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An Experimental Study on Confluence of Two Two-Dimensional Jets*

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An experimental study on the confluence of two two-dimensional incompressible turbulent jets, issuing from nozzles with angle θ_0 against the nozzle exit plane, has been presented. The axial momentum and the energy flux of the jets along the z-axis perpendicular to the exit plane are calculated by using the measured values of velocity and pressure distributions. It has been shown that the axial momentum flux along the z-axis decreases owing to the confluence, that the energy flux also decreases remarkably owing to the confluence, and that the decreased values are approximately 40% of the efflux energy of the jet, independent of the ratio of a distance between two nozzles to the nozzle width and θ_0 . A method of predicting the velocity distribution also has been proposed. The predicted values agreed well with the experimental results.

1. Introduction

Phenomena of confluence of jets are encountered in fluidics, rocketry and boiler combustion chambers, and in a variety of other industrial fields. Therefore, it is desirable to clarify the basic characteristics of the confluence phenomena of jets.

Three-dimensional studies on axisymmetrical annular jets which were related to the confluence phenomena were reported by Taga and Akagawa *et al.*⁽¹⁾⁻⁽³⁾ and one on the confluence of two two-dimensional parallel jets was reported by Tanaka⁽⁴⁾⁽⁵⁾.

In this report, an experimental and an analytical study on the flow characteristics of velocity and pressure distributions, the decay of the total momentum flux, and the decay of the energy flux in confluence of jets with various discharge angles were presented. It was shown that the total momentum of the pressure momentum flux and the velocity momentum flux of each jet were not conserved and decreased remarkably owing to the confluence, and that the velocity energy flux and the pressure energy flux also decreased remarkably owing to the confluence. Furthermore, a method of predicting the velocity distribution in confluent jet was proposed.

Notations

- K : diffusion coefficient of the jet
 l : Prandtl's mixing length
 n : number of nozzles
 P : imaginary point of the source of the confluent jet
 u : time average velocity in z-direction

- v : time average velocity in y-direction
 u' : fluctuation velocity component of u
 v' : fluctuation velocity component of v
 U : value of u at z=0(at nozzle exit)
 V : absolute velocity vector, $(u^2+v^2)^{1/2}$
 V_c : value of V on the main stream line
 V_s : value of V at z=0(at nozzle exit)
 V' : fluctuation velocity component of V
 P₀ : static pressure in the pressure chamber

2. Experimental apparatus and procedures

The experimental apparatus is shown schematically in Fig. 1. Air from a blower

- ① Pressure chamber ⑤ Traverse device
 ② Pressure tap for velocity
 ③ Nozzle measurement
 ④ End plate ⑥ Micro traverse device

Fig.1 Experimental apparatus

Fig.2 Shape of nozzles

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is discharged from the rectangular nozzles (3) installed in the pressure chamber (1) and flows into a two-dimensional test passage, composed of an upper and a lower end plate (4) separated by 200mm. The height of nozzle is 200mm, the width b is 9.7mm, and the width of nozzle in a single nozzle run is 19.4mm which is equal to $2b$. The shape of nozzles is shown in Fig. 2.

The experiment has been carried out under the conditions shown in Table 1. The range of discharge velocities is from 5m/s to 27.8m/s. The distribution of air velocities was measured by a single hot wire probe and by a couple of disc type static pressure tubes and a pitot type total pressure tube which are equipped to a traverse equipment (6). The fluctuation velocity V' was measured by the single hot wire probe made by Nihon Kagaku Industry Co.

3. Flow characteristics and static pressure distributions

3-1. Flow characteristics near the nozzle exit and confluent point

An example of velocity profiles is shown in Fig. 3. Two jets issuing from each nozzle are confluent at some distance from the nozzles and become a single jet with a velocity profile which is similar to that of an ordinary single jet. And a recir-

Table 1 Dimensions of nozzle

θ_0	a mm				
	40	170	200	230	300
0	○	○	○	○	○
15		○		○	○
30		○		○	○
45		○			
0	Single jet

Fig. 3 Velocity distribution near nozzle

Fig. 4 Static pressure distribution near nozzle

ulation zone surrounded by two jets and the nozzle wall exists in which a reflux toward the nozzle wall along the z -axis occurs.

