532.526

Turbulent Boundary-Layer along a Streamwise Bar of a Square Cross Section Placed on a Flat Plate*

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Turbulent boundary layer developing along a long streamwise bar with a square cross section placed on a flat plate was investigated experimentally. Measurements were made on the distributions of mean velocities, turbulence intensities and wall stresses. Main results obtained are as follows; Near the streamwise edge and corner, secondary currents are generated whose directions near the bisectors are away from and toward the wall respectively. The wall region still exists near the edge and the corner, but the logarithmic straight part in the defect law representation undergoes an upward or downward shift. The Preston tube measurements showed that the mean value of the wall stress in the region affected by the bar agrees well with that in the two-dimensional region, which makes it possible to calculate the total frictional drag of this wall from the flat plate formula. Usefulness of the approximated momentum integral equation in this case was examined using the experimental results.

1. Introduction

Streamwise corners are often encountered in many fluid conduits and in the transporting machines such as an air plane and a ship. Also for some cases in the heat augmentation techniques, it becomes necessary to adopt streamwise fins which inevitably form the streamwise corners. The flow along a long streamwise bar protruding from a wall is thus related to many interesting problems of engineering. A correct evaluation of the effects of a streamwise protrusion requires a detailed inspection of the flow near the protrusion, but so far this problem has not attracted the interest of the workers.

Streamwise corners are, in general, divided into those protruding outward and those dented inward. In the former the fluid occupies an angle larger than 180° (referred to simply as the "edge" in the rest of this paper) and in the latter the fluid occupies an angle less than 180° (referred to as the "corner"). In the practical flow geometries, the edge and the corner usually appear in pair, especially for the streamwise protrusions such as the heat transfer fins they stand close to each other, giving the flow a more complicated nature than those along a corner

or an edge alone.

The flow on the surface described above is essentially three-dimensional, and moreover when the flow is turbulent a secondary current of Prandtl's second kind arises near the corner or the edge. This secondary current which is recognized to originate from the non-uniformity of the Reynolds stress is inherent to the turbulent flow along a streamwise corner or edge and distinguished from those generated in a curved pipe due to the centrifugal force (the first kind) and those around an oscillating obstacle (the third kind).

Although the secondary current of the second kind is much smaller than the first kind, its effects on velocity, Reynolds stress and the wall stress distributions are considerable since it arises near the wall surface where the velocity gradient is very large.

Probably Nikuradse⁽¹⁾ was the first to investigate the flow accompanied with the secondary current considered here, making comprehensive experiments of the turbulent flow through pipes with various non-circular cross sections, including very peculiar ones. Since then, a relatively large number of works were made on the flow in ducts. In recent years, most of these⁽²⁾ concern the relation between the turbulent structure and the secondary currents, where the hot wire technique is effectively utilized for the measurements of the Reynolds stress components. Some researchers endeavor to attack the problem using multi-equation models of turbulence.

As for the boundary layer flow in this wall configuration, theoretical studies have been actively made by many workers for the laminar condition, although the predicted isotaches near the corner does not agree with the experimental re-

* Received 29th August, 1973.

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sults⁽³⁾ being obviously affected by the secondary flow whose direction is opposite to the turbulent condition.

Few investigations can be found on the turbulent case and the problem is far from being solved. The wall stress near a corner or an edge decreases or increases respectively, compared with that in the two-dimensional region far from them. In relation to the total drag of this wall shape, Gersten⁽⁴⁾ carried out the measurements of the boundary layer along a 90° corner and analyzed the results applying the momentum integral equations. Elder⁽⁵⁾ investigated the flow past a flat plate of finite width and proposed an empirical equation for the total drag, where he found a pair of secondary currents near the edge. The momentum integral equations which account for the effects of a secondary current were proposed by Eichelbrenner⁽⁶⁾ and Toan⁽⁷⁾. The wall and the defect laws of the velocity distribution in the corner bisector were derived in a paper of Bragg⁽⁸⁾.

Thus there are investigations of the flow along a corner or an edge, but the flows where an interference of the edge and corner occurs have not been examined yet. Considering these, the present study intends to clarify a general character of the flow along a streamwise protrusion by measuring the boundary layer developing along a long bar having a square cross section and placed such that its longitudinal axis coincides with the free stream direction. Measurements were made about mean velocity, wall stress and turbulence intensity distributions.

