On the Ball Indentation Hardness for Plastics*

By Sadao YAMASHIRO **

By applying the extended Meyer's law proposed by the author to the relation among the depth of indentation $h$, the initial load $P_0$, the major load $P_m$ and the diameter of steel ball $D$, the following equation was obtained analytically:

$$h = A^2/n (P_0^{2/n} - P_m^{2/n}) D^{(n-1)/8},$$

where $A$ and $n$ called Meyer's index are constants for the material and $P_t = P_0 + P_m$. The relation was verified as correct experimentally.

Furthermore, the effects of errors in $P_0$, $P_m$ and $D$ on the hardness $H_K$ were calculated from the above relation and verified, and the total error $\Delta H_K$ due to each permissible error of the tester can be restricted to $\Delta H_K = H_K^{1/2} \times 25\%$ in the worst case.

1. Introduction

Following the development of mechanical materials, many kinds of testing methods of hardness which is one of the mechanical properties have been developed. The ball indentation hardness is essentially the same as the Brinell hardness in metals and it is defined as the quotient of the load on the ball indenter by the surface area of the indentation caused by the ball indenter. But, the Brinell hardness $H_B$ for metals is calculated from the chordal diameter $d'$ of the residual indentation after unloading, namely

$$H_B = 2/P_0 D (D - d'/2),$$

where $P_0$ kgf is a load and $D$ mm is a diameter of the ball indenter. However, in the case of plastics it is difficult to measure the diameter of the residual indentation. Moreover, since the greater part of the indentation recovers elastically, it does not fit the actual feel, for instance no residual indentation occurs with rubber. Therefore, the ball indentation hardness $H_K$ is calculated from the depth of indentation $t$ mm under the load $P_0$ namely

$$H_K = P_0/nt,$$

As far as the same ball indenter is used, the law of similarity concerning indentations does not hold for $H_B$ or $H_K$ in general. Namely even for the same material, its $H_B$ or $H_K$ is dependent on the load.

Moreover, in the case of $H_K$, the depth of indentation $t$ cannot be exactly measured, since an elastic deformation of the apparatus containing the indenter exists and the contact position between the test surface and the indenter is vague.

In order to avoid these defects, in Draft ISO (Draft ISO Recommendation No.2039) the initial load $P_0$ kgf is applied and the depth measuring device is set to zero, thus the vagueness of the contact position is excluded. Furthermore, the major load $P_m$ kgf is chosen from the specified values to obtain the following nearly constant depth of indentation:

- **Method A** (executed in France, small load): between 0.07 and 0.10 mm (the standard is 0.08 mm, $D = 5$ mm).
- **Method B** (executed in Germany, large load): between 0.15 and 0.35 mm (the standard is 0.25 mm, $D = 5$ mm).

The apparent depth of indentation $h$ mm under load $P_m$ kgf is measured and the true depth of indentation $h'$ mm is obtained by correcting the elastic deformation of the frame of the apparatus $h_2$ mm, i.e., $h = h_1 - h_2$, and the reduced test load $P_r$ kgf which causes a reduced depth of the indentation $h'_r$ mm is calculated as follows:

- Method A: $P_r = P_m (h_1/h'_r)^{1.23}/(5/D)^{0.63}$
- Method B: $P_r = 0.21 P_m (/h - h_r + 0.21)$

where $h_1 = 0.08$ mm, the effect of the difference of the diameter of the indenter is excluded by applying the law of similarity.

Thus, the ball indentation hardness $H_K$ kgf/mm$^2$ is calculated from the following equation:

$$H_K = P_0/5n.$$  

In the above equation, a constant number ($5$) means $D = 5$ mm. In this way, by keeping $h'_r$ constant and by mediating the reduced test load $P_r$, Draft ISO avoids the changing of $H_K$ by the load and the diameter of indenter (only in Method A), and makes the hardness universal. As mentioned above, it may be understood that $H_K$ is the same as $H_m$ essentially.

Because the ball indentation hardness adopted in Draft ISO will be established as JIS (Japanese Industrial Standard) in the future, this paper tries to develop the analytical result by H.K. Racké and T. Pett (5), whose result in based upon Draft ISO Method B, and another purpose of this paper is to make clear the effects of errors in the initial load $P_0$, the major load $P_m$ and

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the diameter \( D \) on the ball indentation hardness \( H_K \).