The static pressure distributions near the nozzles are shown in Figs. 4 and 5. As shown in these figures, the pressure in the recirculation zone is sub-atmospheric, and on the z -axis the pressure is increased by the confluence, takes the maximum value at a short distance upstream from the confluent point, is decreased due to the flow acceleration downstream from the confluent point, takes the minimum values, and finally approaches the atmospheric pressure.

The confluent point which is an important point for analyzing the confluence phenomena was defined by the intersection of two extra-polated main stream lines of jets as shown in Fig. 3. The nondimensional confluent distance z_j/a is shown in Fig. 6 against $a/2b$. As shown in the figure the confluent distance decreases with an increasing θ_0 and increases with an increasing $a/2b$.

3-2. Flow characteristics downstream from the confluent point

3-2-1. Velocity profiles and the diffusion coefficient

The non-dimensional velocity profiles, u/u_c , in each cross section perpendicular to the z -axis downstream from the confluent point are shown in Fig. 7. The experimental points fell on curves of velocity profile of a single jet expressed by a polynomial

Fig. 5 Static pressure on z -axis

Fig. 6 Confluent point

Fig. 7 Velocity distribution

equation⁽⁶⁾ and Gauss's formula. Namely, the velocity profile downstream from the confluent point is similar to that of a single jet independently of θ_0 and $a/2b$. The iso-velocity lines in the measured velocity profiles are expressed by straight lines as shown in Fig. 8. This means that the jet width increases in linear proportion to the distance z , that is, the width is expressed by

$$y_{0.5} = K \cdot Z \quad (1)$$

where Z is a distance from an imaginary source of the jet P shown in Fig. 8. The diffusion coefficient, K , can be determined by the above experimental results from the following equation which is obtained from Eq. (1) and Gauss's formula

$$(y/Z)^2 = -2K^2 \log_e(u/u_c) \quad (2)$$

The values of K are shown in Fig. 9. It can be seen from the figure that the larger θ_0 and a/b are, the larger the values of K are, and the values of K are larger than the 0.08 to 0.09 in a single free jet, especially in the case of large angle of $\theta_0 = 30^\circ$, the values are two or three times as large as that of a single jet. The reason for the value of K being larger is due to the pressure rise by the confluence of two jets, that is, the two jet repulse each other by the higher pressure near the confluent point, and a confluent single jet spreads with higher rate than the ordinary single jet in downstream region from the confluent point.

In Fig. 10 the velocities along the main stream lines (main stream velocity) are presented. It should be noticed that the absolute velocity in the confluent jets decreases remarkably compared with that of a single jet, and that the tendency of decay of the velocity far downstream from the confluent point is similar to that of a single free jet. The velocity decreases remarkably near the confluent point owing to the conversion of the kinetic energy to the pressure energy, and the velocity in-

Fig. 8. Iso-velocity lines.

Fig. 9. Diffusion coefficient.

creases owing to the reverse conversion and take the maximum value and then decays with a diffusion of jet which is similar to that in the free jet. The decrease rate of the velocity downstream from the confluent point increases with θ_0 and the width, "a", as seen in the figure. This characteristics may be estimated from the tendency of the values of K in Fig. 9, because the jet having a larger value of K spreads with a larger angle.

4. Discussion on decrease of momentum flux and energy flux, and on decay of main stream velocity

4-1. Momentum of the time average velocity and integration of the static pressure

In a two-dimensional single jet the momentum conservation equation is given by the following equation⁽⁷⁾:

$$\int_{-\infty}^{\infty} \left(\frac{u}{U} \right)^2 dy + \int_{-\infty}^{\infty} \frac{p}{\rho U^2} dy + \int_{-\infty}^{\infty} \frac{u^2}{U^2} dy = \text{const.}$$

In the region not very far from the nozzle exit, the turbulent term is negligibly small and the conservation law for the sum of the velocity term and the pressure term holds in the present experiment as shown in Fig. 11. Also, in this study on the confluent jets, the conservation of the total momentum of the velocity and the pressure term was discussed neglecting the turbulent term. Three elements of momenta were defined as Eqs. (3), (4) and (5) respectively.

$$J_k = \rho \int_{y_L}^{y_{Lmax}} V^2 \cos^2 \theta_1 dy + \rho \int_{y_R}^{y_{Rmax}} V^2 \cos^2 \theta_1 dy \quad (3)$$

Fig. 10. Distribution of the velocities on the main stream line.