2. Nomenclature

x	: distance measured downstream from the leading edge
y	: distance normal to test plate
z	: distance normal to x - and y -directions measured from center line of test plate
z'	: distance in spanwise direction measured from side wall of the bar = $ z - b/2$
U, V, W	: mean velocities in x -, y - and z -directions respectively
U_1	: free stream velocity
u_T	: friction velocity
τ_w	: wall stress
δ^*	: displacement thickness
θ	: momentum thickness
h	: height of bar
b	: breadth of bar
A_w	: symbol indicating upper surface of bar
B_s, C_s	: symbols indicating side wall of bar
B_w, C_w	: symbols indicating flat plate surface
Π	: subscript indicating that variables are those in two-dimensional region far from bar

3. Experimental apparatus

For the purpose of this study it was considered essentially important to obtain a reasonably two-dimensional flow in a test section, since the effects of very slight secondary flows generated near the corner or the edge in the boundary layer have to be examined. A low speed open-circuit wind tunnel was designed and constructed for this experiment, carefully considering so far reported results⁽⁹⁾ on the factors affecting the two-dimensionality of the flow. The tunnel has an overall length of about 11 m as shown in Fig. 1. The area ratio of the diffuser part is almost 5. In the settling chamber a honey-comb and four screen gauzes were set. The honey-comb consisted of densely packed aluminum pipes of 30 mm in diameter and 150 mm in length. The mesh number and the wire diameter of the screen gauzes were properly chosen, considering the open area ratio. The mesh numbers and diameters employed were 0.683:8, 0.697:12, 0.697:12 and 0.649:18 respectively from upstream to downstream direction.

The contraction nozzle has a contraction ratio of about 6.5 and the curve was calculated from the equation proposed by Shima⁽¹⁰⁾ for the two-dimensional symmetrical nozzle. But the profile in the large end part was slightly modified to obtain an allowable total length. Special efforts were directed to a fine finishing of the surface of the small end part. A foundation paint was utilized for this purpose.

Just downstream from the nozzle exit which had a 336 mm x 1125 mm rectangular cross section, the test section shown in Fig. 2 was placed. On every side of its entrance tip, knife edges were mounted and in addition flap plates were attached to

Fig. 1 Open-circuit low speed wind tunnel.

Fig. 2 Test section and measuring plate.

the nozzle exit in order to stabilize the flow near the leading edges. The lower wall of the test section was an aluminum plate of 2.5 m long and 1.0 m wide which provides a measuring surface. The upper wall was made flexible by connecting eleven plates by hinges so as to be able to adjust the pressure gradient in the test section. Both side walls were made of lucite and were openable to facilitate probe setting.

The measuring plate used was provided by placing square (20 mm x 20 mm) cross-sectioned brass bar along the center line of the lower wall. A wooden blunt-nose body was set at the leading edge of the bar to avoid the flow separation there.

Measurements were made at four streamwise stations of $x = 650, 950, 1400$ and 1700 mm under the condition of a zero pressure gradient and a constant unit Reynolds number of $U_1/\nu = 1.40 \times 10^6$ 1/m which corresponds to the free stream velocities of $U_1 = 19 \sim 22$ m/s. Turbulence intensity in the free stream of the test section was about $0.3 \sim 0.4$ percent at $U_1 = 21$ m/s and the critical Reynolds number was about 8×10^5 .

Velocity measurements were made by a hypodermic tube with Göttingen manometer. The wall stress was measured by means of a Preston tube where the static pressure was obtained at the wall static hole of 0.4 mm in diameter. The measurements of turbulence intensity were done using a constant temperature hot wire anemometer.

4. Experimental results and discussion

Transverse variations of the non-dimensional velocity in the boundary layer obtained at $x = 950$ and 1700 mm on the flat plate with a trip wire are shown in Fig. 3. The deviation of the velocity from the mean value at each y -position does not go beyond ± 1 percent of the mean value. Not even a small scale distortion of the profiles could be observed, showing the flow was very nearly two-dimensional. On the plate without a trip wire, the flow was found to exhibit some instability in the transition region, but upstream or downstream from it the deviation was also less than ± 1 percent of the mean. The following results, therefore, could be considered to be obtained in a satisfactory two-dimensional

Fig. 3 Verification of two-dimensionality of the flow on a flat plate.

Fig. 4 Coordinate system and symbols.

Fig. 5 Static pressure distribution around the bar.

boundary layer.

As stated in the preceding section measurements were made at four streamwise sections, but the flow behavior in each section showed almost the same trend, hence mainly the results at $x = 1700$ mm will be described for convenience. In the following, each part of the measuring plate will be referred to by the symbol shown in Fig. 4, where z' shows the distance in the z -direction measured from the side wall of the bar.