2. Analysis between load and indentation depth

The relation between the diameter of indenter \( D \), the depth of indentation \( t \) from the original surface and the chordal diameter of indentation \( d \) on the original surface is shown as \( (t-\delta)=d^2/(2d^2) \) by simple geometry. Therefore:

\[
t = (D - D^2/2 - (D^2/4)^2)/2 + d^2/4 - d^2/16d^2 + d^2/32d^2 + \ldots.
\]

(1)

The most general relation between \( P \), \( D \) and the chordal diameter \( d' \) of remaining permanent indentation is shown as \( P = A'd^m/P^n \), which is known as Meyer's law, where \( A' \) is a characteristic constant for each material and \( n \) is called Meyer's index is a characteristic constant whose value lies between 2 and 3 for each material.

Suppose this Meyer's law is applicable even to \( D \) on the original surface under loading and suppose Meyer's index \( n \) with the load on has the same value as that when the load is off, then Meyer's law can be extended to the state under loading by using \( d' \) and then under the initial load \( P_0 \)

\[
P_0 = A'd^m/P^n.
\]

(2)

Under the total load \( P_t = (P_0 + P_m) \),

\[
P_0 = A'd^m/P^n - P_m = A'd^m/P^n - P_0.
\]

(3)

where \( A \) is another characteristic constant different from \( A' \) which depends on the material. From Eq. (1),

\[
t_0 = \delta d^2/4h_0,
\]

(4)

\[
t_0' = \delta d^2/4h_0,
\]

(5)

where \( t_0 \) and \( t_0' \) denote the depths of indentation under \( P_0 \) and \( P_m \) respectively.

Denoting the increase in the depth of indentation under \( P_0 \) to \( P_t \) by \( h \) mm, as \( h = (P_t - P_0)/h_0 = (P_t - P_0)/h_0 \)

(6)

If \( h \) corresponding to the reduced test load \( P_0 \) is denoted as \( h_r \) (\( h_r = 0.25 \) mm for Method B),

\[
h_r = A'2/n[(P_0 + P_m)2/n - P_02/n]n - (P_02/n - P_02/n) = P_02/n - P_02/n.
\]

(7)

From Eqs. (6) and (7),

\[
h_r = (P_02/n - P_m2/n)/(P_0 + P_m)2/n.
\]

(8)

Substituting \( P_m = 1 \) kgf and \( h_r = 0.25 \) mm,

\[
P_m = 0.25(P_02/n - P_02/n)(P_0 + P_m)^2/n = (P_02/n - P_02/n)(1 + 1/n)^2/n.
\]

(9)

Since \( P_0 \) is a constant, from Eq. (7),

\[
\delta h_r/\delta P_0 = (2/n)(P_02/n)(1 - (P_02/n - P_02/n))/(P_0 + P_m)^2/n.
\]

(10)

Substituting \( h_r = 0.25 \) mm,

\[
\delta h_r/\delta P_0 = (1/(2n))(1 - (P_02/n - P_02/n)) = a.
\]

(11)

Therefore \( \delta h_r/\delta P_0 = a(P_02/n)/(P_0 + P_m)^2/n) \)

(12)

In the neighborhood of \( h_r = 0.25 \) mm, the relations between \( h \) and \( P_m \) are nearly linear as shown in the following figure 3, therefore:

\[
h = h_r + \delta(a(P_02/n)/(P_0 + P_m)^2/n),
\]

(13)

where \( \delta \) is the slope of lines. Then,

\[
P_m = a(P_0 - (h - h_r)))/(h - h_r).
\]

(14)

Taking \( a = 2.3 \) kgf and \( P_m = 98 \) kgf, the values of \( a \) and \( P_m \) are calculated from Eq. (11) and shown in Table 1, and they lie within the range of 0.21±0.01 for harder materials whose \( P_m \) is over 36.5 kgf and \( h_r \) is over 2.5 when \( P_m \) is in the order of 13.5 kgf.