Fig. 11. Distribution of momentum fluxes.

$$J_p = \int_{y_L}^{y_{Lmax}} p dy + \int_{y_R}^{y_{Rmax}} p dy + \int_0^{y_L'} p dy + \int_0^{y_R'} p dy \quad (4)$$

$$J_r = \rho \int_0^{y_L'} V r^2 \cos^2 \theta_1 dy + \rho \int_0^{y_R'} V r^2 \cos^2 \theta_1 dy \quad (5)$$

Here, J_k is the velocity momentum of the positive direction flow, J_r is that of the reflux, and J_p is the pressure momentum. Thus, the total momentum J_z is expressed as follows.

$$\frac{J_z}{J_0} = \frac{J_k}{J_0} + \frac{J_p}{J_0} + \frac{J_r}{J_0} \quad (6)$$

The time average velocity momentum, J_k/J_0 , increases just after the nozzle exit and takes a larger value than that of the nozzle exit owing to the acceleration induced by the sub-atmospheric pressure in the recirculation zone (A_k). The momentum decreases remarkably owing to the change of the flow direction and also the confluence of the two component jets, and takes the minimum value (B_k), and then approaches a maximum (C_k) owing to the pressure decrease, but does not exceed the initial value because of the confluence loss, and finally decreases gradually.

The pressure momentum J_p/J_0 which is the integration of the pressure is negative and the tendency of the value along the z-axis is symmetric with respect to that of the velocity momentum. The reflux momentum in the recirculating zone is small in comparison with the other momentum.

The total momentum flux consisting of the above three terms decreases monotonously with the distance from the nozzle exit to the confluent point. Downstream of the confluent point, the total momentum flux has a constant value as shown in the figure, that is, the conservation law of the momentum flux holds. This tendency can be seen in every experimental run as shown in Fig. 12. It should be noticed that the

Fig. 12. Decay of the momentum flux

Fig. 13. Flow model

momentum flux decreases remarkably in the recircular zone and takes a constant value in the downstream zone.

4-2. Change of momentum flux along the z-axis

In confluent jets, the momentum flux along the z-axis, J_z/J_0 , decreases remarkably in the upstream region from the confluent point shown in Fig. 11, because each z-component of two jets decreases due to the change of its flow direction. This fact may be explained by a flow model shown in Fig. 13 as follows. The static pressure in the recircular zone, p_b , and that in the surrounding air at rest, p_∞ , are assumed to be constant, then the momentum flux along the s-axis, J_0 , is constant. Therefore, the momentum flux balance along the z-axis is expressed by Eq. (7)

$$J_0 \cos \theta_0 - J_z = \int_{S_0}^{S_1} (p_b - p_\infty) \sin \theta_1 ds \quad (7)$$

Furthermore, it is assumed that the locus of the main stream line of a jet is expressed by a circular arc of a radius, r , the centrifugal force of the curved jet, J_0/r and the static pressure difference, $(p_b - p_\infty)$, interact on the s-axis, and they balance each other. From the geometrical relation shown in Fig. 13, the following equation holds

$$\int_{S_0}^{S_1} \sin \theta_1 ds = r (\cos \theta_0 - \cos \theta_1)$$

Then, the momentum flux along the z-axis is obtained by Eq. (8) from the above equations

$$\left. \begin{aligned} J_z &= J_0 \cos \theta_1 \\ \cos \theta_1 &= \sqrt{1 - \left(\frac{z}{r} + \sin \theta_0 \right)^2} \end{aligned} \right\} \quad (8)$$

$$z = r \sin \theta_1 - r \sin \theta_0$$

This means that the decrease of the momentum flux along the z-axis upstream from the confluent point is predicted by the above equation.

