532.59:532.542; 532.59

Pressure and Velocity Distributions in Pulsating Turbulent Pipe Flow

Part 2 Experimental Investigations*

By Munekazu OHMI,** Tateo USUI†, Osami TANAKA and Masao TOYAMA††

Pressure and velocity of pulsating turbulent air and water flows in circular pipes generated by rotary valves were measured by the use of a strain-gauge-type pressure transducer and a hot-wire anemometer, respectively ($0.0432 \leq f \leq 48.0$ Hz, $7740 \leq Re \leq 95900$).

Comparisons of experimental data with approximate solutions derived from a four-region model for a pulsating turbulent flow in a circular pipe containing a slightly compressible fluid show that:

(i) Cross-sectional distribution of time-averaged velocities agrees well with that of steady flow velocities.

(ii) Cross-sectional distribution profiles of oscillating velocity coincide well with the above-mentioned theoretical ones, and therefore, depend only on the Reynolds number and the dimensionless angular frequency $\omega' = R^2\omega/\nu$, where R is pipe radius, ω angular frequency, and ν kinematic viscosity.

(iii) Axial distributions of oscillating pressures and velocities on the pipe axis also agree well with the above-mentioned theoretical ones within the frequency range of $0.1 \leq f \leq 48.0$ Hz.

1. Introduction

In a previous paper⁽¹⁾ approximate solutions for a slightly compressible turbulent flow are derived on the basis of five models for eddy viscosity profile. Comparison of these solutions shows that the four-region model based on von Karman's one is rather reasonable enough to calculate them. In the present work the pressure and velocity of a pulsating turbulent flow in a rigid tube are measured in order to examine the validity of these solutions based on the four-region model.

The cross-sectional distribution of pulsating velocities and the axial distribution of pulsating pressures have been measured only for liquid (water or aqueous solution) flow by Schultz-Grunow⁽²⁾, by Franke⁽³⁾, by Brown, et al.⁽⁴⁾, by Gerrard⁽⁵⁾, and by Mizushima, et al.⁽⁶⁾ Influences of Reynolds number, frequency, and amplitude on these distributions are, however, not examined systematically in these experiments.

Hence, the pressure and the velocity of pulsating air and water flows in a circular pipe were measured within Reynolds number, frequency, and amplitude ranges as wide as possible for the experimental apparatus used and compared with the approximate solutions.

2. Boundary conditions

The approximate solutions for oscillating pressure p_{os} , oscillating velocity

$u_{j,os}$ and wave propagation constant γ based on the four-region model are given in the previous paper⁽¹⁾ by

$$p_{os} = (Ae^{\gamma x} + Be^{-\gamma x})e^{i\omega t}, \quad \dots(1)$$

$$u_{j,os} = \frac{\gamma}{i\omega\rho_0} (Ae^{\gamma x} - Be^{-\gamma x}) \{C_j I_0(z_j) + D_j K_0(z_j) - 1\} e^{i\omega t},$$

$$(j = 1, 2, 3, 4) \quad \dots(2)$$

and

$$\gamma = \frac{i\omega/c}{\sqrt{1 - \{2/(R\sqrt{i\omega/\nu})\} \{C_1 I_1(z_1(R)) - D_1 K_1(z_1(R))\}}}, \quad \dots(3)$$

where x and z_j are the axial coordinate and the transformed variable of radial coordinate r , the subscripts j denote the quantities in the j 'th region, R is the radius of the pipe, t and ω are time and the angular frequency, ρ_0 and ν are the density and the kinematic viscosity of fluid, c is the isentropic speed of sound, I_n and K_n are the modified Bessel functions of the first and the second kind of order n ($n=0,1$), and C_j, D_j are the integration constants determined analytically.

Integration constants A and B may be determined, for instance, in the following way⁽⁷⁾:

(i) $P_1 P_2$ -type: The constants are determined by the use of experimental data of oscillating pressure at any two arbitrary stations separated from each other by intervals of L . In other words, we apply the boundary conditions

$$p_{os} = P_1 e^{i\omega t} \text{ at } x=0 \text{ and } p_{os} = P_2 e^{i\omega t} \text{ at } x=L \quad \dots(4)$$

to Eq.(1). In this way the oscillating velocity can be estimated from pressure measurements.