4.1 Static pressure around a square bar

The static pressure distributions on the plate surface are shown in Fig. 5, where the reference pressure is one in the midst of A_w . The profiles are nearly symmetrical concerning the center line of the plate. A characteristic feature observed is a slightly high value at the midst of A_w and both corner points. Although the deviation is less than ± 0.25 percent of the dynamic pressure in the free stream, it is closely related to the secondary currents and their directions which will be mentioned in a later section.

4.2 Mean velocity fields and secondary currents

Velocity distributions U/U_1 vs. y at various transverse positions of A_w , B_w and C_w are shown in Fig. 6 (a) and (b), where the profile designated by the solid symbol is one at the corresponding station on the plate without a bar which was found to co-

incide well with that in the two-dimensional region on the plate with a bar. The streamwise variations of the profile at $z = 0$ mm of A_w and $z' = 5$ mm of C_w are shown in Fig. 7 and Fig. 8. Near the bar the profiles exhibit a two step variation due to the constraining effect of the side walls. At the transitional region near the bar height $h = 20$ mm there exists a narrow region where the velocity decreases with an increasing y . This can be interpreted as the effects of the secondary currents which transport the fluid with low velocity near the wall of A_w , B_s and C_s into these portions. Effects of the secondary currents on the velocity profile are also found in A_w where the velocity near the wall is very large, resulting in a small displacement thickness.

Isotach curves around the bar obtained from above described results are shown in Fig. 9(a) and (b) together with that at $x = 650$ mm. As shown in these figures,

Fig. 6(a) Velocity profiles on B_w and C_w .

Fig. 6(b) Velocity profiles on the upper surface of the bar.

the flow is nearly symmetrical concerning the center line. The distortions of these isovels suggest that the secondary currents near the bisectors direct toward the corner or the free stream in the corner or the edge region respectively. The effects of the secondary currents can be seen more distinctly on lines of a constant turbulence intensity shown in Fig. 10 which was obtained in the same section as in Fig. 9 (b). Distortions qualitatively similar to but more prominent than those in isovels are apparent in the vicinity of the wall.

Concerning the origin of these secondary currents, many plausible explanations have been made thus far, based on the momentum, vorticity and energy points of view. Despite of these, the question which of them describes the phenomena most properly seems to be open.

Fig. 7 Streamwise variation of the velocity profiles in the midst of A_w .

Fig. 8 Streamwise variation of the velocity profiles near the bar on C_w .

Hinze⁽¹¹⁾ proposed, using his experimental results in partly roughened rectangular ducts, that the turbulence energy equation be approximated by the following:

$$V \frac{\partial}{\partial y}(\bar{q}^2) + W \frac{\partial}{\partial z}(\bar{q}^2) = -\bar{u}v \frac{\partial U}{\partial y} - \bar{u}w \frac{\partial U}{\partial z} - \epsilon \dots (1)$$

where $\bar{q}^2$ and ϵ denote the total turbulence energy $(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)/2$ and the viscous dissipation respectively. Equation (1) implies that in a locally symmetrical flow region ($W = 0$), the direction of the secondary current (or the sign of V) can be determined depending on whether there is an excess production or dissipation of the turbulence energy. Application of Eq. (1) to the present results shows there is an excess dissipation in the corner region and conversely an excess production in the edge region.

The change in the secondary flow patterns due to the wall configuration and the boundary layer thickness on it is shown schematically in Fig. 11. In the left-most Elder's results⁽⁵⁾ for the plate of a finite width are combined with those in the corner boundary layer, since it can be con-

sidered an extreme case of the wall shapes investigated here. In the present case of a comparable protrusion height and width with the boundary layer thickness, the flow pattern does not differ from that in the corner boundary layer but the direction of the secondary currents in the symmetry plane of Aw is opposite to that of a plate with a finite width.

The direction of the secondary current inferred from the isovels are consistent with the distribution of the static pressures of Fig. 5, which shows a slightly high pressure at the midst of Aw and at both corner points where the implied secondary currents are considered to stagnate.