4-3. Momentum flux in various discharge conditions downstream from confluent point

As illustrated in Sec. 4-2 the axial momentum flux J_z decreases along the stream line upstream from the confluent point, while the momentum flux J_0 along the s-axis is conserved. Since each component jet turns to the z-direction downstream of the confluent point, the value of J_z would recover to the initial value if there were no momentum loss by the collision at the confluent point. However, the recovery of momentum does not take place as can be seen in every experimental run.

The momentum loss was expressed by the momentum conservation coefficient η_z as Eq. (9)

$$J_z = \eta_z \cdot J_0 \quad (9)$$

The values of η_z are shown in Fig. 14, and η_z is in the range of 0.2 to 0.45. This

means that 55 to 80 percent of the momentum is lost by the confluence of two jets.

η_z is affected mainly by a/b except in case of $\theta_0=0$, and the effect of θ_0 is relatively low. In the parallel jets ($\theta_0=0$), the effect of a/b on η_z is little, because the shape of the locus of the main stream line of the jet is approximately similar and at the collision angle θ_0 is very small and is almost constantly independent of a/b .

In case of the confluence jet with the discharge angle θ_0 , the larger a/b is, the smaller η_z is, because the length of the flow path up to the confluent point increases with "a" and the collision angle increases.

4-4. Decrease of energy flux

An important feature of the confluent jets is a remarkable energy decay due to the turbulent shearing stress and the collision of the component jets. Here, as the energy of the confluence jet the kinetic energy E_k and the pressure energy E_p are considered, which are defined as follows

$$E_k = \frac{1}{2} \rho \int_{y_L}^{y_{Lmax}} u^2 dy + \frac{1}{2} \rho \int_{y_R}^{y_{Rmax}} u^2 dy + \frac{1}{2} \rho \int_{y_L}^{y_L'} u^2 dy + \frac{1}{2} \rho \int_{y_R}^{y_R'} u^2 dy$$

$$E_p = \int_{y_L}^{y_{Lmax}} |u| \cdot p dy + \int_{y_R}^{y_{Rmax}} |u| \cdot p dy + \int_{y_L}^{y_L'} |u| \cdot p dy + \int_{y_R}^{y_R'} |u| \cdot p dy$$

These two values normalized by the kinetic energy at the nozzle exit, $E_0 = \rho b V_0^2$, are presented in Fig. 15. The kinetic energy of each component jet issuing from two nozzles is decreased by the diffusion, and near the confluent point a part of the kinetic energy changes into the pressure energy, so E_k/E_0 is decreased remarkably, and downstream from the confluent point the pressure energy is partially converted to the kinetic energy, then E_k/E_0 is increased, and finally the kinetic energy is decreased gradually by the free diffusion. Consequently the curve of E_k/E_0 takes a minimum and a maximum value as shown in

Fig. 14 Coefficient of momentum decrease

Fig. 15 Kinetic and pressure energy fluxes

Fig. 15. The pressure energy E_p takes the maximum value and decreases with the conversion to the kinetic energy and then increases gradually to a value corresponding to the atmospheric pressure.

The sum of $(E_k + E_p)/E_0$, therefore, decreases monotonously with "z" owing to the reverse tendency of both energies, as shown in Fig. 16, and the decay is significant in the upstream region from the confluent point.

In case of the small nozzle distance; $a=40$, and $\theta_0=0$, the energy loss is low and the experimental data are between an experimental curve of the ordinary single jet and the theoretical value of two parallel jets calculated with the assumption of no interaction, because in this particular case two jets are confluent immediately near the nozzles and their collision angle is small, so the loss is small. Except in the above case, the rate of decay of the energy flux is very high in the upstream region from the confluent point, and at a distance of $z/2b = 16$ downstream from the confluent point, the conserved energy flux is in order of 0.01 to 0.15 E_0 . It can be seen that the loss of energy due to the recirculation, and that due to the collision are in order of 0.4 E_0 as compared with the conserved energy flux of 0.5 E_0 in a single jet. The conserved energy at $z/2b = 36$ under every experimental condition is shown in Fig. 17. Little effect of θ_0 and a little effect of width "a" on the conserved energy are recognized.