<table>
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<tr>
<th>( n )</th>
<th>2.3</th>
<th>2.4</th>
<th>2.5</th>
<th>2.6</th>
</tr>
</thead>
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<tr>
<td>( P_m )</td>
<td>5 kgf</td>
<td>0.275</td>
<td>0.268</td>
<td>0.263</td>
</tr>
<tr>
<td>( a )</td>
<td>36.5</td>
<td>0.241</td>
<td>0.233</td>
<td>0.227</td>
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<td>98</td>
<td></td>
<td>0.219</td>
<td>0.212</td>
<td>0.205</td>
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<td></td>
<td></td>
<td>0.217</td>
<td>0.208</td>
<td>0.200</td>
</tr>
</tbody>
</table>

(15)

| Equation (15) agrees with what H. H. Racke and T. Fett had proposed by obtaining the constant 0.21 from experiment directly that is used in Draft 150 Method B. Moreover, when \( P_m \) or \( P_m \) is over 36.5 kgf, the denominator of Eq. (11) tends approximately to unity, and then the value of \( a \) is regarded as a reciprocal number of twice Meyer's index approximately (see Table 1). |

3. Analysis of the effect of errors in loads and diameter of indenter on \( H_K \)

Denoting the errors of the initial load \( P_0 \), the major load \( P_m \), the total load \( P_t = (P_0 + P_m) \) and the diameter of indenter \( D \) as \( \delta P_0 \), \( \delta P_m \), \( \delta P_t \), and \( \delta D \), respectively, and also denoting the error of the depth of indentation \( h \) affected by them as \( \delta h \), and differentiating Eq. (6) by each term,

\[
(\delta h/\delta n)P_0 = 2/(n)(P_0 + P_m)(2/n - 1)\delta P_0/\delta P_t.
\]

(16)

\[
(\delta h/\delta n)P_m = 2/(n)(P_0 + P_m)(2/n - 1)\delta P_m/\delta P_t.
\]

(17)

\[
(\delta h/\delta n)P_t = 2/(n)(P_0 + P_m)(2/n - 1)\delta P_t/\delta P_t.
\]

(18)

\[
(\delta h/\delta n)D_n = -2/(n)(P_0 + P_m)(2/n - 1)\delta D/\delta P_t.
\]

(19)

Therefore, after obtaining Meyer's index by experiment, \( \delta h \) affected by \( \delta P_0 \), \( \delta P_m \) and \( \delta D \) can be calculated. Also, from Eq. (15), taking \( h_r = 0.25 \) mm,

\[
\delta h_r/\delta D = \delta h_r/\delta P_0 = \delta h_r/\delta P_m = \delta h_r/\delta P_t = 0.047.
\]

(20)

The ball indentation hardness \( H_K \) is defined as follows,

\[
H_K = P_t/\delta h_r.
\]

(21)
Hence, \( \Delta h / H_0 = \Delta F / F_p \). 
(22)
Calculating \( \Delta h_1 / H_1 \) and \( \Delta h_2 / H_2 \) is obtained from Eq. (20) and then \( \Delta h_1 \) for \( H_1 \) can be calculated from Eq. (22).

4. Experiments and results

4.1 Specimens and experimental methods

The specimens used are shown in Table 2. An Akashi Seisakusho Ltd.-made tester improved from the Rockwell hardness tester to conform to Draft ISO Method B was used. To examine the effect of the error in the initial load, the initial load could be changed from 1 kgf to 11 kgf or 1.2 kgf. The major loads 8, 20 and 60 kgf were added to its nominal load 5, 13.5, 36.5 and 98 kgf, and also when the initial load was set at 1.1 or 1.2 kgf, it was possible to decrease the major load by 0.1 or 0.2 kgf and to keep the total load at its nominal load. The accuracy of \( F_0 \) and \( F_m \) was kept within ±0.5 % and the accuracy of the depth measuring device which contains a magnifying mechanism with a lever was kept within ±0.0005 mm.

All experiments were carried out in an air-conditioned room and the depths of five or ten indentations were read under the same condition. The major load was applied at a constant velocity and after 30 seconds since the major load has been applied perfectly, the apparent depth of indentation \( h_1 \) under load was read. The spot anvil of the diameter 10 mm was used for this measurement.

4.2 On the elastic deformation of the test containing the indenter

Since the depth of indentation \( h \) i.e., the object of measurement must be read under \( F_m \), the pointer of the measuring device indicates the true depth \( h \) plus the elastic deformation \( h_2 \) of the frame, the ball chuck and the steel ball. Therefore, the elastic deformation \( h_2 \) should be determined in advance and by subtracting \( h_2 \) from the reading of pointer \( h_1 \) the true depth \( h \) should be obtained.

