Non-Steady Characteristics of Orifices

By Atsushi YAMAGUCHI**

The non-steady characteristics of orifices have been analyzed on the assumption that the free streamline forms an ellipse on the hodogram.

The coefficient of contraction is greater than that for steady flow during the period of acceleration, and smaller during the period of deceleration. In sinusoidal pulsating flow, the mean value tends to be slightly smaller than the steady flow value.

The opposite is true for the coefficient of discharge, defined by the instantaneous flow rate and the pressure difference.

The experimental results, in which the amplitude of the pulsating flow rate was less than 0.5 and frequency less than 50 Hz, agreed well with the theoretical calculations.

1. Introduction

To design fluid power systems it is sometimes important to understand the non-steady characteristics of orifices. In measuring a pulsating flow rate, the time averaged values are often related experimentally to the "root square error" of the pressure difference $\Delta p$. This approach is based on one-dimensional quasi-steady flow conditions assuming that the streamlines do not change with time.

However, to deal with instantaneous values of flow rates one-dimensional unsteady flow analyses are involved. In these cases the concept of the equivalent length of an orifice is introduced $(2)$, $(4)$-$(6)$, but even this technique is approximate and not necessarily suitable for predicting the flow characteristics of orifices.

The purpose of this paper is to analyze the unsteady flow of incompressible fluid through an orifice by applying the free stream line theory and to verify the result experimentally.

There are several papers which apply the free stream line theory to unsteady flow $(7)$, $(8)$, and there is another dealing with numerical methods of solution $(11)$. There seems to be, however, no study which applies this theory to unsteady flow through a restrictor. The method of this paper is different from the previous studies.

2. Nomenclature

\begin{align*}
D & : \text{pipe diameter} \\
F & : \text{complex potential} = U L F \\
i & = \sqrt{-1} \\
:\text{coefficient in Eq. (36)} \\
L & : \text{characteristic length} \\
L w U & : \text{Strouhal number} \\
\Delta p & : \text{pressure} \\
\Delta \phi & : \text{pressure difference} = (p/2) U^2 \\
\psi & : \text{strength of source} = U L \phi \\
Q & : \text{flow rate} = -U L Q \\
\gamma & : \text{radius in} \xi \text{ plane} \\
R & : \text{radius of free stream line in} \zeta \text{ plane} \\
\tau & : \text{time} = \tau / w \\
\phi & : \text{velocity component in} \xi \text{ direction} \\
\psi & : \text{velocity component in} \zeta \text{ direction} \\
U & : \text{characteristic velocity} \\

\end{align*}

\begin{align*}
w & : \text{complex velocity} = (a - i b) U = a U \\
\alpha & : \text{co-ordinate} = L x \\
\beta & : \text{co-ordinate} = L y \\
\epsilon & : \text{approximate fluid} \\
\alpha & : \text{angle in} \gamma \text{ plane} \\
\beta & : \text{fluid density} \\
\tau & : \text{angular speed of pulsation} \\
\phi & : \text{velocity potential} = U L \phi \\
\psi & : \text{stream function} = U L \phi \\
\omega & : \text{angular frequency of pulsation} \\
\beta & : \text{average value of a period of pulsation} \\
\epsilon & : \text{steady state value corresponding to average flow rate, or initial flow rate} \\
\gamma & : \text{nondimensional quantity} \\
\beta & : \text{theoretical analysis} \\
\end{align*}

We consider a two-dimensional irrotational motion of the flow through a restrictor. We assume an incompressible fluid, then, both velocity potential $\phi(x, y, \tau)$ and stream function $\psi(x, y, \tau)$ satisfy the Laplace equation.
From the Bernoulli equation extended to unsteady flow, we obtain
\[ z \left( \frac{\partial U}{\partial t} + (pU)\frac{\partial U}{\partial z} + \frac{\partial p}{\partial z} \right) = \frac{\partial \Phi}{\partial z} - 2 \rho \]  
(1)

where the effect of gravity is omitted, and \( \Phi = \Phi(z) \) is an arbitrary function. On the free stream line, \( p = 0 \), then
\[ \frac{\partial \Phi}{\partial t} = 2 \rho \]  
(2)

The boundary conditions are:
1) At the upstream surface of the restrictor (AB in Fig. 1)
\[ \frac{\partial \Phi}{\partial z} = 0 \]  
(3)

2) The flow is symmetric with respect to axis.