4-5. Prediction of main stream

velocity downstream from the confluent point.

In confluent jets, the flow near the confluent point is so complex that the theoretical analysis of the velocity distribution is very difficult. However, in the downstream region from the confluent point the static pressure has approximately a constant value, and the axial momen-

Fig. 16 Decrease of the energy flux

Fig. 17 Energy decrease rate

tum flux is conserved. Therefore, the analysis of the velocity profile can be done by applying the ordinary free jet theory. For the analysis, however, the coefficient of the time average velocity momentum, η_k , the proportional constant, κ , and the jet width at the confluent point, $y_{\max(0)}$ (ref. in Sec. 4-5 (a)), are required. The energy equation of confluent jets in the downstream region from the confluent point is expressed by Eq. (10)⁽⁸⁾ neglecting the fluctuation velocity energy

$$\frac{d}{dz} \int_0^{y_{\max}} \rho u^3 dy - 2 \int_0^{y_{\max}} u \frac{\partial \tau}{\partial y} dy = 0 \quad (10)$$

and the momentum equation is expressed by Eq. (11)

$$\int_0^{y_{\max}} \rho u^2 dy = \eta_k \rho b V_s^2 \quad (11)$$

where the term $\rho b V_s^2$ is the momentum flux at the nozzle exit and η_k is the ratio of the momentum of the time average velocity in the downstream region from the confluent point to the nozzle exit momentum flux. The shearing stress in Eq. (10) is assumed to be expressed by $\tau = \rho \kappa u_c y_{\max} (\partial u / \partial y)$. Then the substitution of the polynomial equation of the velocity profile in Fig. 7 and the above equation of τ into Eq. (10) yields Eq. (12), and the substitution of the polynomial equation into Eq. (11) yields Eq. (13)

$$0.251 \frac{d}{dz} (u_c^3 y_{\max}) = -2.314 \kappa u_c^3 \quad (12)$$

$$0.316 \rho u_c^2 y_{\max} = \eta_k \rho b V_s^2 \quad (13)$$

From Eqs. (12) and (13), an equation of the width of jets, $y_{\max}$, is obtained as follows

$$d/dz (y_{\max} - \frac{1}{2}) = -9.219 \kappa y_{\max}^{-\frac{3}{2}}$$

Solving the equation, using the initial value $y_{\max(0)}$ at $z' = 0$, one obtains

$$y_{\max} = 18.44 \kappa z' + y_{\max(0)} \quad (14)$$

The main stream velocity u_c/V_s is obtained by substituting Eq. (14) into Eq. (13) as Eq. (15)

$$\frac{u_c}{V_s} = 1.779 \left(\frac{b}{18.44 \kappa z' + y_{\max(0)}} \right)^{\frac{1}{2}} \eta_k^{\frac{1}{2}} \quad (15)$$

where $y_{\max(0)}$, η_k , and κ in Eq. (15) are experimentally determined constants shown as follows.

Fig. 18. Jet width.

(a) $y_{\max(0)}$ The jet width $y_{\max}/a$ is plotted against the distance of z/a in Fig. 18. These experimental curves cross at a point where z/a is 1.2, and $y_{\max}/a$ is 0.55, independently of θ_0 and "a". Thus the point seems to be an imaginary initial point where a jet begins to spread with a constant rate. Therefore, at a location of $z' = 0$, width $y_{\max(0)} = 0.55a$ and $z = 1.2a$.

(b) η_k The momentum decrease coefficient of the time average velocity, η_k , at $z' = 0$ is plotted against "a" and θ_0 in Fig. 19. η_k is affected by the distance, "a", and little affected by θ_0 as shown in the figure.