4.3 The lateral extent of the flow region affected by the bar

For the flow on the wall geometry considered here, the estimation of a lateral extent of the flow region affected by the bar is one of the important problems. The simplest way of evaluating it is to consider parameters δ_A , δ_C and δ_E defined as follows, i.e., δ_A , δ_C and δ_E are the lengths of the center line, the corner and edge bisectors respectively cut away by the wall and the isovels of $U/U_1 = 0.99$. As stated in the earlier section, the velocity profiles in the region far enough from the bar coincide well with those at the corresponding station on the flat plate without a bar. This region will be referred to as the two-dimensional region and variables in the region are identified with the subscript Π .

The streamwise variations of the parameters δ_A , δ_E and δ_C nondimensionalized by the boundary layer thickness in the two-dimensional region δ_Π are shown in Fig. 12,

Fig. 9 Isotach curves of velocities.

Fig. 10 Lines of constant $\sqrt{u^2}/U_1$, $x = 1700 \text{ mm.}$

Fig. 11 Secondary flow patterns for different wall geometries.

Fig. 12 Streamwise variations of δ_A/δ_Π , δ_E/δ_Π and δ_C/δ_Π .

which indicates almost constant values of δ_A/δ_H , δ_E/δ_H and δ_C/δ_H . Consequently, the lateral extents thus defined are nearly constant multiples of the boundary layer thickness in the two-dimensional region within the region of the present experiments, implying the bar would not be buried in the boundary layer.

4.4 On the law of the wall and the velocity defect law

It seems very interesting to investigate the applicability of the universal laws of velocity profile which are well established for the turbulent boundary layer on a flat plate to the profiles obtained here. The velocity profiles in A_w and B_w are plotted using parameters of the law of the wall in Fig. 13, where the friction velocities were those obtained by Preston tube. The curve in the figure shows the logarithmic straight line with the constants proposed by Sarnecki since Patel's formula was employed for the calculation of the wall stress τ_w from the Preston tube reading.

Pitot tubes used were the hypodermic tubes of a nominal outer-diameter of 0.6 mm. Wall-proximity correction of Pitot tube was made for the readings in the re-

gion of $y \leq 10$ mm, applying MacMillan's equation ⁽¹²⁾ $\Delta y = 0.15d$. The displacements of an effective Pitot tube center Δy were 31 and 1 percent at $y=0.30$ and 10.0 mm respectively.

The logarithmic straight part can be observed for all profiles including those very near the corner and edge points which were properly expected to undergo some deviations from the two-dimensional law of the wall. In this connection Hinze ¹¹ stated that the thickness of the wall region becomes smaller there where a secondary flow directs toward the wall. Following almost the same similarity considerations and making use of a characteristic velocity U_k which was determined from the wall stress distribution near the corner, Bragg ⁸ derived the following wall similarity law in the corner bisector plane.

$$U/U_k = G \log U_k y / \nu + G \log U_k z / \nu + H \dots (2)$$

where G and H are experimental constants.

According to Eq. (2), the logarithmic straight part of the profile undergoes a downward shift when the wall stress is decreased, i.e., when approaching the corner. In this experiment, as shown in Fig. 13, the systematic downward shifts expected from Eq. (2) could not be observed.

The same profiles are presented in terms of the velocity defect law parameters in Fig. 14, where the curve has the same slope as that in Fig. 13. The profiles in the far region from the bar well agree with this line, while the profiles near the bar exhibit marked upward or downward shift which formally corresponds to the two-dimensional boundary layer profiles with adverse or favorable pressure gradient respectively. The shape factor by Clauser

$$G = \int_0^\infty \left(\frac{U_1 - U}{u_\tau} \right)^2 dy \bigg/ \int_0^\infty \left(\frac{U_1 - U}{u_\tau} \right) dy$$

for the profiles in A_w was about 4, a value which has never been encountered in so far reported cases of the favorable pressure gradient.

4.5 Distributions of wall stresses

The distribution of the wall stresses τ_w obtained by traversing a Preston tube across the wall is shown in Fig. 15. Calculation of τ_w was made by the following Patel's formula,

$$y^* = 0.8287 - 0.1381x^* + 0.1437x^{*2} - 0.006x^{*3} \dots (3)$$

$$y^* = \log \tau_w d^2 / 4 \rho \nu^2$$

$$x^* = \log \Delta p d^2 / 4 \rho \nu^2$$

where d is the outer diameter

Fig. 13 Wall law representation of velocity profiles, $x = 1700$ mm.

Fig. 14 Defect law representation of velocity profiles, $x = 1700$ mm.

of Preston tube and Δp is the difference between the total pressure on Preston tube and the static pressure. Since most universal functions $y^* = f(x^*)$ including Eq. (3) have been verified for the two-dimensional boundary layer, the validity of Eq. (3) for the present case is not self-evident, but the results of Fig. 13 can be considered to justify adoption of Eq. (3).