3) If the free surface is given by
\[ G(x, y, z) = 0 \]  
(4)

the following equation must be satisfied.
\[ \frac{\partial G}{\partial t} + \frac{\partial (U G)}{\partial x} + \frac{\partial (V G)}{\partial y} + \frac{\partial (W G)}{\partial z} = 0 \]  
(5)

The condition 3) is approximately satisfied by using \( \Phi = \Phi(x, y, z, \tau) - \Phi(x, y, z) \) for the free stream line, where \( \Phi(x, y, z) \) is an arbitrary function.

Now, we analyze this flow using the hodograph plane. We assume that the velocity along the free stream line is expressed by an ellipse on the hodograph, that is, by taking advantage of symmetry, the flow shown in Fig. 1 is given by the flow in one quadrant of an ellipse as shown in Fig. 2. This treatment will correspond to the case where the effect of unsteadiness is not severe, because the free stream line for steady flow is given by a circle on the hodograph.

The relationship between the physical plane (z plane) and the hodograph plane (w plane) is given by
\[ z = \int \frac{dw}{w} \]  
(6)

We map the w plane on the z plane by a conformal transformation
\[ w = Re^{it} \]  
(7)

where R is an unknown parameter. We transform it again into the T plane by
\[ T = \frac{z}{R} + (a/T) \]  
(8)

and then, transform it into the \( \zeta \) plane by
\[ \zeta = R^{-1} \]  
(9)

where \( \zeta \) and \( R \) are also unknown parameters. Finally, the flow in the w plane is transformed to the flow within one quadrant of a circle of radius \( R \) in the \( \zeta \) plane (Fig. 3). Here, both \( R \) and \( a \) are functions of time, but this method is applicable because time is included as a parameter in the conformal mapping theory.

By the method of images, the flow in the \( \zeta \) plane corresponds to the case with a source at the origin and sinks at the points \( \pm R \). We use the same characteristic value as that for steady flow. That is, in case of the pulsating flow, we take the velocity at the vena contracta for the time average flow rate as equal to unity. Therefore, \( R = 1 \) for this characteristic velocity, the flow rate is \( \Pi \), and the width of the restrictor is \( \epsilon + 2 \) in the dimensionless form.

The complex potential in the \( \zeta \) plane is given by
\[ \Phi = \Phi_0 - \ln(\zeta + R)/(\zeta - R) + i \gamma \]  
(10)

and the free stream line is given by
\[ \zeta = Re^{i\gamma} \]  
(11)

From Eqs. (7) - (9) there is the relationship between the w plane and the \( \zeta \) plane such that
\[ \omega = R^2 e^{i\gamma} \]  
(12)

Then, from Eqs. (6), (10), (12) we obtain
\[ \omega = \Phi \left[ \frac{1}{R} + \frac{1}{R} \right] \]  
\[ + i \Phi \left[ \frac{1}{R} - \frac{1}{R} \right] + i \gamma \]  
(13)

where \( \Phi \) is an arbitrary function of \( \tau \). As \( \zeta \) is expressed by
\[ \zeta = Re^{i\gamma} \]  
(14)

then, by substituting Eq. (14) into Eq. (13) and separating into the real part and the imaginary part, we obtain
\[ x = \frac{1}{R} \cos \theta + \frac{1}{R} \cos \theta + \frac{2}{R} \cos \theta \]  
\[ + i \frac{1}{R} \cos \theta + 2 R \cos \theta \]  
(15)