(c) κ The constant κ in Eq. (14) is determined from the experiments as follows. The shearing stress is expressed by $\tau_{(9)} = -\rho u'v'$, where $u' = l_y du/dy$ and $v' = l_v dv/dy$, then $\tau = \rho l_u l_v (du/dy)^2$. Furthermore, l_u is nearly equal to l_v according to Abramovich⁽⁹⁾ $l_u \approx l_v \approx l$, then $\tau = \rho l^2 (du/dy)^2$ and $\kappa = l^2 (du/dy) / (u_c y_{\max})$, where l can be obtained from the experimental values of u' and du/dy as $l = \sqrt{u' / (du/dy)}$. Therefore, the value of κ could be calculated. Here, the value of $\sqrt{u'^2}$ obtained from a single hot wire probe was used instead of $\sqrt{u'^2}$. Examples of the values of l are shown against y in Fig. 20. From the figure, it can be seen that the value of the mixing length l is constant in the region of $y/y_{0.5} > 1$. Thus the value of κ in the region far downstream from the

Fig. 19. Momentum decrease coefficient of time average velocity.

Fig. 20. Mixing length.

Fig. 21. Proportional constant.

Fig. 22. Prediction of the velocity on the main stream line.

confluent point obtained from the above values in Fig. 21 is in the range of 0.02 to 0.05 and the value of κ increases with θ_0 and $a/2b$. The predicted values of u_c/V_s obtained from Eq. (15) using the above values of $y_{\max(0)}$, $\eta\kappa$, and κ agree well with the experimental values as shown in Fig. 22.

5. Conclusions

From the experiment on the confluent jets of two jets, the following conclusions were obtained.

(1) The velocity profile in the downstream region far from the confluent point is similar to that of the ordinary single jet. However, the values of the diffusion coefficient of confluent jets increases with an increasing $a/2b$ and θ_0 .

(2) The axial momentum flux decreases along the main stream line in the upstream region from the confluent point owing to the change of the flow direction toward the z -axis and the decreased momentum is not recovered downstream from the confluent point. This shows that the loss is caused by the confluence.

(3) The coefficient of the momentum decrease $\eta\kappa$ is in the range of 0.2 to 0.45 and decreases with a/b and is little af-

ected by θ_0 .

(4) The loss of the energy flux in confluent jets is increased remarkably owing to the existence of the recirculation flow and the confluence, and the value of loss is approximately $0.4E_0$ in this experimental range of " a " and θ_0 . However, in the particular case of small " a " and $\theta_0 = 0$, the loss of energy owing to the confluence is so small that the energy flux distribution is approximately similar to that of the ordinary single jet.

(5) The velocity on the main stream line of confluent jets decays remarkably as compared with that of a single jet. The velocity distribution could be predicted by Eq. (15) using the jet width $y_{\max(0)}$, the momentum decrease coefficient of the time average velocity $\eta\kappa$ and the proportional constant for the turbulent shearing stress κ which had experimentally been determined.

References

- (1) Akagawa, K. and Taga, M., Trans. Japan Soc. Mech. Engrs. (in Japanese), 31-221(1965-1), 105.
- (2) Taga, M. and Akagawa, K., Trans. Japan Soc. Mech. Engrs. (in Japanese), 36-283(1970-3), 385.
- (3) Taga, M. and Akagawa, K., Trans. Japan Soc. Mech. Engrs. (in Japanese), 36-283(1970-3), 397.
- (4) Tanaka, E., Trans. Japan Soc. Mech. Engrs. (in Japanese), 35-274(1969-6), 1257.
- (5) Tanaka, E., Trans. Japan Soc. Mech. Engrs. (in Japanese), 39-327(1973-11), 3386.
- (6) Abramovich, G. N., The theory of turbulent jets, (1963), 180, M.I.T. Press.
- (7) Sedat, Sami., *et al.*, J. Fluid Mech., Vol. 27, Part 2(1967), p231.
- (8) Shin-I. Pai, Fluid dynamics of jets, (1954), 115, D. Van Nostrand Co., Inc.
- (9) Abramovich, G. N., The theory of turbulent jets, (1963), 43, M.I.T. Press.