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Motion of Pistons in Piston-Type Hydraulic Machines*

(2nd. Report : Experiment)

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In this report the results of test carried out with a rig modeled after a piston pump to verify the theoretical analysis are presented. The friction at the connecting-rod bearings is shown to affect the motion of the piston considerably. It is concluded that the calculated values including the friction coincide fairly well with the test results excepting the low rotational speed-case where the piston comes into contact with the cylinder wall.

1. Introduction

In this report the results of test carried out with a rig modeled after a piston pump are presented. There seems to be no experimental study on the motion of pistons as described in the 1st. report.⁽¹⁾

2. Nomenclature

We use the same symbols as used in the 1st. report. The following are the additional ones used in this report.

- C_1 : fluid film thickness at measuring position of high pressure side
- C_2 : fluid film thickness at measuring position of low pressure side
- f_0 : coefficient of friction of connecting-rod bearing
- n : rotational speed of cam

3. Test rig and method

Fig.1 shows the main part of the test rig. The piston is driven by a driving mechanism with an eccentric cam and a lever. We add suffix 0 to values when the piston is farthest along the z axis, and in this condition the lever is made perpendicular. We define this condition as the origin of time, i.e., the rotational angle of the cam $\tau=0$.

Using the symbols shown in the figure, radial thrust to the y direction (we define the y axis upwards on the paper) W_y is

$$W_y = -p_b \pi R^2 \tan \varphi \quad (1)$$

, where it is assumed that p_a is atmospheric pressure. Let φ be the angle between the center line of the cylinder bore and that of the connecting-rod, then we obtain

$$\sin \varphi = H/l$$

$$H = H_0 + l_1(1 - \cos \theta) - d_1 \sin \theta$$

, where θ is the inclined angle of the lever to the perpendicular line, $l = \overline{BP}$, $l_1 = \overline{O_s A}$. $P_c(\tau, \xi)$ denotes the contact point between the lever and the cam, then

$$\xi = d_2 + R_c(1 - \cos \theta) + E(1 - \cos \tau)$$

$$\eta = -E \sin \tau - R_c \sin \theta$$

$$(l_2 - \eta) \tan \theta = \xi - (d_2 / \cos \theta)$$

, where $R_c = \overline{P_c O_c}$, $E = \overline{O_c O_M}$ and $l_2 = \overline{O_s O'_s}$. In this rig $\theta < 0.17$, then we omit terms of higher order than θ^3 . Then, we obtain

$$\varphi = (H_0 + 0.5 l_1 \theta^2 - d_1 \theta) / l$$

$$\theta = \frac{-\{l_2 + E \sin \tau\} + [(l_2 + E \sin \tau)^2 - 2E \times \{1 - \cos \tau\}(R_c + d_2 + E - E \cos \tau)]^{1/2}}{R_c + d_2 + E(1 - \cos \tau)} \quad (2)$$

The z coordinate of the small end of the connecting-rod z_D is given by

$$z_D - z_{D0} = -l_1 \sin \theta - d_1(1 - \sin \theta) - l(\cos \varphi + \cos \varphi_0)$$

, where z_{D0} is z_D when $\tau=0$. Therefore, the reciprocating velocity of the piston w_D is given by

$$w_D = dz_D/dt \quad (3)$$

Table 1 shows dimensions of the rig, and Fig.2 shows φ .

The cam was driven by an electric motor and a belt type power transmission. The outlet pressure p_b of a gear pump acted on the piston head and was kept nearly

Fig.1 Schematic diagram of test rig

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constant in spite of the reciprocating motion of the piston by a relief valve and an accumulator used in the hydraulic line. The clearance between the piston and the cylinder wall was measured by the capacitance type gap transducers, which were set on the yz plane shown in Fig.1. The z coordinate of the piston was measured by a differential transformer and, for this purpose, a small iron bar was fitted to the piston. Pressure was measured by a strain-gage type transducer. The test conditions were as follows: $n = 470, 960, 1460$ rpm, $p_b = 30, 40, 50$ atg, and oil temperature $= 40^\circ\text{C}$ (turbine oil, viscosity: 2.77×10^{-7} kgs/cm²).