Fig. 1 z plane

Fig. 2 Hodograph plane

Fig. 3 \( \zeta \) plane
\[ y = \frac{1}{R} \left( \frac{\sin \theta}{R^2 - r^2} \right) + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \]

\[ y = \frac{1}{R^2} \left[ -\sin \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right] + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \]

From the boundary condition, the real axis \( \theta = 0 \) corresponds to the flow along the \( x \) axis in the \( z \) plane, then \( y = 0 \) in Eq. (16) when \( \theta = 0 \). That is, we obtain \( y = 0 \). Also, the imaginary axis \( \theta = \pi/2 \) corresponds to the flow along the \( y \) axis (the flow along \( AB \) in Fig. 1), then \( \bar{y} = 0 \) in Eq. (15) when \( \theta = \pi/2 \). That is, \( \bar{y} = 0 \).

The free streamline is derived from Eqs. (15), (16) by putting \( r = R \). Then,

\[ \bar{y} = \frac{R}{R^2} \left[ \cos \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right] \]

\[ \bar{y} = \frac{R}{R^2} \left[ \cos \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right] \]

The free streamline is drawn from the edge of the boundary (point \( B \) in Fig. 1) on the \( z \) plane. Therefore, \( \bar{y} = 0 \) and \( \bar{y} = (R/2) \)

\[ \frac{\theta}{\bar{y}} = \frac{\pi + 2}{\pi - 2} \]

The coefficient of contraction \( c_2 \) is defined as the ratio of \( \bar{y} \) at \( \bar{z} = \infty \) to \( \bar{y} \) at \( \bar{z} = 0 \), that is,

\[ c_2 = \frac{R}{R^2 - R^2} \]

\[ c_2 = \frac{R}{R^2 - R^2} \]

If we substitute \( R = \frac{y}{R} \) into Eq. (24), we obtain

\[ \frac{R}{\bar{y}} = \frac{1}{R^2} \left( \cos \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right) \]

An \( R \) and \( \bar{y} \) are independant of \( \theta \), the relationship \( R = \bar{y} \) is not true in the case of \( L \omega / \gamma + \bar{y} \), that is, for unsteady flows. We may assume, however, that the ratio \( R / \bar{y} \) is close to unity. Then, we write

\[ R / \bar{y} = 1 + \frac{1}{R^2} \left( \cos \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right) \]

and rewrite Eq. (24). That is,

\[ \frac{U}{2 \omega} \left( \cos \theta + \frac{1}{R^2} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right) \]

where

\[ \theta = \cos \theta \]

\[ \gamma = \cos \gamma \]

\[ \varphi = \frac{1}{R^2} \left( \cos \theta + \frac{1}{R} \tan \theta \cdot \frac{2R \sin \theta}{R^2 - R^2} \right) \]

In order that Eq. (26) can be satisfied regardless of \( \theta \) in the range of \( \theta = 0 \) to \( \pi/2 \), it is necessary to put all the coefficients in \( \varphi \) equal to zero when we expand the coefficients of Eq. (26) which include \( \varphi \) into a power series of \( \varphi \) and arrange them with respect to \( \varphi \). From the relation of unknown numbers between equations and equations, however, only one condition is available for this purpose. Then, we adopt an approximate treatment. That is, we multiply both sides of Eq. (26) by \( \varphi \) and express \( \varphi \) as follows:

\[ \theta = \theta_0 + \delta \theta (1 + 0.348) \]

where \( \theta_0 \) and \( \delta \) are constants. By using optimal approximation (mini-max approximation), we obtain

\[ \theta_0 = 1.97, \delta \theta = 4.65, \delta = -6.50 \]

As shown in Fig. 4, the errors due to this treatment are \( 10 \sim 15\% \) of the maximum value of \( 10 \delta \theta \).

