together with the interference factor \( m \). As the actual displacements can be represented from preliminary complex displacements as follows:

\[ \xi_i = \tilde{\xi}_i e^{at} \sin(a(x+c-t)) \] .......................... (14)

\[ \eta_i = \tilde{\eta}_i e^{at} \sin(a(x-c-t)) \]

where \( B = ac \), one can obtain the mutual relations of \( \tilde{\xi}_i \) and \( \tilde{\eta}_i \) corresponding to each mode. The form of the displacements for each mode is sketched in Fig. 2. Modes 1, 2, 3 and 4 correspond to symmetric displacements, and modes 5, 6, 7 and 8 to antisymmetric displacements for the inner and outer pairs of vortex sheets, while the form for each pair of the sheets composed of the sheets 1 and 2 or 3 and 4 is quasi-symmetrical or quasi-antisymmetrical as the result of the mutual interaction. The horizontal displacement leads to a periodic pattern of increasing and decreasing vortex strength along the sheets. The growing variation of vortex strength along each pair of sheets in quasi-antisymmetrical form (mode 3 or 7) gives the first indication of the
arrangement of two parallel vortex streets. Thus, focusing attention on modes 3 and 7 (named the symmetrical mode and the anti-symmetrical mode respectively), we shall explain the results of the analysis.

It is shown in Fig. 3 that for the symmetrical mode the initial growth rate $\delta$ divided by $U$ decreases with a decrease in the value of the gap distance ratio $(g/h)_{3}$ at any wave number, while for the antisymmetrical mode it increases just a little in spite of a wide variation of the value of the ratio $(g/h)_{3}$ from infinity to zero. Meanwhile, as shown in Fig. 4, corresponding to such variation of the gap distance ratio, the phase velocity $c_{p}$, divided by $U$, increases for the symmetrical mode, but it decreases for the antisymmetrical mode. In the Karman idealization of a vortex sheet, the neutrally stable situation of the alternate rows of point vortices was found to be $h/a_{w} = 0.281$, with $h$ the distance between rows of vortices separated by a distance $a$. The wave number $\alpha_{w}$ corresponding to this spacing ratio is $1.76/h$ as marked on the abscissas in Figs. 3 and 4; while there is nothing in particular about disturbances at $\alpha_{w}$ from that of infinite gap distance appears when the value of $g/h$ is smaller than 2.0 for both modes.

The interference factor is shown in Fig. 5. At a value of $g/h$ larger than unity, the factor takes a maximum value for the symmetrical mode, or a minimum value for the antisymmetrical one, and asymptotically approaches unity with an increase of wave number. This tendency suggests that the interaction between two pairs of vortex sheets is small for a disturbance of higher wave number (higher frequency). On the other hand, at a value of $g/h$ smaller than unity, the interference factor monotonously increases for the symmetrical mode or decreases for the antisymmetrical one. However, it is impractical that the higher the wave number of a disturbance, the stronger the interaction is. The impracticality is attributed to such an assumption as the existence of two pairs of vortex sheets at any gap distance. Therefore, as the result of this linear analysis, it is shown paradoxically that two pairs of parallel vortex sheets can not exist at a value of the gap distance ratio $(g/h)$ smaller than unity.

2.4 Non-linear analysis of symmetric or antisymmetric disturbance

Rosenhead and Abernathy calculated the non-linear growth of initial periodic disturbances in a vortex sheet and in two parallel vortex sheets respectively, assuming that the vortex sheet could be represented by a row of point vortices of finite uniform strength. In the same way, one can examine the growth of the initial disturbances in two pairs of vortex sheets corresponding to the modes 3 and 7.

As shown in Fig. 6, there are four rows of vortices in an inviscid incompressible fluid, originally located in parallel to the x-axis and initially given a small disturbance. The free stream velocity in the absence of the vortex rows is taken to be $U$. The strength of the vortices in the rows 1 and 3 is $-2U/n$ and in the rows 2 and 4, $2U/n$, where $n$ is the number of vortex elements per wave-length and positive vorticity is counterclockwise. When the co-ordinates of the j-th vortex element in the row (i) are represented by $x_{j}(t), y_{j}(t)$, and the velocity components of this element by $u_{j}(t), v_{j}(t)$ at time $t$, the differential equations which govern the motion of the element may be written as follows:

$$\frac{dx_{j}(t)}{dt} = u_{j}(t), \quad \frac{dy_{j}(t)}{dt} = v_{j}(t). \quad \text{(15)}$$
The velocity components can be determined by considering the velocity induced at \((x_j, y_j)\) by all other vortices of the system and by the free stream motion. The components of the induced velocity \(u_{ij}\), \(v_{ij}\) are written by