(ii) $P_1 U_1$ -type: The boundary conditions by the use of experimental data of oscillating pressure and oscillating velocity along the centreline of the pipe at any station, say $x=0$:

* Received 11th October, 1973; 19th October, 1973.

** Professor, Faculty of Engineering, Osaka University.

† Assistant, Faculty of Engineering, Osaka University, Yamada-Kami, Suita, Osaka.

†† Engineer, Kobe Steel Making Co., Kobe.

Fig.1 Schematic of pulsating flow apparatus.

①, ②, ... denote the positions of measuring station; dimensions in mm.

$$p_{os} = P_1 e^{i\omega t}, (u_{os})_{r=0} = U_1 e^{i\omega t} \quad \dots (5)$$

are applied to Eqs. (1) and (2) to determine the values of A and B.

Dimensionless oscillating velocity is expressed as

$$U = 1 - C_j I_0(z_j) - D_j K_0(z_j) \quad \dots (6)$$

(j = 1, 2, 3, 4)

which depends on the dimensionless angular frequency $\omega' = R^2 \omega / \nu$, Reynolds number, and r/R (1).

3. Experimental apparatus and method

The experimental setups for air and water flows are shown in Fig.1. Both test sections were horizontally installed and made of brass tube having a wall of 5 mm thickness. The butterfly valves of which diameters were 49.7 mm in setup A and 75.0 mm in setup B, both having a shaft of 15 mm diameter, were employed to generate a pulsating flow. The measuring and the recording equipments used were described in the previous paper (7) for a pulsating laminar flow.

It was shown by the dynamic calibration of the pressure transducer that the magnitude and phase angle of oscillating pressure measured by a transducer usually set 25 mm apart from the pipe inner wall (p_{25}) are nearly the same as those by one flush mounted (p_0) within the frequency range $0 \leq f \leq 50$ Hz in air flow; and those by one set 50 mm apart from the pipe inner wall (p_{50}) are, on the other hand, different especially at rather high frequencies (see Fig.2).

The output voltage from the film probe (DISA 55A83) often drifts in a turbulent water flow due to the adherence of bubbles, dust, etc. Fortunately it decreases linearly with the time exposed, so that the output voltage is corrected by the use of this linearity character. The film probe is calibrated by setting it along the pipe axis and exposing it to the known water flow rates. The hot-wire probe (DISA 55A25) is calibrated by the use of calibration equipment (DISA 55D41/42). Agreement between the velocity distributions measured by these

Fig.2 Magnitude ratio and phase lag of the fundamentals $[p_{25}/p_0]_1$, $[p_{50}/p_0]_1$ at the station ⑭ of setup A.

○, $[p_{25}/p_0]_1$; ●, $[p_{50}/p_0]_1$

probes and by Pitot tube is very good in a steady flow.

4. Experimental results and discussion

Table 1 shows the experimental conditions and the values of the main parameters adopted for a pulsating flow. The Reynolds number Re and the friction velocity u^* are defined by $Re = 2Ru_{m,ta}/\nu$, $u^* = u_{m,ta}\sqrt{\lambda/8}$ where $u_{m,ta}$ is the cross-sectional mean and long-time averaged value of the pulsating flow velocity profile measured, and λ is the coefficient of fluid friction and

$$1/\sqrt{\lambda} = 2 \log_{10}(Re\sqrt{\lambda}) - 0.8.$$

In setup A experiments were carried out within the frequency range of $0.0432 \leq f \leq 48.0$ Hz and the Reynolds number range of $7740 \leq Re \leq 95900$. This frequency range is wider than the range of $10 \leq \sqrt{\omega'} \leq 100$ in which the cross-sectional distribution of U defined by Eq.(6) is changing remarkably (1).