For that purpose, the following five kinds of ball chuck were prepared as sockets, one with concave spherical surface of whose radius of curvature is 2.5 mm (Symbol S) and four with concave conical surfaces whose top angles of cone are 90°, 105°, 120° and 135° (V90, V105, V120, V135). Three steel balls each one part of which was planed flat by 0.2, 0.3, 0.4 mm (Symbol: B0.2, B0.3, B0.4) in radial direction were set with each planed part placed on the specimen (See Fig. 1).

Using a piece of tungsten carbide as the specimen, after having applied \( F_0 \) and adjusted the depth measuring device to zero and applying \( F_m \) then the indicating value of the pointer was read. Since a plastic deformation was not caused on the surface of the specimen, neglecting the elastic deformation of the contacting part under load, the indicating value of the pointer was regarded as the elastic deformation \( h_2 \) of the frame, the ball chuck and the steel ball. The results are shown in Table 3. The symbol F means the elastic deformation of the frame alone without the ball chuck.

| Table 3 Elastic deformation of the tester containing indenter: unit \( \mu m \) |
|----------------|---|---|---|---|
| \( P_m \) kgf | 5 | 13.5 | 36.5 | 98 |
| \( F \) | 2.5 | 6.5 | 16.0 | 38.9 |
| \( F + S + B0.2 \) | 2.9 | 7.1 | 16.9 | 41.9 |
| \( F + S + B0.3 \) | 2.8 | 6.3 | 15.7 | 38.8 |
| \( F + S + B0.4 \) | 2.6 | 6.1 | 15.1 | 38.5 |
| \( F + V90 + B0.3 \) | 2.9 | 6.8 | 16.6 | 41.4 |
| \( F + V105 + B0.3 \) | 2.8 | 6.9 | 16.6 | 41.0 |
| \( F + V120 + B0.3 \) | 3.0 | 7.5 | 16.9 | 41.4 |
| \( F + V135 + B0.2 \) | 3.0 | 7.6 | 17.9 | 43.8 |
| \( F + V135 + B0.3 \) | 2.8 | 7.0 | 16.7 | 41.2 |
| \( F + V135 + B0.4 \) | 2.9 | 6.9 | 16.1 | 40.4 |
| mean \( \) | 2.8 | 6.9 | 16.5 | 40.7 |
| max - min \( \) | 0.5 | 1.5 | 2.8 | 5.3 |

From the results, the elastic deformation of the frame alone was not always a minimum. It would seem that the end of the plunger rod is not parallel to the surface of the specimen. Of course, when the

| Table 2 Specimen, hardness and Meyer's index |
|----------------|---|---|---|---|---|
| Specimen | Symbol | Size \( \) mm | Thickness \( \) mm | \( P_m \) kgf | \( H_p \) kgf | \( H_p/mm^2 \) (1) (2) | Meyer's index \( n_1 \) \( n_2 \) mean |
| PMMA | A | 70X60 | 10 | 98 | 72.7 | 71.8 | 18.5 | 18.3 | 2.54 | 2.40 | 2.47 |
| PVC 1-3 | B | 75X70 | 5 | 36.5 | 69.7 | 65.3 | 17.7 | 16.6 | 2.29 | 2.31 | 2.30 |
| PVC 1-3 | C | 80X80 | 6 | 36.5 | 60.2 | 60.0 | 15.3 | 15.3 | 2.55 | 2.55 | 2.55 |
| PVC 1-1 | D | 80X80 | 4 | 36.5 | 55.9 | 58.4 | 14.2 | 13.9 | 2.35 | 2.35 | 2.35 |
| HIPS | E | 70X70 | 4 | 36.5 | 48.9 | 49.0 | 12.7 | 12.5 | 2.34 | 2.40 | 2.37 |
| PP | F | 80X80 | 5 | 36.5 | 35.6 | 35.5 | 9.1 | 9.0 | 2.31 | 2.42 | 2.32 |
| PA | G | 50X50 | 12 | 13.5 | 20.4 | 20.4 | 5.2 | 5.2 | 2.60 | 2.65 | 2.63 |
| PTFE | H | 50X50 | 4 | 13.5 | 11.3 | 11.3 | 2.9 | 2.9 | 2.49 | 2.60 | 2.55 |
| PE | I | 75X75 | 6 | 5 | 6.4 | 6.3 | 1.6 | 1.6 | 2.60 | 2.48 | 2.54 |
same ball chuck was used, the value of \( h_0 \) with a larger planed ball was smaller. Also, in the case of this experiment, the values of \( h_0 \) with the ball chuck of the socket \( D \) were minimum, but it seems to be accidental that the concave sphere touched the steel ball closely, and considering the difficulty of accurate manufacture of the concave spherical surface, it is dangerous to choose the spherical surface.