In the figure a slight difference of the distributions between B_w and C_w is attributable to the lack of a strict symmetry of the flow. The wall stress τ_w is zero at the corner point, increasing abruptly with distance from the bar on the plate and approaching a constant value in the two-dimensional region. On the bar surface also observed is an abrupt increase in τ_w until it reaches the maximum value in the midst of A_w . The ratio of the maximum wall stress $(\tau_w)_{\max}$ to $\tau_{w\pi}$ at each streamwise station was almost constant at about 1.4.

Nondimensional distributions of τ_w on the plate near the bar were presented in Fig. 16 in the form of $u_\tau/u_{\tau\pi}$ vs. $u_{\tau\pi}z'/v$. As shown, each profile agrees well with each other and the logarithmic straight distribution of the same slope as those found by Bragg⁽⁸⁾ in the turbulent corner flow can be observed. The intercept of the line for present case, however, much differs from Bragg's. A small difference even exists between B_w and C_w , which shows a strong dependence of the position of the logarithmic straight distribution of τ_w on the wall geometry and the magnitude of the secondary current.

Fig. 15 Distribution of wall stresses, $x = 1700$ mm.

Fig. 16 Nondimensional distribution of friction velocities.

5. Total frictional drag

From the practical point of view it is important to compare the frictional drag of the present wall with a flat plate case. For the evaluation of the frictional drag of the wall investigated here, it becomes necessary to know the extent of the flow region affected by the bar. As stated earlier in section 4.3, it is possible to decide it from the isovels or the distribution of τ_w . Here it was considered more reasonable to determine it from the lateral distribution of the momentum thicknesses near the bar.

In Fig. 17 the momentum thickness θ divided by θ_π is plotted against $\log z'$. The profiles at various stations of x are seen to be approximated by straight lines intersecting rather sharply the line of $\theta/\theta_\pi = 1.0$. The values of z' at the intersection were adopted as the edge of the bar affected flow region and designated by δ_z . In the z -coordinate it equals to the range of $-\delta_z - b/2 \leq z \leq \delta_z + b/2$. The thickness thus determined almost coincides with δ_c or that obtained from the wall stress distribution.

The mean value of the local frictional coefficients over the above stated interval of z i.e.,

$$\bar{C}_f' = \frac{1}{2\delta_z + h + b/2} \times \int_{-(\delta_z + h + b/2)}^{(\delta_z + h + b/2)} \left(\tau_w / \frac{1}{2} \rho U_1^2 \right) ds$$

(ds is the line element along a wetted perimeter) was calculated from the distribution of τ_w such as Fig. 15. The results are plotted against Reynolds number in Fig. 18. In the figure the value denoted by the symbol $\bullet$ or $\odot$ shows that in the two-dimensional region and the symbol $\circ$ or $\ominus$ shows the mean local frictional coefficient $\bar{C}_f'$ defined above, where the value by the symbol $\circ$ or $\bullet$ was calculated by Patel's formula for τ_w and others by NPL formula. As seen from the figure, the mean frictional coefficient $\bar{C}_f'$ agrees well with the value in the two-dimensional region. Referring to Fig. 15, this result indicates that there happens to occur an increase of τ_w in the edge region which is almost equal to a decrease in the corner region. The curve in the figure shows Schoenherr's equation for the flat plate, thus implying it is more adequate to use NPL's formula for the calculation of the frictional drag.

Fig. 17 $\theta/\theta_\pi \sim \log z'$.

It is not difficult to infer that the total frictional coefficient $\overline{C_F}$ as an integrated value of C_f' in Fig. 18 will be almost equal to the flat plate value. Here the relation between the momentum loss and $\overline{C_F}$ will be discussed. The three-dimensional boundary layer equation in the x -direction

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = U_1 \frac{dU_1}{dx} + \frac{\partial}{\partial y} \left(\nu \frac{\partial U}{\partial y} - \overline{uv} \right) + \frac{\partial}{\partial z} \left(\nu \frac{\partial U}{\partial z} - \overline{uw} \right) \dots (4)$$

is integrated in the region affected by the bar, i.e., the region of $0 \leq y \leq \delta$ and $-\delta_z' - b/2 \leq z \leq \delta_z' + b/2$, where δ_z' is the edge of the bar affected region in the z -direction, resulting in the following equation.