In this experiment, measurement of the film thickness was most important. Therefore, to measure it accurately electrodes of the gap transducer were set as shown in Fig.3. The distance between the surface of electrodes and the cylinder wall was about 0.25 mm so that the output of the transducer could be nearly proportional to the film thickness. The gap transducers were calibrated by the assumption that the difference between the outputs when the piston was pressed to the upper wall of the cylinder at $p_b = 5$ atg and to the lower wall was equal to double the radial clearance C . Moreover, there was scarcely any change in the output when p_b was changed from 30 atg to 50 atg.

4. Test results and comparison with theory

Fig.4 shows an example of test results. The scale given to C_1 and C_2 is relative scale to express fluctuation of the radial displacement. Arrows show the direction of increase of the film thickness. Difficulty in measuring absolute values including their calibration is the reason why only relative values are given. The following are found out in the figure. The record of C_1 differs considerably from that of C_2 ; C_1 increases at the top dead center but C_2 decreases; the amplitude of C_2 is greater than that of C_1 .

Now, let us compare these results with the theory. In this test rig $\bar{\omega} = 0$ and both C_1 and C_2 are derived from $\bar{Y}$ and $\bar{\phi}_x$, as mentioned later. The spherical bearings at the ends of the connecting-rod, however, are operated in a boundary lubricating condition like usual piston pumps.⁽³⁾ Then, it is necessary to consider the effect of friction on the motion.

The frictional condition at the small end of the connecting-rod changes from kinetic friction to static friction when $\phi_x + \varphi = \text{const.}$, i.e.,

$$\frac{d\phi_x}{d\tau} + \frac{R}{C} \frac{d\varphi}{d\tau} = 0 \quad (4)$$

as shown in Fig.5. On the contrary, transition from static friction to kinetic friction occurs when the next relation is valid, i.e.,

$$\left| M_s + \frac{a}{L} \bar{F}_s \right| > \frac{R}{L} |\bar{F}_{1s}| \quad (5)$$

and this is derived from consideration of the moment around the small end. $\bar{F}_s$ is the frictional force shown in Fig.6 and by using a coefficient of friction f_s , is given by

$$|\bar{F}_s| = \bar{p}_s \pi f_s R / (2L \cos \varphi) \approx \bar{p}_s \pi f_s R / (2L) \quad (6)$$

and suffix s shows static friction.

Table 1 Dimensions of test rig (unit: cm)

$l = 6.8$	$E = 0.45$
$l_1 = 10.0$	$R_1 = 0.70$
$l_2 = 5.0$	$R_2 = 1.10$
$d_1 = 1.4$	$R = 1.099$
$d_2 = 1.1$	$C = 2.5 \times 10^{-2}$
$H_0 = 0.475$	$L = 5.72$
$R_c = 5.0$	$a = 0.346$

Fig.2 Variation of φ with τ

Fig.3 Set-up of gap transducer electrode

Fig.4 Example of test results

For the bearing at the large end, transition from kinetic friction to static friction occurs when $\angle O_2BD = (\pi/2) - \theta + \varphi + \tan^{-1}(d_1/L_1) = \text{const.}$, i.e.,

$$\frac{d\varphi}{d\tau} - \frac{d\theta}{d\tau} = 0 \quad (7)$$

On the contrary, transition from static friction to kinetic friction occurs when the following conditions are valid: In case of static friction at the small end;

$$\left| \frac{\pi R l}{2L^2} \varphi \bar{p}_b + \bar{M}_z - \frac{l-a}{L} \bar{F}_y \right| > \frac{R_2}{L} |\bar{F}_{2x}| \quad (8)$$

In case of kinetic friction at the small end;

$$\left| \frac{\pi R l}{2L^2} \varphi \bar{p}_b - \frac{l}{L} \bar{F}_y - \frac{R_1+l}{L} \bar{F}_1 - \frac{l}{L} \bar{F}_1' \right| > \frac{R_2}{L} |\bar{F}_{2x}| \quad (9)$$