If the permissible error of the ball diameter should be within 0.05 mm according to Draft ISO, the radius of curvature of the socket should be 2.525 mm, and if the steel ball whose diameter is 4.950 mm is set in the socket, according to Hertz's equation and putting \( P_m = 5 \cdot 98 \) kgf, the elastic deformation due to contact between the socket and the steel ball will be between 0.9 and 6.4 \( \mu m \). Considering the experiment changing the diameter of steel ball as mentioned later, the socket of its top angle 120\(^\circ\) was used in this experiment and \( h \) was obtained by subtracting \( h_0 \) shown in Table 3 from the reading of pointer \( h_1 \).

Also, in the case of the frame alone shown as \( F \) in Table 3, if the major load is removed from the total load, the pointer should ideally return to the initial set position, but it does not in general. The difference between the initial set position and the returned one is called 'the residual deformation of frame' and is shown in Fig. 2 for the tester used. When \( P_m = 98 \) kgf, the value of the residual deformation was 0.4 \( \mu m \) and this value could be neglected.

4.3 Relation between the major load, the diameter of indenter and the depth of indentation

When the diameter of indenter \( D \) is 5 mm, Fig. 3 shows the relations between the major load \( P_m \) and the depth of indentation \( h \) for each plastics. Within the range of \( h = 0.15 \) to 0.35 mm, the relations between \( P_m \) and \( h \) can be regarded as linear. As an example, the comparison of Eq. (6) with the observed values shall be made for Polypropylene (F). Taking Meyer's index \( m = 2.42 \) as mentioned later and the constant \( A = 9.51 \) from the experiment, Eq. (6) is shown as follows:

\[
 h = 9.51 \cdot \frac{2}{2.42} \left( \frac{P_m + 1}{2.42} - 1 \right) \times \frac{1}{2.42/4} 
\]

(23)

In Fig. 3, the values calculated by the above equation are shown by black circles. The calculated values agree with the observed ones shown by the solid line and the white circles within the range of 0.15 \( \mu m \).

Changing the diameter of steel ball to \( 3/16 \) in., 5 mm, 7/32 in., and 1/4 in. under \( P_m \) shown in Table 2 for each specimen, the values of \( h \) are shown in Fig. 4 on logarithmic coordinates. Within the limits of this experiment, the relations between log \( D \) and log \( h \) are linear. Therefore, \( h = kD^{-m} \) (24) where \( k \) and \( m \) are characteristic constants for each specimen. For Polypropylene (F), taking \( P_m = 36.5 \) kgf, the comparison of Eq. (6) i.e. Eq. (23) with the observed values shall be made. Equation (23) is shown as follows:

\[
 h = 9.51 \cdot \frac{2}{2.42} \left( \frac{P_m + 1}{2.42} - 1 \right) \times \frac{1}{2.42/4} 
\]

(25)

Equation (25) is shown by the broken line in Fig. 4. Close agreement between the observed values and the calculated ones is obtained within the range of 0.15 \( \mu m \).

Fig. 2 Residual deformation of the frame

Fig. 3 Relations between major load and depth of indentation \((D = 5 \text{ mm})\)

Fig. 4 Relations between diameter of steel ball and depth of indentation

From the above facts, it seems that Eq. (6) sufficiently satisfies the relation among \( P_m \), \( D \), and \( h \), and it can be supposed that Eq. (6) satisfies the relation between \( P_0 \) and \( h \) similarly.
The values of \( P_r \) obtained by Eq. (15) and (9) are shown in Table 2. Except for Polyvinylchloride 1-3 (B), both agree with each other. Also, the ball indentation hardness \( H_k \) calculated from Eq. (21) using each \( P_r \) is shown in Table 2 as \( H_k (1) \) and \( H_k (2) \).