By substituting Eq. (26) into Eq. (26) and equating the coefficients in \( \theta \) and \( \theta^2 \) to zero, we can derive the equation for \( \alpha \) and \( \beta \). That is, \( \alpha \) and \( \beta \) are the following equations.

\[ \alpha = 0 \]

\[ \beta = 0 \]
Now, let us derive the pressure at the point \( x = x_p, \dot{y} = 0 \). From Eq. (15), the relationship between \( r \) and \( x_p \) is given and if we omit terms of higher order than \( r/R \), we obtain

\[
r = \left( \frac{x_p}{y} \right) + \sqrt{\left( \frac{x_p}{y} \right)^2 + 4 \left( \frac{R^2}{y^2} \right)} \quad (4/R^2) + (2R^2)^{-1/2} \]

From the velocities at the point \( x = x_p, \dot{y} = 0 \) we obtain

\[
(\dot{r})^2 = \left( \frac{rR^2}{r^2 + ar^2} \right) = r^2 \left( 1 - \frac{a}{R^2} \right) \quad (33)
\]

and

\[
\dot{y} = 2r \dot{R} \left( \frac{1}{R} \right) = 2 \left( \frac{R}{r} \right) \left( \frac{R}{R} \right) = 2 \left( \frac{R}{r} \right) \left( \frac{R}{R} \right) \quad (34)
\]

By substituting the above equations into Eq. (1), we can obtain the pressure at the point \( x = x_p, \dot{y} = 0 \).

With regard to the flow coefficient, we adopt in this report the instantaneous coefficient of discharge defined by

\[
c = d \frac{\sqrt{3}}{R^2} \dot{R} \quad (35)
\]

Then, we discuss these results by numerical examples. In order to clarify the effect of unsteadiness on the characteristics of restrictors, calculated values are shown as ratios; the values for unsteady flow being divided by those for steady flow. In these calculations we set

\[
q = 1 - \cos \theta, \quad L = 0, U = 1 \quad (36)
\]

unless otherwise stated. Fig. 5 shows the change in \( \alpha \) during one period of pulsation. Fig. 6 shows the ratio of the coefficients of contraction \( C_{c1}/C_{c2} \). The ratio \( C_{c1}/C_{c2} \) changes in the same manner as \( \alpha \) and increases with the flow rate. As \( \alpha \) increases, the mean value of \( C_{c1}/C_{c2} \) decreases only slightly. Fig. 7 shows the transient response of \( C_{c1}/C_{c2} \) to the change of flow rate given by

\[
t = 0 \quad \rightarrow \quad q = 1
\]

\[
0 < t \leq \pi/2 \quad \rightarrow \quad q = 1 + \sin t
\]

\[
t = \pi/2 \quad \rightarrow \quad q = 1 + 1
\]

It is clear that \( C_{c1}/C_{c2} > 1 \) at \( q > 0 \) and \( C_{c1}/C_{c2} < 1 \) at \( q < 0 \). This means that in the former case, a loss in the restrictor is smaller than for steady flow and vice versa in the latter case. This explains the experimental results of Dailey et al. (30)

Fig. 8 shows the change of \( \partial \dot{y}/\partial \dot{x} \). In the calculation of \( \partial \dot{y}/\partial \dot{x} \), we use \( \dot{x} = 0 \) as the downstream pressure, and the pressure at

\[
x = -\dot{x}/2 \quad (k = 1-4) \quad (38)
\]

as the upstream pressure. In this figure the results for \( k = 1, 2, 3, \) and 4 are shown, but for \( k = 2, 3, \) and 4 intermediate values are obtained. \( \dot{y} = 1 \) is given by

\[
\dot{y} = 1 - \left( \frac{\dot{x} + \sqrt{\dot{x}^2 + 8}}{16} \right) \quad (39)
\]
Figs. 9–11 show the ratio of the instantaneous coefficients of discharge $c_i/c_s$, $c_i/c_s < 1$ at $\frac{\dot{u}}{U} > 0$ and $c_i/c_s > 1$ at $\frac{\dot{u}}{U} < 0$. The pressure required to accelerate or decelerate the fluid surrounding the restrictor is included in the definition of Eq. (35). Fig. 9 shows the effect of the measuring position of the pressure on $c_i/c_s$. As $k$ (that is, $\frac{\dot{u}}{U}$) increases, the fluid mass surrounding the restrictor increases, then, the change in $c_i/c_s$ increases. The mean value of $c_i/c_s$, that is, $\bar{c}_i/c_s$ increases also. Fig. 10 shows the effect of Strouhal no. and Fig. 11 shows the effect of amplitude of flow pulsations.