\[
\begin{align*}
  u_{ij} &= \frac{U}{a} \frac{(-1)^{m+k}}{n+k} \times \\
          &\frac{\sinh(2\pi(y_j-y_i)/a) \cos(2\pi(x_j-x_i)/a) \cosh(2\pi(y_j-y_i)/a)\cos(2\pi(x_j-x_i)/a)}{\cosh(2\pi(y_j-y_i)/a) \cos(2\pi(x_j-x_i)/a)}
\end{align*}
\]

\[
\begin{align*}
  v_{ij} &= \frac{U}{a} \frac{(-1)^{m+k}}{n+k} \times \\
          &\frac{\sin(2\pi(x_j-x_i)/a) \cos(2\pi(x_j-x_i)/a) \cosh(2\pi(y_j-y_i)/a)\cos(2\pi(x_j-x_i)/a)}{\cosh(2\pi(y_j-y_i)/a) \cos(2\pi(x_j-x_i)/a)}
\end{align*}
\]

where, the terms of \(\ell = j\) must be excepted at \(k = 1\). Therefore, the velocity components of the vortex element are represented by

\[
\begin{align*}
  u_j^{n} &= U + u_{ij}, \quad v_j^{n} = v_{ij} \quad (17)
\end{align*}
\]

Meanwhile, considering the initially given symmetric or antisymmetric disturbance in the rows of vortices, one will obtain the following relations of the co-ordinates;

for the symmetric disturbance,

\[
\begin{align*}
  x_j^{n} &= x_j^{0}, \quad x_j^{n} = x_j^{0}, \quad y_j^{n} = y_j^{0}, \quad y_j^{n} = y_j^{0}
\end{align*}
\]

(18)

for the antisymmetric disturbance,

\[
\begin{align*}
  x_j^{n} &= x_j^{0} - /2, \quad x_j^{n} = x_j^{0} /2, \quad y_j^{n} = y_j^{0}, \quad y_j^{n} = y_j^{0}
\end{align*}
\]

(19)

Thus, it is not necessary to calculate the co-ordinates for the rows 3 and 4.

The initial displacements of the individual elements of the vortex rows are written by using the result of the linear analysis,

\[
\begin{align*}
  x_j^{(0)} &= (j-1)a/n + \xi_j \text{sin}(2\pi(j-1)/n) \\
  y_j^{(0)} &= (h+3/2) + \eta_j \text{sin}(2\pi(j-1)/n)
\end{align*}
\]

(20)

Introducing the relations of \(\xi_j\) and \(\eta_j\), corresponding to the mode 3 or 7 shown in Table 1 into Eqs. (20), one can calculate the nonlinear growth of the disturbance by numer-
ically integrating Eqs. (15).

2.5 Results of numerical integration

The typical results of the calculations of the positions of vortices at various times after the initial disturbance have been plotted in Figs. 7 to 12. In these sketches, the solid lines were drawn to suggest continuous vortex sheets and the dotted lines drawn to connect the regions of vortex concentration to the solid line. On the other hand, as clear vortex sheets no longer existed in the last stages of the development of vortex streets, a solid line was simply drawn surrounding the individual clouds which were made from the vortex concentration, with arrowheads indicating the sense of rotation of the net vorticity within the individual clouds. The parameters used in the numerical integration are written in the captions of the figures, and the elapsed time from the original perturbation to each later configuration sketched is shown immediately to the right of each drawings. The open and solid marks in the drawings represent vortices with clockwise and counterclockwise senses of rotation respectively.

The value of the spacing ratio $h/d$ was taken to be the Karman spacing ratio ($0.281$), except for Fig. 12. It should be noted that with the number of vortices per wave-length and the time step, $n = 24$ and $\Delta t = 0.005U/A$ instead of $21$ and $0.01U/A$ respectively, still no noticeable differences in the results of the calculation were detected.

In Figs. 7 and 9, the developments of vortex streets are shown by focusing attention on only one pair of vortex sheets (1 and 2). As shown in Fig. 7, where the initial disturbance is symmetric at $h/h = 5.0$, it is
clear that a vortex street is independently formed from each pair of vortex sheets without mutual interaction. In the case of the initially antisymmetric disturbance also at \( g/h = 5.0 \), which is not shown here, the development of vortex streets was the same as shown in Fig.7. As \( g/h = 2.0 \) for the initially symmetric disturbance, the process of the development of vortex streets does not differ so much from that at \( g/h = 5.0 \) (compare Fig.7 with Fig.8). However, also at \( g/h = 2.0 \) but for the antisymmetric disturbance, the interaction appears in the intermediate stage of the development, and some vortices in a pair of vortex rows are displaced to the other pair of rows across the center line, as shown in Fig.9. When the value of \( g/h \) is taken to be 1.0 and the initial disturbance is symmetric, the vortices are never displaced across the center line, as shown in Fig.10. On the other hand, also at \( g/h = 1.0 \) but for the initially antisymmetric disturbance, as shown in Fig.11, the displacements of vortices across the center line appear in the early stage of the vortex concentration, and as the result of these vertical displacements the system tends to form only one vortex street instead of two streets. At the smaller spacing ratio \( h/a \) as shown in Fig.12, this tendency appears in the earlier stage, and a pair of rows of the clouds is distorted to a pair of vortex layers in the later stage.