The values of \( a \) calculated from Eq. (11) (Table 1) and obtained from the experiment are shown in Fig. 5. It seems that both agree with each other approximately.

4.4 Meyer's index \( n \)

Using the same diameter of steel ball and two total loads \( P_r' \), \( P_r'' \) which indent until near \( h_m=0.25 \) mm, and denoting the depths of indentation \( h \) as \( h' \) and \( h'' \) respectively, from Eq. (6),

\[
h'/h'' = (P_r' / P_r'')(n_1 / n_2),
\]

(26)

Changing the value of \( n \), the right side of Eq. (26) is calculated and comparing it with the left side of Eq. (26) obtained by the experiment and the values of \( n \) which agree with each other are denoted as \( n \), for the material. Also, from the experiment in which the diameters of steel ball under the same major load (Fig. 4) are changed, the slopes \( m \) of the straight lines are obtained, and from Eq. (6) as follows,

\[
m = (n - 1) / n.
\]

(27)

Meyer's index \( n_2 \) are obtained from Eq. (27). \( n_1 \), \( n_2 \) and the mean value \( n \) are shown in Table 2. Since \( n \) falls in the range from 2.3 to 2.6, \( n_2 \) falls in the range from 2.3 to 2.6 and mean values of both are 2.46, Meyer's index for the material is determined by the mean value \( n \).

4.5 Relation between the ball indentation hardness and the major load

The relation between the ball indentation hardness \( H_k \) and the major load \( P_m \) for each specimen is shown in Fig. 6. \( H_k (1) \) (Draft ISO) calculated by using \( P_r \) of Eq. (15) are shown by the white circles and the solid line, and \( H_k (2) \) calculated by using \( P_r \) of Eq. (9) are shown by the black circles and the broken lines. The former grows larger as \( P_m \) decreases and taking \( h = 0.04 \) mm, \( H_k (1) \) will grow to infinity. The latter becomes smaller as \( P_m \) decreases, as if the relation between the Brinnell hardness and the load (2) for metals is much. However, under \( P_m \) making the depth of indentation in the range from 0.15 to 0.35 mm (marked by square brackets in Fig. 6), both hardnesses agree nearly and \( H_k \) is regarded as constant being independent of \( P_m \). Therefore, it seems that Eq. (15) found by Racke and Pett is simple and just exact within the limits of 0.15 to 0.35 mm.

4.6 Effects of error in the load and the diameter of steel ball on the depth of indentation

To investigate the effect of the error in the initial load \( P_0 \) alone, \( P_0 \) was changed from 1.0 kgf to 1.1 kgf or 1.2 kgf, and accordingly, the major load \( P_m \) was decreased by 0.1 kgf or 0.2 kgf from the nominal load and then keeping the total load \( P_0 \) constant, the changes of the depth of indentation \( \Delta h \)

Fig. 5 Relation \( P_r \) and \( a \)

Fig. 6 Relations between \( P_m \) and \( H_k \)

were investigated. In the results, \( h \) decreased linearly as \( \Delta P_0 \) increased. Taking \( \Delta P_0 / P_0 = 10 \% \), the observed \( h / h \) are shown by the white circles in Fig. 7. They agree fairly well with Eq. (16) mentioned above. Taking \( n = 2.5 \), from Eq. (16),

\[
(\Delta h / h)_P = -0.8[(P_m + 1)^0.8 - 1]^{-1} \Delta P_0 / P_0.
\]

(28)

The effect of the error in the major load \( P_m \) was obtained from the relation between \( P_m \) and \( h \) near \( h = 0.25 \) mm as shown in Fig. 3. Taking \( \Delta P_m / P_m = 1 \% \), the observed results are shown by the white circles in Fig. 8. The observed results agree comparatively well with the calculated results by Eq. (16). Taking \( n = 2.5 \), from Eq. (18),

\[
(\Delta h / h)_{P_m} = 0.8[(P_m + 1)^{-0.8} - 1] \Delta P_m / (P_m + 1).
\]

(29)

The relations between \( D \) and \( h \) were already shown in Fig. 4; since the observed results agree fairly well with the calculated results, the relation between \( \Delta D \)
5.1 On the permissible error of the initial load