Fig. 12 shows the relation between $\frac{\delta p_m}{\delta p_b}$ and $\chi$. In measuring a pulsating flow rate, $\frac{\delta p_m}{\delta p_b}$ is used as the correction factor to the root square error and is given by

$$\frac{\delta p_m}{\delta p_b} = 1 + 0.5\chi$$

In this theory the effect of Strouhal no. is insignificant in the range of $\frac{\dot{u}}{U} \leq 1$, and the value of $\frac{\delta p_m}{\delta p_b}$ almost agrees with Eq. (40). Fig. 13 shows the relation between $\frac{\sqrt{\delta p_m}}{\delta p_b}$ and $\chi$. In this case the effect of Strouhal no. is clearly recognized and we can not calculate $\delta p$ for larger values of $\chi$ than that shown in the figure. In measuring a pulsating flow rate Schiltz–Grunow (9) showed experimentally that we could treat the flow as a quasi-

![Image](attachment:image1.png)

Fig. 7 Transient response of coefficient of contraction

![Image](attachment:image2.png)

Fig. 8 Pressure difference (effect of $k$ in Eq. (38))

![Image](attachment:image3.png)

Fig. 9 Instantaneous coefficient of discharge (effect of $k$ in Eq. (38))

![Image](attachment:image4.png)

Fig. 10 Instantaneous coefficient of discharge (effect of Strouhal no.)

![Image](attachment:image5.png)

Fig. 11 Instantaneous coefficient of discharge (effect of amplitude of flow pulsation)

![Image](attachment:image6.png)

Fig. 12 Mean value of pressure difference

![Image](attachment:image7.png)

Fig. 13 Mean value of square root of pressure difference
steady flow if Strouhal no. \( \text{d}(\omega/2\pi)/u \) was less than 1/510 (where \( u \) =mean velocity in a pipe). From the result in Fig.13, however, we can neglect the effect of unsteadiness in case of much larger Strouhal numbers. The difference due to \( k \) is insignificant in Figs. 12 and 13 (in the range of \( k = 1 \sim 4 \)).

4. Experiment

The above theory was derived for two-dimensional flow, but the experiment was done on JIS standard orifices (JIS 88762) meeting the ISO standard, as there seemed to be no particular difference between the effects of unsteadiness on two-dimensional flows and on axi-symmetrical flows, and because practical interest mainly concerns axi-symmetrical flows. The test orifice was set up at the end of the pipe line as shown in Fig.14.

Two orifice plates were used in the experiment, one being \( \beta = 0.45, c_\alpha = 0.628 \) and the other \( \beta = 0.55, c_\alpha = 0.643 \). \( c_\alpha \) was obtained by the method described later. The orifice set up to measure flow rates had \( \beta = 0.40 \), and the vena contracta in the test section was adopted to both orifice meters. To generate pulsating flows, a piston in an air cylinder was driven by an electro-hydraulic servo valve. Two air chambers were set up for straightening and prevention of the propagation of the pulsation to the upstream side. There was a straightening nozzle in one chamber and filter elements (pore sizes 5 and 10 \( \mu \text{m} \)) were set up at the end of the pipes in chamber 2. The steady state velocity distribution on the horizontal and the vertical diameter at the end of the test pipeline (the point where the test orifice meter set up) was checked and found to conform with the one-seventh law.

A strain gauge type pressure transducer was used to measure the pressure upstream of the test orifice (capacity 20 g/cm², resonant frequency 500 Hz), and a constant-temperature type hotwire anemometer was used to measure velocities (probe: X and I type, tungsten wire, dia. 5 \( \mu \text{m} \), effective length 1.0 mm). Both were calibrated under steady state conditions, and the latter was calibrated for every experimental run.