$\bar{F}_2$ is given by

$$|\bar{F}_2| = \bar{p}_b f_2 R / (2L) \quad (10)$$

where f_2 is a coefficient of friction. Considering balance of frictional force acting on the connecting-rod, we obtain

$$\bar{F}_1' = (R_2 \bar{F}_2 - (R_1 + l) \bar{F}_1) / l \quad (11)$$

The direction of frictional force is given as follows:

$$\bar{F}_1 \geq 0 \text{ according to } d\bar{\phi}_x/d\tau + (R/C) d\varphi/d\tau \geq 0$$

$$\bar{F}_2 \geq 0 \text{ according to } d\theta/d\tau - d\varphi/d\tau \geq 0$$

The motion of the piston is, therefore, described in four ways according to the frictional condition:

(1) in case of kinetic friction at both ends. From Eqs.(22) and (23) of the 1st. report, we obtain

$$2B \left(\frac{RQ}{V} \right) \frac{d\bar{\phi}_x}{d\tau} = \frac{R}{L} (\bar{W}_y + \bar{F}_y) - \bar{T}_z \quad (12)$$

$$2A \left(\frac{RQ}{V} \right) \frac{d\bar{Y}}{d\tau} - A \left(\bar{w}_0 + 3 \frac{\bar{p}_b - \bar{p}_a}{L/R} \right) \bar{\phi}_x = \bar{W}_y + \bar{F}_y \quad (13)$$

where inertia terms are omitted. Moreover, we obtain

$$\bar{F}_y = -(\bar{F}_1 + \bar{F}_1'), \quad \bar{T}_z = (R_1/L) \bar{F}_1 \quad (14)$$

(2) in case of static friction at the small end but kinetic friction at the large end. Because the piston and the connecting-rod move as a rigid body, we obtain

$$\bar{\phi}_x = K_1 - (R/C) \varphi \quad (15)$$

where K_1 is a constant decided by $\bar{\phi}_x$ and φ when the frictional condition at the small end changes to static friction. Eq.(13) is valid for $\bar{Y}$.

(3) in case of kinetic friction at the small end but static friction at the large end. Because the connecting-rod and the lever move as a rigid body, we obtain

$$\varphi = K_2 + \theta \quad (16)$$

Table 2 Test conditions

n rpm	470	960	1460
Ω rad/s	49.2	100.5	152.9
$\bar{p}_b$			
$\bar{p}_b = 30$ atg	—	1.243	0.819
$\bar{p}_b = 40$	3.39	1.659	1.091
$\bar{p}_b = 50$	—	2.07	1.365

Fig.5 Relationship between ϕ_x and φ

Fig.6 Frictional force acting on connecting-rod

Fig.7 Variation of calculated $\bar{Y}$ and $\bar{\phi}_x$ with τ

Fig.8 Comparison of theory and experiment on fluctuation of fluid film thickness

where K_2 is a constant decided by θ and φ when the frictional condition at the large end changes to static friction. Eq.(12) is valid for $\bar{\phi}_x$. On the contrary, as the small end moves around the point Q_3 , $\bar{Y}$ is given by

$$\bar{Y} = \bar{Y}_0 - (\theta - \theta_0) \frac{R}{C_1} \frac{l-a}{R} - \frac{\sqrt{l_1^2 - d_1^2}}{R} \sin\left(\theta - \tan^{-1} \frac{d_1}{l_1}\right) \quad (17)$$

where $\bar{Y}_0$ and θ_0 are $\bar{Y}$ and θ when the frictional condition at the large end changes to static friction.

(4) in case of static friction at both ends. Eqs.(15)-(17) are valid in this case.

Using $V = |\omega_0|_{\max} = 0.82 \Omega$ (cm/s), $\Omega = 2\pi n/60$ (rad/s) as the characteristic value, we obtain values shown in Table 2. In the following calculations it is assumed that $f_1 = f_2 = f_0$ and that there is no difference in f_0 between kinetic and static friction.