The permissible error of the initial load is within ±1% for 1 kgf in Draft ISO and ±0.2 kgf for 10 kgf in JIS (H). If the initial load of the ball indentation hardness tester is applied by a spring compressed in the same way as the commercial Rockwell tester, the accuracy of the initial load cannot be expected to be so great. Also, as shown in Table 4, when ∆P₀/P₀ = ±1%, ∆H₀ = 0.00 ~ 0.01 kgf/mm² for H₀ = 0.9 ~ 18 kgf/mm², ∆H₀ is too small, comparing with the effects by other permissible errors. Therefore, it is proposed that the permissible error of the initial load be set at less than ±1%.

5.2 On the permissible error of the major load

The permissible error of the major load is within ±1% for each load in Draft ISO and less than ±1% as 60±0.5, 100±0.65 and 150±0.9 kgf in JIS (H). Taking ∆P₁/P₁ = ±1%, the effect on H₁ is nearly 1% (ΔH₁/H₁), it is possible to keep the permissible error within ±1% and it is quite proper. Therefore, Draft ISO should be recommended.

5.3 On the permissible error of the diameter of steel ball

In Draft ISO the permissible error is ±1% for 5 mm, but in JIS (H) it follows JIS B 1501 (on steel ball for ball bearing). According to JIS B 1501, the tolerance is within ±0.015 mm for 3/16 in., or 7/32 in. However, according to steel ball makers, the tolerance in ball diameter is generally within ±0.003 mm. Since the permissible error of the diameter of steel ball ±0.050 mm is too large, it is proposed that the permissible error be set at less than ±0.003 mm. The value of ∆H₁ introduced by ∆D = ±0.003 mm is in the order of ±0.036 %, ∆H₁/H₁ is in the order of ±0.05 % and it can be neglected.

5.4 On the permissible error of the measuring device

In Draft ISO and JIS (H), the permissible error is within ±0.001 mm. The value of ∆H₁ introduced by the error ±0.001 mm falls in the range from ±0.3 to ±0.9 % and then Draft ISO should be recommended, but it is necessary to select the measuring devices from batches carefully.

5.5 On the trouble in the ball chuck

It is preferable in idea that the socket of the ball chuck is concave spherical, but a change of the chuck introduces the erratic hardness readings, because of difficulties of manufacture. In order to make the difference of Rk small, i.e., to obtain good reproducibility, it is proposed that the socket itself be made a concave conical surface. In the case of the conical surface, any conclusion cannot be drawn due to an insufficient number of experiments about what difference of Rk (i.e., ΔRk) is caused between the chucks. If there are differences of 0.00005, 0.0001, 0.002 or 0.003 mm between the chucks for Pm = 9, 13.5, 36.5 or 98 kgf respectively, as shown.
Table 4 Permissible errors of main parts of tester and their effects $\Delta H_k$; Absolute values, ($\delta$) for Draft ISO

<table>
<thead>
<tr>
<th>Permissible error</th>
<th>$P_O =1$ kg</th>
<th>$P_O =12.5$ kg</th>
<th>$P_O =36.5$ kg</th>
<th>$P_O =98$ kg</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P_O = \pm 0.3$%</td>
<td>$\pm 0.04$%</td>
<td>$\pm 0.06$%</td>
<td>$\pm 0.11$%</td>
<td>$\pm 0.14$%</td>
<td>$\pm 0.15$%</td>
</tr>
<tr>
<td>$\Delta D = \pm 0.005$ mm</td>
<td>$\pm 0.005$ mm</td>
<td>$\pm 0.005$ mm</td>
<td>$\pm 0.005$ mm</td>
<td>$\pm 0.005$ mm</td>
<td>$\pm 0.005$ mm</td>
</tr>
<tr>
<td>Error by ball chuck</td>
<td>$0.00025$ mm</td>
<td>$0.00025$ mm</td>
<td>$0.00025$ mm</td>
<td>$0.00025$ mm</td>
<td>$0.00025$ mm</td>
</tr>
<tr>
<td>Total $\sum H_k$ error ($\Sigma H_k$)</td>
<td>$0.05$</td>
<td>$0.10$</td>
<td>$0.15$</td>
<td>$0.20$</td>
<td>No limit</td>
</tr>
<tr>
<td>$\Sigma D/H_k$ %</td>
<td>$2.00$</td>
<td>$2.00$</td>
<td>$2.00$</td>
<td>$2.00$</td>
<td>$2.00$</td>
</tr>
<tr>
<td>$\Delta D/H_k$ %</td>
<td>$1.00$</td>
<td>$1.00$</td>
<td>$1.00$</td>
<td>$1.00$</td>
<td>$1.00$</td>
</tr>
<tr>
<td>$\delta P/O$</td>
<td>$0.005$</td>
<td>$0.010$</td>
<td>$0.015$</td>
<td>$0.020$</td>
<td>$0.025$</td>
</tr>
<tr>
<td>$\delta D/O$</td>
<td>$0.005$</td>
<td>$0.010$</td>
<td>$0.015$</td>
<td>$0.020$</td>
<td>$0.025$</td>
</tr>
<tr>
<td>$\delta H_k/O$</td>
<td>$0.005$</td>
<td>$0.010$</td>
<td>$0.015$</td>
<td>$0.020$</td>
<td>$0.025$</td>
</tr>
</tbody>
</table>