$$\begin{aligned} \frac{d}{dx} (U_1^2 \Theta) + \Delta U_1 \frac{dU_1}{dx} - 2\Theta U_1 \frac{d\delta_z'}{dx} + 2U_1^2 \Theta_{xz} \\ = \int_{-(\delta_z' + b/2)}^{(\delta_z' + b/2)} (\tau_w / \rho) ds \\ - 2 \int_0^{\delta} \{ (\tau_{xz})_{II} / \rho \} dy \dots (5) \end{aligned}$$

where,

$$\Theta = \int_{-\delta_z' - b/2}^{\delta_z' + b/2} \int_0^{\delta} \frac{U}{U_1} \left(1 - \frac{U}{U_1} \right) dy dz$$

$$\Delta = \int_{-\delta_z' - b/2}^{\delta_z' + b/2} \int_0^{\delta} \left(1 - \frac{U}{U_1} \right) dy dz$$

$$\Theta_{II} = \left[\int_0^{\delta} \frac{U}{U_1} \left(1 - \frac{U}{U_1} \right) dy \right]_{z=\delta_z' + b/2}$$

$$\Theta_{xz} = \left[\int_0^{\delta} \frac{W}{U_1} \left(1 - \frac{U}{U_1} \right) dy \right]_{z=\delta_z' + b/2}$$

$$(\tau_{xz})_{II} = \left(\nu \frac{\partial U}{\partial z} - \overline{uw} \right)_{z=\delta_z' + b/2}$$

For the present case $dU_1/dx = 0$, therefore the second term in the left hand side of Eq. (5) does not contribute. In addition, the third term makes no contribution either since $\overline{C_F}$ is evaluated from the rectangular

Fig. 18 Mean local frictional coefficient.

wall surface of the length x' and the width $2\{(\delta_z')_{x=x'} + h + b/2\} = 2B$, whose center line coincides with that of the bar.

For convenience we replace δ_z' by δ_z in $x = x'$ and neglecting the fourth term in the right hand side and the second term in the left hand side of Eq. (5), we can obtain the following momentum equation similar to the two-dimensional flow:

$$\frac{d\Theta}{dx} = \int_{-B}^B (\tau_w / \rho U_1^2) ds \dots (6)$$

The coefficient $\overline{C_F}$ can be obtained by integrating Eq. (6) with respect to x .

$$\begin{aligned} \Theta/Bx &= \frac{1}{2Bx} \int_0^x \int_{-B}^B (\tau_w / \frac{1}{2} \rho U_1^2) ds dx \\ &= \frac{1}{x} \int_0^x \overline{C_f'} dx \dots (7) \end{aligned}$$

The approximated momentum equation (7) may be considered the first order approximation which neglects the momentum change taking place on both sides of the control surface enclosing the bar.

The usefulness of Eq. (7) was examined in Fig. 19, where $\overline{C_F}$ calculated from the velocity profiles (the left side of Eq. (7), denoted by the symbol $\odot$) is compared with that calculated from the wall stress distributions (the right hand side of Eq. (7), denoted by the symbols $\circ$ and $\otimes$) in the region except near the leading edge. The discrepancy is about 10 percent when Patel's formula was used but it was about 3 percent if NPL's formula was employed. These results combined with those in Fig. 18 show that the approximation made for the derivation of Eq. (7) is adequate.

6. Concluding remarks

Results of the present investigation on the turbulent boundary layer developing along a long streamwise bar with a square cross section placed on a flat plate are summarized as follows.

Near the streamwise edge and corner secondary currents are generated, whose directions near the bisector are away from and toward the wall respectively. Corresponding variation of the static pressure on the bar surface was observed.

Fig. 19 Total frictional coefficient.

The wall region where the velocity profile is governed by the logarithmic law still exists near the edge and corner, but the logarithmic straight part in the velocity defect representation undergoes an upward or a downward shift caused by secondary currents.

The Preston tube measurements showed that the mean value of the wall stress in the region affected by the bar agrees well with that in the two-dimensional region, which makes it possible to calculate the total frictional coefficient of this wall from the flat plate formula, taking account of a constant ratio of the extent affected by the bar to the boundary layer thickness in the two-dimensional region.

Usefulness of the approximated momentum integral equation (7) was examined using the experimental results. A more strict evaluation of the validity of the approximation requires a more accurate determination of the wall stress than that by Preston tube method.

Acknowledgment

The authors would like to express their thanks for the partial financial support of the HATAKEYAMA FOUNDATION in execution of this work.

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