621.9-52

On the Dynamic Behaviour of Feedback Controlled Feed Drive*

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In the present paper, to improve the controllability of machine tool feed drive, the dynamics is studied by the method of linear analysis. The results are represented by a special pole-zero locus graph by which the effects of the components on the dynamic behaviour of the drive can easily be understood. By means of this graph, the design of drive with higher performance is discussed and particular attention is paid to the problems caused by the low load damping in this class of drive. Also, the introduction of a specific tachometer feedback network is suggested.

1. Introduction

Ever since numerical control system was first introduced in machine tools, increasing efforts have been made to improve feed drives. In conventional feed drives, since tool movement is not in operation while positioning is in progress, the only decisive factors are positioning accuracy and positioning time. Efforts were therefore made to reduce the inertia masses in gear transmission, to develop more accurate leadscrews and to improve antifriction slideways properties⁽¹⁾⁽²⁾.

Recently there has been an increasing demand for machine tools capable of controlling tool movement while cutting is in progress. Typical examples of workpieces for which the machining requires such equipment are, turbine blades, propellers, casting moulds and forging dies. The feed drive of the machine tools now being demanded must have a much more severe dynamic performance, because workpieces accuracy is more directly affected by the transient response of the drive system than by the feed drive with only positioning controls.

Thus, more efforts were rendered to investigate problems⁽³⁾ arising in controlled feed drive for NC machine tools and to develop faster feed motors⁽⁴⁾ with higher response. Also, for machine tools operating with position feedback, an investigation⁽⁵⁾ into the stability was described with the consideration of compliance and backlash in the feed drive.

However, as the properties of the whole positional servomechanism are almost solely determined by the properties of the feed drive mechanism, to constitute the most suitable drive system the relation between dynamic behavior of the drive system and function of its assembled elements

must first be studied in detail on a linearized model.

The purpose of this paper is to illustrate the dynamic performance of the feed drive mechanism with a change in the following parameters: load on the table; gear ratio; damping factors at the load and in the hydraulic motor; stiffness of the leadscrew and the parts supporting it. And the other purpose is to provide synthesizing data on feedback controlled drive systems.

Nomenclature

T_i : torque at the point i
 Θ_i : angular displacement at the point i
 F_i : force at the point i
 X_i : displacement at the point i
 e_{in} : input voltage
 i : driving current
 n : gear ratio = Z_1/Z_2
 Z_1 : no. of input gear teeth
 Z_2 : no. of output gear teeth
 J_a : actuator inertia
 J_1 : pinion inertia
 J_2 : gear inertia
 m : table mass
 k_s : torsional stiffness of leadscrew
 k_b : stiffness of the element supporting the leadscrew
 L : lead of leadscrew
 D : damping constant between the table and the bed
 S : Laplace operator
 K_a : amplifier gain
 K_x : position loop gain constant
 K_v : tachometer feedback loop gain constant
 r_g : internal resistance of electrohydraulic motor
 ω_n : load-side natural frequency
 ω_a : motor-side natural frequency
 ζ_a : damping factor of motor-side
 ζ : damping factor of load-side
 Ω : frequency ratio
 v : inertia ratio
 J_m : equivalent inertia mass of the table mass
 D_m : equivalent damping constant for D
 J : total inertia mass of motor-side

* Received 8th November, 1974.

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2. The dynamics of a feed drive system

2.1 Composition and mathematical model of the feed drive system

2.1.1 Composition of the drive system.

The feed drive for a numerically controlled machine tool, normally employs either a hydraulic cylinder drive package or a motor drive package. The hydraulic cylinder drive has the merit of providing simple, economical performance, and has been thoroughly studied with the introduction of the servo control method⁽⁶⁾, but its area of application is limited by its narrow range of operations.