in Table 4, the values of $\Delta H_k/H_k$ introduced by them will grow from 1 to 3% and these values are larger than the values of $\Delta H_k$ by the above mentioned errors. If any trouble happens with $H_k$ in future, it would be due to the chucks. Draft ISO does not refer to this trouble.

5.6 On the total error

The total error introduced by each permissible error of the tester may be the total sum $\sum H_k$ in the worst case and can be restricted to from $0.03$ to $1.48$ kgf/mm² (or from $\pm 2\%$ to $\pm 5.5\%$) for $H_k=0.9-48$ kgf/mm². However, since the effects of plus and minus may partly cancel each other in general, taking the square root of the sum of the square of each effect as the total error, the total error $\Delta \sum H_k$ can be restricted to from $0.02$ to $1.48$ kgf/mm², in other words about $\pm 3\%$ and if so, the ball indentation hardness seems to be very useful as one of hardness tests.

6. Conclusions

On the ball indentation hardness used for plastics, the relation among the initial load $P_O$, the major load $P_m$, the diameter of steel ball $D$ and the depth of indentation $h$ i.e. the hardness $H_k$ has been studied analytically from the extended Meyer's law and it was verified by the experiment.

The obtained results are as follows:

1. From the extended Meyer's law, the relation among $P_O$, $P_m$, $D$ and $h$ can be expressed by following equation,

$$ h = A^{-2/3} \left( \frac{P_m}{P_O} \right)^{2/3} \frac{D^2}{h^2} $$

where $A$ is a characteristic constant for material, $n$ called Meyer's index is a characteristic constant for material and $P_m = P_O + P_C$.

2. The above equation has been ascertained to agree fairly well with the observed results.

3. The reduced test load $P_r$ derived from the above equation is shown as follows:

$$ P_r = \frac{0.25(P_m - h) + 1}{h^{1/2}} - 1 $$

and this equation has been compared with the equation proposed by Racke and Fett and adopted in Draft ISO Method B, i.e. $P_r = 0.21P_m/(h - 0.021)$, where $h = 0.25$ mm. As a result, both equations have fairly well agreed with each other within the limits of $0.15\%$ to $0.35\%$, therefore it seems that the equation by Racke and Fett is a simple and exact one as far as the above limits are kept. Also, a constant 0.21 that Racke and Fett has obtained from the experiment could be approximated to 1/2m and the physical meaning can be given to it.

4. In the case that $h$ is beyond the above mentioned limits, the equation by Racke and Fett cannot be used and one by the author is rather proper.

5. From the equation shown in the conclusion (1), the errors of $h$ introduced by the error of $P_O$, $P_m$ or $D$ are shown analytically as follows:

$$ (\Delta h)(P_m)^{2/3} = (\Delta P/O)(P_m)^{2/3} - (\Delta D/O)(P_m)^{2/3} $$

and these have agreed with the observed results.

6. The permissible errors of the main parts of the tester have been examined and it has been made clear that $\Delta P/O = h/AD$, $\Delta D/O = h/AD$, and $\delta H_k/O = (h - n)/h$, which have agreed with the observed results.

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References