4.1. Cross-sectional velocity distribution

Figure 3 shows an example of the cross-sectional distribution of low frequency pulsating velocities at twelve equi-interval points in one period, starting from the standard time mark which corresponds to a certain opening angle of the valve through a run. The instantaneous velocity distribution is the same as the steady flow ve-

$\bar{u}_{st}/u^* = 5.75 \log_{10} y^* + 5.5$,
 where $\bar{u}_{st}$ is the steady flow velocity and
 $y^* = u^*(R-r)/\nu$.

The Fourier expansion is applied to pulsating pressure or velocity wave forms recorded on the photo oscillograph, as mentioned in the previous paper (7). The n 'th components of magnitude and phase angle are identified by subscripts n .

Figures 5~7 show qualitative comparison between cross-sectional oscillating velocity distributions obtained from Eq.(6) and from the experiments in terms of magni-

(a) Run A1 $f = 0.0432$ Hz ($\sqrt{\omega'} = 3.29$)
 $Re = 67500$ ($u^* = 1.053$ m/s)

(b) Run A5 $f = 6.0$ Hz ($\sqrt{\omega'} = 39.8$)
 $Re = 47600$ ($u^* = 0.742$ m/s)

(c) Run A14 $f = 48.0$ Hz ($\sqrt{\omega'} = 112$)
 $Re = 68300$ ($u^* = 1.025$ m/s)

Fig.5 The influence of frequency on the cross-sectional distribution of the oscillating velocities. —, theoretical curves. Experiment: O, magnitude; ●, phase angle.

tude and phase angle. Only the first component of the experimental values is adopted in the velocity profile analysis. The theoretical curve is calculated as $|U|$ multiplied by a certain value (m/s) and $\angle U$ added with a certain value (deg). Figure 5(a) corresponds to Fig.3.

Figure 5 shows the influence of frequency, to be exact, dimensionless angular frequency, on the distributions in setup A at the Reynolds number of several thousand. The distribution profiles of experimental results agree well with those of Eq.(6). At low frequencies the magnitude profile is nearly the same as the steady flow velocity profile and the phase angle profile is flat, so that the flow is considered to be in quasi-steady state (see Fig.5(a)). At high frequencies the magnitude profile has a peak and the phase angle advances near the wall, and it is flat in the central region of the pipe (see Fig. 5(c)).

Figure 6 shows the influence of Reynolds number on the distributions in setup A at the same frequency. The distribution profiles of experimental results nearly agree with those of Eq.(6). It is seen from Figs. 5 and 6 that the distribution in the case of low frequencies and small Reynolds numbers is similar to that in the case of high frequencies and large Reynolds numbers.

Figure 7 shows the distribution in the case of the water flow in setup B. The film probe used is much larger than the hot-wire probe and the calibration is delicate as mentioned before, so that the comparison between the measured and the calculated distribution profiles is rather inaccurate. The distribution profiles in the case of the water flow are, however, considered to

(a) Run A7 $f = 24.0$ Hz ($\sqrt{\omega'} = 78.0$)
 $Re = 7740$ ($u^* = 0.1557$ m/s)

(b) Run A12 $f = 24.0$ Hz ($\sqrt{\omega'} = 79.6$)
 $Re = 69700$ ($u^* = 1.020$ m/s)

Fig.6 The influence of Reynolds number on the cross-sectional distribution of the oscillating velocities. Notation as in Fig.5.

agree with those of Eq.(6).

By the way the second component of the oscillating velocity in run A6 is quite large and its distribution profiles also agree well with those of Eq.(6). The frequency, Reynolds number, and oscillating velocity amplitude ranges in which these conclusions concerning time-averaged and oscillating velocity are drawn are in setup A only regarding the frequency $0.0432 \leq f \leq 48.0$ Hz, the Reynolds number $7740 \leq Re \leq 95900$, and the amplitude of oscillating velocity on the axis $0.8 \leq |u_{0,os}| \leq 23$ m/s.

4.2. Pressure and velocity distributions along the pipe axis

Figures 8~13 show axial distributions of magnitudes and phase angles of oscillating pressures and pipe centreline velocities. Theoretical curves are obtained as mentioned in section 2 by use of the data at stations ⑥ and ⑫ for setup A and ③ and ⑨ for setup B ($L=3$ m) in the case of P_1P_2 -type, and at the station ⑩ for setup A and ⑦ for setup B in P_1U_1 -type. Approximation curves are obtained using the relations $\gamma \approx i\omega/c$ and $U_{r=0} \approx 1$, which are available only at high frequencies.