In this paper, therefore, an electrohydraulic motor and a number of other machine elements, in which energy and information flow take place, are used in the drive system. A simple diagram of the system composition is shown in Fig.1. The table is displaced by a ballscrew driven by the motor through the gear box. The ballscrew is flexibly mounted at both ends of the bed, a nut being fixed to the table. Drive backlash is reduced to a minimum. Force F_0 is an external force resulting from the cutting force or load acting on the table in the longitudinal direction. In order to reduce friction forces, hydrostatic guideways are introduced on the upper horizontal surfaces. However, the use of hydrostatic guideways results in reduced damping at the table, and thus controllability of the drive system is adversely influenced by low damping. A typical application of feedback compensation is a system with tachometer feedback provided on the motor shaft, in which the effect of the tachometer feedback loop in addition to the position loop is examined to see how it copes with improved controllability.

2.1.2 The system signal flow diagram. The first step in the process of analysis of the system is to establish a mathematical model by means of a signal flow diagram which is capable of providing a more detailed illustration of a complex system than a block diagram. It is assumed that the system under study moves linearly over the range of operations, therefore, referring to Fig.1, the linear equations of the relationships between the electrohydraulic servo motor and other machine components are developed. Thereafter taking Laplace transform with zero initial condition for

Fig.1 Schematic view of a feed drive system

these equations, we express equations by rearranging them as follows:

(a) Equations for electrohydraulic motor. Linearizing for small perturbations of T_{in} , i and $\dot{\theta}_p$ about some operating point on the motor torque-speed characteristics and neglecting compressibility of oil,

$$\mu G(s)i - r_s \dot{\theta}_p = T_{in} \quad (1)$$

$$T_{in} - T_s = J_s S^2 \cdot \theta_p \quad (2)$$

where $G(s)$ represents a normal transfer function of the servovalve and T_{in} is a motor torque acting upon the actuator.

(b) Equations for gear transmission. Neglecting deformation of gear teeth, the gear transmission consisting of inertial elements and a pair of gears is expressed by the following three transfer matrix representations;

$$\begin{bmatrix} T_{s1} \\ \theta_{s1} \end{bmatrix} = \begin{bmatrix} 1 & J_1 S^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{s1}' \\ \theta_{s1}' \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} T_{s1}' \\ \theta_{s1}' \end{bmatrix} = \begin{bmatrix} -n & 0 \\ 0 & -1/n \end{bmatrix} \begin{bmatrix} T_{s2}' \\ \theta_{s2}' \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} T_{s2}' \\ \theta_{s2}' \end{bmatrix} = \begin{bmatrix} 1 & J_s S^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{s2} \\ \theta_{s2} \end{bmatrix} \quad (5)$$

where $n = Z_1/Z_2 =$ no. of input gear teeth/no. of output gear teeth.

(c) Equation for a leadscrew. The leadscrew which lies between the gear box and the nut is treated as a shaft. Then the dynamics of shaft is represented by the next transfer matrix consisting of a transcendental function,

$$\begin{bmatrix} T_{s1} \\ \theta_{s1} \end{bmatrix} = \begin{bmatrix} \cos \beta l & -GI\beta \sin \beta l \\ (1/GI\beta) \sin \beta l & \cos \beta l \end{bmatrix} \begin{bmatrix} T_{s2} \\ \theta_{s2} \end{bmatrix} \quad (6)$$

where $\beta = \sqrt{\rho/G} \cdot \omega$, G = shear modulus of elasticity for shaft material, l = effective length of shaft, I = polar moment of inertia of shaft, ρ = density for shaft material.

By making Taylor's expansion of the function $\sin \beta l$ and $\cos \beta l$ and retaining only the first terms of series, the transfer matrix derived in the foregoing is approximated by a lumped parameter system as follows,

$$\begin{bmatrix} T_{s1} \\ \theta_{s1} \end{bmatrix} = \begin{bmatrix} 1 & J_s S^2 \\ 1/k_s & 1 \end{bmatrix} \begin{bmatrix} T_{s2} \\ \theta_{s2} \end{bmatrix} \quad (7)$$

where $k_s = GI/l$ = torsional stiffness,

$J_s = I\rho l$ = shaft inertia.