Fig.7 shows the calculated results of $\bar{Y}$ and $\bar{\phi}_x$ when $n = 960$ rpm, $p_0 = 40$ atg. In this figure the sign of the value $d\bar{\phi}_x/d\tau + (R/C)\bar{\phi}_x$ is shown, because static friction at the small end occurs when the sign changes. Static friction at the large end occurs at $\tau = 0.0$ and 3.3 . The value of f_0 , 0.70 is theoretically derived from the condition that $\bar{\phi}_x$ changes periodically, and the value of $\bar{\phi}_x$ at $\tau = 0$ is also decided from the same condition about $\bar{Y}$. The value of $\bar{Y}$ at $\tau = 0$ is given as an initial condition, but in this figure it is chosen such that $(\bar{Y})_{\tau=0} = -0.5$, because we are examining only a relative value. From this result it is clear that the contact between the piston and the cylinder wall can be prevented if frictional force is of a suitable value.

C_1 and C_2 are expressed by $\bar{\phi}_x$ and $\bar{Y}$ as follows.

$$C_1 = C[1 - \bar{Y} - (L_1/R)\bar{\phi}_x]$$

$$C_2 = C[1 - \bar{Y} - (L_2/R)\bar{\phi}_x]$$

where L_1 and L_2 are the distances between the electrodes and the piston center and given by

$n = 960$ rpm, $p_0 = 40$ atg, $f_0 = 0.70$

Fig.9 Comparison of theory and experiment on fluctuation of fluid film thickness

$$L_1 = 1.74 - 0.82 \cos \tau \quad (\text{cm})$$

$$L_2 = -1.26 - 0.82 \cos \tau \quad (\text{cm})$$

Figs. 8-12 show the comparison of the theory and the experiment on the fluctuation of C_1 and C_2 , where the scale of the axis of ordinates shows a relative value. Figs.8-10 show the effects of the rotational speed on the fluctuation, and Figs. 9, 11 and 12 show the effect of the pressure. Theoretically, these effects are expressed in $\bar{P}_b$ and as $\bar{P}_b$ increases, the fluctuation of the fluid film increases.

$n = 1460$ rpm, $p_0 = 40$ atg, $f_0 = 0.72$

Fig.10 Comparison of theory and experiment on fluctuation of fluid film thickness

$n = 960$ rpm, $p_0 = 50$ atg, $f_0 = 0.70$

Fig.11 Comparison of theory and experiment on fluctuation of fluid film thickness

$n = 960$ rpm, $p_0 = 30$ atg, $f_0 = 0.70$

Fig.12 Comparison of theory and experiment on fluctuation of fluid film thickness

Fig.8 shows the case $n=470\text{rpm}$ and $p_b=40\text{atg}$. Qualitatively the calculated values coincide with the test results, but quantitatively there are considerable discrepancies. That seems to be mainly caused by the contact between the piston and the cylinder wall. To clarify the contact, the electric resistance between the piston and the cylinder-block was measured. As the result, the contact was recognized during the greater part of its one period, but not in case of $n=960$ and 1460 rpm .

The calculated values coincide fairly well with the experimental results in Figs.9-12. The high frequency fluctuations included in the test results seem to be caused by the fluctuation of f_o and contaminants in oil. The value of f_o shown in figures is greater than the usual coefficient of boundary friction between steel and steel. This is considered to come from the fact that, in the experiments, no pressurized oil was led to the spherical bearings. (If oil pressurized to p_b was led, fluid lubrication would not be realized but the condition of boundary lubrication would differ from the case without a pressurized oil.)

5. Conclusions

To verify the theoretical analysis, some experiments on motion of a piston were done, and the following results were obtained:

- (1) It is possible to keep a piston from coming into contact with a cylinder wall, if the coefficient of friction at the both ends of a connecting-rod is of a suitable value.
- (2) The calculated values including the above mentioned friction coincide fairly well with the test results.

The stabilizing methods described in the previous paper and this, however, are not easy to realize in actual piston-type hydraulic machines, then other methods for stabilization will be discussed in the following report.

References

- (1) Yamaguchi, A. and Tanioka, Y., Bull. of the JSME, Vol.19 No.130(1976-4), p. 402.
- (2) Soda, N., Friction and Lubrication (in Japanese), (1954), p.262, Iwanami.
- (3) Yamaguchi, A., J. Japan Hydraulic Pneumatic Soc., Vol.1, No.1 (1970-10), p.10