Figures 8~10 show the influence of frequencies on the axial distributions in setup A at the Reynolds number of several ten thousand. In the case of low frequencies, Fig.8, the pulsating flow behaves as an incompressible one, and the theoretical curves agree generally with the experimental results but the approximation curves especially velocity ones a_u based on P_1P_2 -type calculation far deviate from them. In the case of high frequencies, Figs.9 and 10, wave propagations are observed, and both curves agree with experimental results. The frequency range for good agreement between theoretical curves and experimental results is $0.1 \lesssim f \lesssim 50$ Hz, and in the case of approximation curves $3 \lesssim f \lesssim 50$ Hz.

The influence of Reynolds number on the axial distributions is examined at the frequency $f = 24.0$ Hz in setup A within the range of $Re = 10800 \sim 95900$ (see Table 1 and Fig.11). In these cases the standing waves are formed in the same way and these experimental results agree well with the theoretical or approximation curves.

Figure 12 shows an example of the axial distributions of oscillating pressures and velocities of water flow in setup B. As

Fig.7 Cross-sectional distribution of the oscillating water flow velocities in run B2 ($f = 1.0$ Hz, $Re = 74100$). Notation as in Fig.5.

Fig.8 Axial distributions of oscillating pressure and pipe centreline velocity in run A1 ($f = 0.0432$ Hz, $Re = 67500$). Experiment: Δ , pressure; $\circ$, velocity. —, theoretical curves; ----, approximation curves.

Type	P_1P_2	Approximation	P_1U_1	Approximation
p_{os}	a	a'	b	b'
$u_{0,os}$	a_u	a'_u	b_u	b'_u

Fig.9 Axial distributions of oscillating pressures and pipe centreline velocities in run A5 ($f = 6.0$ Hz, $Re = 47600$). Notation same as in Fig.8.

a rotary valve is located downstream, the magnitude of oscillating pressure increases linearly in the flow direction. The experimental results in the case of water flow also agree with the theoretical curves.

The higher components of oscillating pressure and velocity play a dominant role in wave propagation phenomena especially in run A6, as mentioned before. Figure 13 shows four components of magnitude and phase angle of oscillating pressure and pipe centreline velocity. Although experimental plots of higher component are rather scattered, the standing waves of second, third, and fourth components are similar to those at $f = 24$ Hz as shown in Fig.11, at 36 Hz, and at 48 Hz as shown in Fig.10, respectively. The theoretical curves are considered to agree rather well with such scattered experimental data, except the P_1U_1 -type calculation for fourth component.

The authors would like to express their thanks to Mr. S. Kyomen of Kure Technical College, Hiroshima, to Mr. Y. Yanai of Osaka University, and to Mr. M. Fukawa of Daikin Kogyo Co., Osaka, for helpful collaborations. This work was supported in part by the Japanese Ministry of Education's scientific research grant. Numerical calculations were carried out by the computer NEAC series 2200 model 700 at Osaka University.

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Fig.10 Axial distributions of oscillating pressures and pipe centreline velocities in run A14 ($f = 48.0$ Hz, $Re = 68300$). Notation same as in Fig.8.

Fig.11 Axial distributions of oscillating pressures and pipe centreline velocities in run A8 ($f = 24.0$ Hz, $Re = 10800$). Notation same as in Fig.8.

Fig.12 Axial distributions of oscillating pressures and pipe centreline velocities in the case of water flow, run B2 ($f = 1.0$ Hz, $Re = 74100$). Notation same as in Fig.8.

Fig.13 Axial distributions of the four components of oscillating pressure and pipe centreline velocity in run A6 ($f = 12.0$ Hz, $Re = 40000$).

Components		1st	2nd	3rd	4th
Experiments		○ ●	□ ■	△ ▲	◇ ◆
Theoretical curves	P_1R_2 -type	—	—	—	—
	P_1U_1 -type	— · —	— · —	— · —	—

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