(d) Equations for load. The longitudinal force F which is translated from the drawing torque T_{s2} must be balanced by the force due to the load and the external force F_0 , hence neglecting the rolling friction by using low-friction recirculating ballscrews,

$$T_{s2} = \frac{d_s}{2} \left(\frac{L + \mu_0 \pi d_s}{\pi d_s + L \mu_0} \right) F \approx \frac{L}{2\pi} F \quad (8)$$

$$F = k_b X_s \quad (9)$$

Also, the force equation for the flexibly mounted ballscrew is given by

$$F = (mS^2 + DS)X_s + F_0 \quad (10)$$

The geometrical relationship among the angular displacement θ_{s2} , the ballscrew displacement measured axially along the bed X_s and the table displacement X_t obeys the following equation;

$$X_t = (L/2\pi)\theta_{s2} - X_s \quad (11)$$

Next, to close the position and velocity feedback the following equation must be added:

The significant characteristics of transient response can be quickly estimated from the pole-zero configuration. The variation in the pole-zero configuration is drawn in detail according to Eq.(16).

Figures 5 and 6 show such a pole-zero configuration using Ω and ζ as parameters, where ω_a and n are predetermined as constant values such as $\omega_a=1$ or $\omega_n=\Omega$ by non-dimensional transformations and $n=5/8$ by dimension using the drive setup. The pole moves along the solid line as Ω varies, and zero moves along the broken line forming an angle of ϕ to the real axis. This is called a "pole zero locus graph". The graph is very useful for dynamic design purposes, since the open loop transfer function of the drive system is easily obtained from the graph. For instance, when it is necessary during design to compare the dynamic characteristic of $\zeta_a=0.3$ with that of $\zeta_a=0.5$, under the condition of $\omega_a=100$ rad/s, $\zeta=0.1$, and $\Omega=0.8$, the open loop transfer functions can promptly be read from Figs.5 and 6, after multiplying respective axes on both figures by reason normalized as $\omega_a=1$. The result is

$$G_2(s) = \frac{(s^2 + 0.16 \times 10^2 s + 0.63 \times 10^4)}{\left\{ \frac{(s + 0.47 \times 10^2)}{\times (s^2 + 0.3 \times 10^2 s + 0.983 \times 10^4)} \right\}} \quad \text{.....(19)}$$

At $\zeta_a=0.5$

$$G_2(s) = \frac{(s^2 + 0.16 \times 10^2 s + 0.63 \times 10^4)}{\left\{ \frac{(s + 0.78 \times 10^2)}{\times (s^2 + 0.38 \times 10^2 s + 0.901 \times 10^4)} \right\}} \quad \text{.....(20)}$$

From the comparison between the two, it is found that ζ_a scarcely affects the oscillation root, but has a large effect on the real root. This may be seen more clearly by drawing a graph with ζ constant and ζ_a as a parameter, as shown in Fig.7. It is remarkable that the pole on the real axis, corresponding to the real root, goes away proportionally from the origin as ζ_a

increases.

To obtain a good controllability and fast response in this kind of system, it is desirable to locate the real root as far as possible from the origin on the S-plane and the oscillation root as far as possible from the origin, and from the imaginary axis. So, in an effort to satisfy the above mentioned condition, we can concentrate in the design problems for the drive system on how to determine suitably the values ω_a , n , and ζ_a as against the values ω_n and ζ which are restricted considerably by given specification.

Judging from this point of view, although the introduction of a hydrostatic guideway with a resulting low load damping worsens the controllability of a system due to the low value of ζ , the controllability may be sufficiently compensated by adjusting the values of ζ_a and ω_a .

However, these values depend upon the setting of r_g , k_s , and J , which are often limited because of various restrictions on the function and structure of the system. Accordingly, a method should be developed to adjust externally the value of ζ_a without affecting the other values. It is noted from Fig.3 that leading the signal from Θ_p into the summing point passing through the filter with transfer function $K_v/G(s)$, adjustment of K_v produces the same effect as the controlling of r_g . A sample of this fact is explained as follows: In a considered system, if $\zeta=0.05$, $\zeta_a=0.2$, and $\Omega=0.6$, the root configuration is easily read from Fig.8-A which is made by reference to Fig.7. In this case, $\zeta_a=0.4$ when K_v is properly selected so as to satisfy the condition $r_g=K_v \cdot K_a \cdot \mu$, and the resulting root configuration is shown in Fig.8-B. As can be clearly understood from this graph, the controllability is obviously improved.