The flow rate was calculated as follows: the outlet area of the uniform orifice was divided into \( \pi \) equal annular areas \( (n = 7 \text{ for } \beta = 0.45, n = 10 \text{ for } \beta = 0.55) \), then the axial velocity was measured at the point which divided each annular into two equal areas. In the case of the central region, the velocity at the center of the orifice was used. Hence, the number of measuring points was 2\( k = 1 \). Then, the velocities at the same distance from the center of the orifice were averaged. By multiplying this mean velocity by the relevant annular area and summing up, the flow rate was obtained. This flow rate coincided with the flow rate obtained by the orifice to measure mean flow rate within an error of 2\%. (This comparison was done to mean flow rate in case of pulsating flows.)

Table 1 shows the experimental conditions. \( \varepsilon \sim \varepsilon_\alpha \) are the amplitudes of the pulsation defined by Eq. (41).

\[
Q = \frac{\varepsilon}{e} \left( \frac{\varepsilon + \varepsilon}{e} \cos 2\theta + \varepsilon \sin 2\theta \right)
\]

(41)

The characteristic velocity \( \bar{U} \) in the Strouhal no. is derived from the velocity at the vena contracta, then

\[
\bar{U} = \frac{\varepsilon}{\varepsilon_\alpha} \frac{1}{\bar{U}}
\]

(42)

and the characteristic length \( L \) is given by

\[
L = \frac{d}{(2 + c)}
\]

(43)

where \( b = d \).

5. Discussion

Figs.15 and 16 show the comparison of the theory with experiment for \( \Delta P/\Delta B \) and \( C/c_\alpha \), respectively. Theoretical values were calculated using the flow rate and the Strouhal no. given in Table 1 and assuming \( k = 2 \). The experimental results are shown at intervals of one-twelfth of one period.

![Schematic diagram of test circuit](image)

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**Table 1 Experimental conditions**

<table>
<thead>
<tr>
<th>No.</th>
<th>( \beta )</th>
<th>( \text{vol} )</th>
<th>( \text{cm}^3 )</th>
<th>( \text{cm}^3/\text{sec} )</th>
<th>( \text{L} )/( \text{L} )</th>
<th>( \varepsilon )</th>
<th>( \varepsilon_\alpha )</th>
<th>( \varepsilon )</th>
<th>( \varepsilon_\alpha )</th>
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</thead>
<tbody>
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<td>1</td>
<td>0.45</td>
<td>30</td>
<td>3.43</td>
<td>2.12 × 10^4</td>
<td>23.3</td>
<td>-0.34</td>
<td>-0.022</td>
<td>0.016</td>
<td>0.004</td>
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<tr>
<td>2</td>
<td>0.55</td>
<td>30</td>
<td>3.42</td>
<td>6.91</td>
<td>23.2</td>
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<td>0.007</td>
<td>0.002</td>
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<tr>
<td>3</td>
<td>0.45</td>
<td>30</td>
<td>1.39</td>
<td>3.17</td>
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<td>-0.33</td>
<td>-0.017</td>
<td>0.006</td>
<td>0.002</td>
</tr>
<tr>
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<td>30</td>
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<td>0.008</td>
</tr>
<tr>
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<td>0.45</td>
<td>30</td>
<td>1.35</td>
<td>3.19</td>
<td>22.3</td>
<td>-0.34</td>
<td>-0.005</td>
<td>0.018</td>
<td>0.012</td>
</tr>
<tr>
<td>6</td>
<td>0.55</td>
<td>30</td>
<td>1.38</td>
<td>3.22</td>
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<td>-0.497</td>
<td>-0.005</td>
<td>0.011</td>
<td>0.012</td>
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The agreement is reasonably good. For other cases shown in Table 1, similar agreement was obtained. Table 2 shows the comparison of the theory with experimental mean values. The agreement is very good for \( \left( \frac{\Delta P}{\Delta t} \right) \text{mean} / \Delta P_0 \), being unaffected by errors in measuring flow rates. It is frequently convenient to use flow coefficients referred to mean flow rates. \( C_{pb} \) is defined by using the mean flow rate and the square of the root of the differential pressure, that is,

\[
C_{pb} = c_v \sqrt{\Delta P_0} / \Delta P_0 \quad \text{(44)}
\]