One may find in Fig.10 and 11 that the movement of a region of inner vortex concentration in the flow direction is accelerated positively for the symmetric disturbance and negatively for the antisymmetric one.

3. Experimental results

3.1 Apparatus and experimental method

The experiments were made in a water tank in a ducted air stream. In the water tank of 60cm(width) \( \times \) 60cm(depth) \( \times \) 500cm(length), two circular cylinders of 8.0mm(diameter) were moved at a constant speed and the wakes behind them were visualized by using water-color paints. In the air stream, two cylinders of 2.0cm(diameter) were located at the test section in the entrance region of the duct whose section was sized 300mm(width) \( \times \) 100mm(depth).

Either in water or in air, two cylinders were variously spaced in the direction perpendicular to the movement direction or the air flow, and the vortex shedding frequencies were observed with two hot-wires.

3.2 Observation of wakes in water

Figure 13 shows the photographs of the wakes behind two parallel cylinders at various values of the gap-diameter ratio \( G/D \), except for the top one of a single vortex street given as a reference. The photographs (b) and (c) show respectively the symmetrical and antisymmetrical developments of two vortex streets in the vortex formation region at \( G/D = 2.0 \). These two patterns of the developments may be compared with Figs. 8 and 9 respectively. The alternation between two patterns occurred at random during a movement of the cylinders at a constant speed, and this may be attributed to the distortion of the periodicity of vortex shedding. The photograph (d) shows the symmetrical development of vortex streets at \( G/D = 1.05 \), which may be compared with Fig.10. The photograph (e) shows the development also at \( G/D = 1.05 \) but just after the start of the movement of cylinders from rest. The pattern of the symmetrical development is almost the same as that shown in Fig.10. Focusing attention on the flow just behind two cylinders also in the photograph (e), one can find a sign of the alternation from the symmetrical development to the antisymmetrical one. However, no completely antisymmetrical development could be observed in this experiment. At a smaller value of the gap-diameter ratio; \( G/D = 0.50 \), as shown in the photograph (f), the development of two vortex streets no longer could be detected, but a single vortex street with longer wave-length has appeared. The wake just behind the upper cylinder is less wide than one behind the lower, and in the smaller wake one can observe the formation of a weak vortex street with shorter wave-length.

3.3 Vortex shedding frequency and phase relation

The non-dimensional frequency of vortex shedding (called Strouhal number) is shown in Fig.14 as a function of \( G/D \) for the Reynolds numbers 662 (in water) and 30,000.
(in air), together with the curve examined by Spivack. At a value of G/D larger than unity, both Strouhal numbers for two cylinders were the same as that for a single cylinder. However, at a value less than unity, a higher Strouhal number was observed in the smaller wake and a lower one observed in the larger wake (refer to the photograph (f) in Fig.13). Figure 14 suggests that at a value of G/D smaller than unity each cylinder no longer sheds independently a pair of vortex sheets. This agrees with the paradoxical result of the linear analysis.

At some values of G/D larger than unity, the summations of time $t_1$ and $t_2$ in a certain long time $T$, during which the phase relations of vortex shedding of two cylinders were symmetrical and antisymmetrical respectively, are shown in Fig.15. Where during the remaining time ($T - t_1 - t_2$) the relation was unjustifiable. It is interesting that as the value of G/D approaches unity, the antisymmetrical phase relation tends to disappear. This tendency seems to be related with such an evidence obtained in the non-linear analysis that the initially antisymmetric disturbance is more apt to cause the interaction of two vortex streets than the symmetric one.

4. Conclusions

The instability of two pairs of infinitely long parallel vortex sheets was investigated by the linear analysis in an inviscid incompressible fluid, and it was shown paradoxically that two pairs of parallel vortex sheets could not exist at a value of the gap distance ratio $G/h$ smaller than unity. By calculating the non-linear growth in time of periodic disturbances in the vortex sheets, the essential features of the formation and interaction of two parallel vortex streets were analysed, and it was explained that the initially antisymmetric disturbance was rather apt to cause the interaction between two vortex streets in their developing stage. These theoretical results almost agreed with the experimental results which were observed in a flow past two circular cylinders spaced in the direction perpendicular to the flow.

Since the existence of two bluff bodies has been ignored in the theoretical analysis, the distance $h$ between vortex sheets in a pair and the gap distance $g$ between two pairs of the sheets do not always correspond to the normal width of a body and the gap distance between two bodies respectively. Thus, it may be interesting to examine still more the theoretical results by observing a flow past two bluff bodies other than circular cylinders.

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