Fig.5 Pole-zero locus using ζ and Ω as parameters

Fig.6 Pole-zero locus using ζ and Ω as parameters

Fig.7 Pole-zero locus using ζ_a and Ω as parameters

2.3 Dynamics of the drive with a flexibly mounted ballscrew

The above mentioned results were obtained as $1/k_b \rightarrow 0$, but it does not necessarily follow that usually the ballscrew is rigidly mounted at both ends of the bed. In this section, therefore, the influence of spring constant k_b on the dynamics of drives is investigated. By reducing Fig.2, the transfer function corresponding to Eq. (13) is obtained as follows,

$$G_1'(s) = \frac{\frac{J_m}{k_s} s^2 + \frac{D_m}{k_s} s + 1}{\left\{ \frac{J_m}{k_s} (J + n^2 J_s \lambda) s^2 + \frac{D_m}{k_s} (J + n^2 J_s \lambda) s + [J + n^2 (J_s + J_m)] s + n^2 D_m \right\}} \quad (21)$$

where $k_s = \frac{k_b (L/2\pi)^2 k_b}{(L/2\pi)^2 k_b + k_s}$, $\lambda = \frac{k_s}{(L/2\pi)^2 k_b + k_s}$

The term including λ can be neglected as $n^2 J_s \ll J$ and $\lambda \ll 1$. Thus, if k_s is replaced by an equivalent spring constant k_e , Eq. (21) will agree with Eq. (13), and above results are completely applicable in this case.

3. Dynamic performance of system under closed loop control

3.1 Analysis and evaluation of dynamic performance by root locus

The root locus method is not often employed in the design of control systems due to the difficulty in drawing the root locus. Ever since drawing became possible with the aid of a computer⁽⁷⁾, both the transient and frequency responses can be immediately determined by designating an optional loop gain to the root locus drawn on the S-plane. The above fact permits the root locus method to be widely utilized in the design of control systems as a favorable method. The pole-zero configuration stated in the preceding chapter is arranged

for direct utilization with the feedback system based on the root locus method. Thus, the method is employed to clarify the dynamic performance of the drive system.

Although response in the control system has many roots in performance of equations, it is normally controlled by the dominant oscillating root. The evaluation of dynamic performance in this paper is therefore based upon M_p criterion and band width of the dominant oscillating root. In the servo systems, the resonance peak M_p is recommended normally about 1.3 in value which corresponds approximately to 0.4 for damping ratio ζ .

In this paper, the specifications for the feed drive system as shown in Table 1, are examined as a practical example. The specifications for the experimental device used in this study are established on the basis of typical drive system data. Also the transfer function of the servo valve used in this study is shown in Fig.2 as $G(s)$, where $T=1/450$ s/rad, $\omega=1570$ rad/s, $\zeta=0.1$.

3.1.1 Dynamic performance of system provided with only position feedback loop. Since the following dimensions can be obtained with the given specifications; i.e., $\omega_a=2.25 \times 10^3$ rad/s and $\zeta_a=0.185$, the pole-zero configuration of this case can be estimated from Fig.6. This figure has been prepared under the condition $\omega_a=1$, with the result that the figure can be widely used if only the scale values of the respective axes are multiplied by 2.27×10^3 . When the table weighs 150 kg, $\omega_n=3.4 \times 10^3$ rad/s, then $\Omega=1.5$. (When the table weighs 1,000 kg, $\Omega=0.56$.) Supposing that $\zeta=0.05$, the pole-zero configuration can be determined immediately.

Fig.9 shows two different kinds of the root loci traced only by the position feedback corresponding to the respective pole-zero configurations. When the table weighs 150 kg, the oscillating root in the

Fig.8 Effect of the specific tachometer feedback network

m kg s ² /cm	0.15
L cm	0.8
n	5/8
k_s kg/cm	0.28×10^5
J_a kg cm s	1.09×10^{-3}
J_1 kg cm s	1.34×10^{-3}
J_2 kg cm s	7.87×10^{-3}
τg kg cm s/rad	4.58

Table 1 Specifications for the used feed drive system

(In the case of $mg=150$ kg) (In the case of $mg=1000$ kg)
Fig.9 Two kinds of the root loci traced only by the position feedback loop

higher zone is determined by ω_n and ζ values. The oscillating root, being far from the dominant oscillating root, exerts little influence on the dynamic performance of the system. When the table weighs 1,000 kg, the oscillating root in the higher zone approaches the original point, while its stabilization tends to be improved by the effect of ζ_a . However, it is noticed that the band width of the dominant oscillating root is reduced, since one of the real roots on the real axis approaches the original point.