\( c_v \) is defined by using the mean flow rate and the square root of the mean differential pressure, that is

\[
c_v = \sqrt{\Delta P_0} \quad \text{(45)}
\]

From Eqs. (44) and (45), we obtain

Table 2 Comparison of theory with experiment

<table>
<thead>
<tr>
<th>No.</th>
<th>( \Delta P_{0a} )</th>
<th>( c_v \Delta P_{0a} )</th>
<th>( \Delta P_0 )</th>
<th>( c_v \Delta P_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.168</td>
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</tr>
<tr>
<td>2</td>
<td>1.098</td>
<td>0.848</td>
<td>0.864</td>
<td>1.000</td>
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<tr>
<td>3</td>
<td>1.035</td>
<td>0.843</td>
<td>0.864</td>
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<tr>
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<td>1.076</td>
<td>0.842</td>
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<td>5</td>
<td>1.090</td>
<td>0.843</td>
<td>0.863</td>
<td>1.000</td>
</tr>
<tr>
<td>6</td>
<td>1.015</td>
<td>0.841</td>
<td>0.863</td>
<td>1.000</td>
</tr>
<tr>
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<td>1.127</td>
<td>0.842</td>
<td>0.947</td>
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</table>

Fig. 15 comparison of theory with experiment

\[
\frac{\Delta P_{0a}}{\Delta P_0} = c_v \sqrt{\Delta P_0} / \Delta P_0 \quad \text{(46)}
\]

The correction factors in Eqs. (44) and (45) are given by the ratio of the differential pressures, and we can use the values in Figs. 12 and 13.

On the basis of one-dimensional unsteady flow theory, the differential pressure of a restrictor is given by

\[
\Delta P = \frac{1}{2} \rho c_t \left( \frac{A}{L} \right) \frac{\Delta P_{0a}}{\Delta P_0} \quad \text{(47)}
\]

where it is assumed that the coefficient of discharge does not change even if the flow is unsteady. In Eq. (47), \( A_t \) : restrictor area, \( L \) : equivalent length of restrictor. For an orifice plate of thickness \( t \), \( L \) is given by \( 0.044 \)

\[
L = 0.5 \times 10^{2} \quad \text{(48)}
\]

To compare this treatment with the present theory, we put into Eq. (48) \( \rho = 0 \) and \( \rho = \rho \). Then, we obtain from Eq. (47)

\[
\Delta P = \frac{c_v}{\pi} + \frac{16 \rho L}{\pi} \quad \text{(49)}
\]

For the numerical result shown in Fig. 8, \( \Delta P / \Delta P_0 \) calculated from Eq. (49) agrees well for the case of \( k = 2 \).

6. Conclusions

The non-steady characteristics of orifices have been analyzed on the assumption that the free streamline forms an ellipse on the hodograph, and the results compared with the experimental data. The following conclusions are drawn.

(1) The coefficient of contraction is greater than that for steady flow during the period of acceleration (increasing flow rate), and smaller during the period of deceleration. In sinusoidal pulsating flow, the mean value tends to be slightly smaller than the steady flow value.

(2) The opposite is true for the coefficient of discharge, defined by the instantaneous flow rate and the pressure difference.

(3) The experimental results, in which the amplitude of the pulsating flow rate was less than 0.5 and frequency less than 50 Hz, agreed well with the theoretical calculations.

(4) It was verified numerically that the unsteady flow characteristics of orifices can be predicted satisfactorily by using the concepts of equivalent orifice length and "root square error".

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Reference


(4) Mosely, D.S., ibid., p.103.