It has been proved in the preceding chapter that the real root can be controlled by selecting the rg/J value. Therefore, the term must be as large as possible in the design of drive systems requiring a faster transient response.

3.1.2 Dynamic performance when tachometer feedback is added. Figure 10 shows the root loci with tachometer compensation given to the two examples shown in Fig.9. The curves in this figure were obtained by increasing the loop gain, after a specified tachometer feedback gain had been provided. The root locus for only tachometer feedback is shown by the chain line. Fig.10-a shows cases of root loci when $K_v = 0, 0.5$, and 1.0 . When the table weighs 150 kg, the oscillating root on the side of the table does not exert any influence on the performance of the system and it is neglected. As shown in this figure, the band width of the dominant oscillating root can be improved to more than twice its original value by adequate tachometer feedback compensation.

Fig.10-b shows the comparison of root loci for $K_v = 0$ and 0.05 when the table weight is 1,000 kg. The figure shows that the response speed is improved similarly to the preceding paragraph. When the oscillating root of the drive system approximates to the dominant oscillating root with an increase of the table weight, the K_v value increases and the root enters the

unstable zone. Accordingly, the response speed is not expected to be greatly improved.

However, the above mentioned method in which a signal from Θ_p is fed back through the filter represented by the transfer function $1/G(s)$, separates the oscillating root from the imaginary axis. The method also permits the real root on the real axis to go away from the original point. Therefore, more effective measures can be expected from use of this method. This compensation method is effective particularly when the table weight increases.

The table acceleration feedback method is widely known as a compensation method for stabilization of the control system being affected by table weight. However, the signal from Θ_p can be easily detected, with the result that the tachometer output signal is more effectively utilized based upon the detailed study described in this paper.

4. Discussion

4.1 Examination and analysis using an analogue computer

4.1.1 Comparison with the result of the root locus method. The following indicates the result of analysis with regard to the data given in Table 1. Utilizing the signal flow diagram shown in Fig.2 which is reduced to the direct simulation by the analogue computer, the results were obtained as a Bode diagram. Fig.11 shows the comparison between variations of load on the table in a system with overall loop gain $K=300$. The load-side resonance located near 650 Hz when $mg=150$ kg moves to the lower frequency side as the load increases until it approaches the resonance point of the servo valve when the load reaches 1000 kg. The resonance point of the servo valve is only slightly affected by the change of load. It can be clearly seen from the phase characteristic that the bandwidth decreases with an increase of load. These facts meet exactly the result of root locus indicated in Fig.9.

Fig.10 The loci with tachometer compensation

Fig.11 The comparison between variations of load mass

Figure 12 indicates the tachometer feedback, when load $mg=150$ kg, so as to compensate the lack of stability in the system resulting from an increased loop gain, and it also corresponds exactly to the result shown in Fig.10. From these analyses in addition to any other methods of experimentation, it is proved that the use of Figs.5, 6 and 7 is effective in understanding the dynamics of the system constructed from our study.

4.1.2 Examination of other main important causes affecting the response characteristic. It is already known that the ballscrew lead controls the equivalent inertia mass of the table. The result of an experiment performed to study the action of the lead of ballscrew on a closed loop system is shown in Fig.13. From the chart it is recognized that the load-side resonance frequency decreases in inverse proportion to an increase of lead from 0.8 cm, and moreover causes a lack of stability due to an increase in loop gain.

In the actual driving system, saturation of motor speeds as a typical nonlinear characteristic occurs by which the correct response characteristic is prevented. In respect to this saturation, an analysis is done with this system having a maximum motor speed of 2500 rpm employed in this

Fig.12 Effect of the tachometer feedback

Fig.13 The action of the lead in a closed loop system

system. The effect of the resulting saturation and the result of a countermeasure using step response of the table are shown in Fig.14.

In this case, the effect of such saturation appears when the displacement of the table becomes about 1 cm. (See Figs. 14-a and 14-b).

When the table is set in any position exceeding the said displacement, the resultant response time is proportional to the input voltage. To expand its linear response range, the step response of the drive providing a lead which is doubled in size is shown in Fig.14-c. As a result, it vibrates for the reason mentioned in Fig. 13. Fig.14-d shows an example of how to prevent such vibratory response by means of tachometer feedback without reduction of the loop gain.

Consequently, to get quick response over a wide driving range of the table, it is necessary to make many-sided examinations so as to cope with nonlinear characteristics after taking necessary measures to provide a linear model. The method of analysis and results mentioned above will be useful for this purpose.

4.2 Experimental results and discussions

Fig.15 shows all apparatus employed for this experiment. The table is supported on a hydrostatic guideway on the bed. Special efforts were made to minimize backlash in each part of the apparatus. Data collected with the apparatus are shown in Table 1.

An inductance-type displacement pick-up with good dynamic characteristics (frequency range is from DC to 500 Hz over the measuring range of 30 cm) was employed to detect the position feedback signal.

Fig.14 Step response of the table

Fig.15 View showing the drive setup

The experiment was done only with a table of 150 kg in weight. Fig.16 shows the frequency response characteristic of the table with a 0.2 V input signal. The result coincides considerably well with the theoretical values in the frequency range below 300 Hz, while two peaks appear in the higher frequency range. One peak at about 450 Hz may be assumed to correspond to the resonance point of table, which deviates from its theoretical value of 650 Hz to this lower frequency side because the ballscrew actually used is elastically supported.

Fig.17 indicates an example of step response of the table. Actually, a linear response characteristic was obtained from our experiment with displacement of the table not exceeding 0.5 cm, but saturation on the response characteristic was observed in the large displacement range. To keep the good response characteristic obtained on the small signal range so that it does not vary even in the larger signal range, it is necessary to improve the performance of the motor employed. Also, selection of the most suitable values for the gear ratio and length of lead in the ballscrew is essential. In this case, it is necessary to consider the stability and speed of response from all aspects as was mentioned previously.

Fig.16 Measured frequency response

Fig.17 Measured step response

5. Conclusions

In this paper, the dynamics of the feed drive system has been examined in detail for the purpose of collecting various data necessary for improved design for quick response and controllability. The results of this study are as follows;

(1) The results of analysis are arranged on the S-plane to make a pole-zero locus chart, from which the relation between the function of elements and the dynamic behaviour of the system is easily understood.

(2) Referring to the chart, the dynamic characteristic of the system with the variation of load on the table becomes clear. It is also found that the low load damping due to the introduction of an anti-friction guideway can be compensated for by reforming a part of the motor.

(3) The use of a unique tachometer feedback method is proposed to get quick response and to improve the stability.

(4) Actual examples are provided indicating the fact that further requirements of a severe dynamic performance can be achieved not only by improvement of the motor performance alone, but also by the examination of all aspects such as the gear ratio, ballscrew lead, inertia mass in rotation, internal resistance of the motor and compensating feedback method.

Acknowledgments

The authors are especially grateful to Dr. N. Tominari, the president of Onlife Research Inc., for many stimulating discussions and to Mr. M. Yamanouchi, a technical official of National Defense Academy, for performing experimentation.

References

- (1) Polacek, M. and Vavra, V., Proc. 8th M.T.D.R. Conf., (1967), p1127.
- (2) De Fraine, J., Proc. 12th M.T.D.R. Conf., (1972), p193.
- (3) Engel, G., Proc. 12th M.T.D.R. Conf., (1972), p180.
- (4) Cessford, S. K. and Bell, R., Proc. 12th M.T.D.R. Conf., (1972), p217.
- (5) Zeleny, J., Proc. 10th M.T.D.R. Conf. (1970), p349.
- (6) Welch, T. R., Trans. ASME Journ. Eng. for Industry, Feb., (1962), p8.
- (7) Nagy, N. and Al-Tikriti, Proc. IEE, 119-2, (1972), p243.