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The Effect of Radiation Heat Transfer in the Measurement of Thermal Conductivity for the Semitransparent Medium*

by

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The effect of radiation heat transfer in the measurement of thermal conductivity for the semitransparent medium is examined, in general, as the parameters of the radiation conduction parameter, the optical thickness and the surface emissivity. The region where the superposition approximation of radiation and conduction can be applied is clarified, and then, the region where the radiation heat transfer can be neglected within an error of 5 % and the region where the optically thick approximation can be applied within an error of 5 % are clarified. Lastly, these examinations are applied to soda-lime glass.

1. Introduction

In a semitransparent medium, there are two heat transfer processes that is conduction and radiation. In the measurement of the thermal conductivity of this medium by the method such as shown in Fig.1, the apparent thermal conductivity, k' , which is calculated by the heat flux including two heat transfer processes of radiation and conduction, is expressed by equation(1).

$$k' = q_t L / (T_h - T_c) \quad (1)$$

This apparent thermal conductivity is not the property of the medium, since it largely depends on the temperature and the thickness of the medium, and the surface emissivity. So, the calculated result yields a large error when the apparent thermal conductivity is used in the heat conduction equation. For example, in case of the soda-lime glass, the apparent thermal conductivity depends on the temperature and the thickness of the glass as shown in Fig.2. At $L=2$ cm and $T=600^\circ\text{C}$, an error of this value increases about 80 %.

Consequently, when the thermal conductivities of the semitransparent media such as glass and gas at high temperature is measured, it is necessary that thermal conductivity be calculated by using the conduction heat flux which is obtained by subtracting the radiation heat flux from the total heat flux. Hence, it is assumed that the superposition approximation of radiation and conduction is applicable.

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Under this assumption, since the total heat flux q_t is the sum of pure conduction and pure radiation, the true thermal conductivity can be obtained by

$$k = q_c L / (T_h - T_c) \quad (2)$$

where

$$q_c = q_t - q_R$$

Up to now, Sugiyama et al.[1] measured the thermal conductivity of the bottle glass, that was divided into two regimes due to radiation and conduction, by using the optically thick approximation. Poltz[2] measured the thermal conductivity of organic solvent, that is also divided into two regimes due to radiation and conduc-

Fig.1 Temperature field for measuring thermal conductivity.

Fig.2 The change of apparent thermal conductivity for soda-lime glass.

tion, by using the concept of radiation conductivity. But since their solutions were based on the optically thick approximation or the gray medium assumption, the results led to a noticeable error in case of large wavelength dependency of absorption coefficient in the medium. Furthermore, they did not examine the applicable region of the superposition approximation of radiation and conduction, which was not always applicable.

In this paper, the applicability of the superposition approximation of radiation and conduction when the thermal conductivity of semitransparent medium is measured is examined by using the numerical solution in problem of combined radiation and conduction; and the effect of radiation heat transfer in the semitransparent medium is examined; and consequently, the region where the radiation heat transfer can be neglected, and the region where the optically thick approximation can be applied are clarified. And lastly, the true thermal conductivity of soda-lime glass is measured, and the effect of radiation heat transfer of this glass is examined. Consequently, the region where the radiation heat transfer can be neglected and the region where the optically thick approximation can be applied are clarified.

Nomenclature

B_1, B_2	Radiosities of walls
$c\rho$	Heat capacity
k	Thermal conductivity
k'	Apparent thermal conductivity
$k_{m,n}$	Mean thermal conductivity of m-th and n-th factors
L	Sample thickness
l	Number of bands in band model approximation
N_R	Radiation conduction parameter
q_t	Total heat flux
q_c	Conduction heat flux
q_R	Radiation heat flux
q_c'	Pure conduction heat flux
q_R'	Pure radiation heat flux
Q_R	Proportion of radiation heat flux
T	Temperature
γ	Refractive index
ϵ	Surface emissivity
κ	Absorption coefficient
ρ	Reflectivity
τ_0	Optical thickness

Subscripts

1	Radiation from left to right
2	Radiation from right to left
g_1, g_2	Internal generated radiation
R_1, R_2	Radiation of the below critical angle reflection
s_1, s_2	External radiation
TR_1, TR_2	Radiation of the total reflection
h	High temperature
c	Low temperature

2. Energy equation for semitransparent medium.

Since the energy equation for semitransparent medium become a complex integrodifferential equation, it is difficult to obtain a strict solution. For this reason, numerical solution by a finite difference equation which has been already reported by authors[3] is used. This numerical solution method is briefly describe here.

The next assumptions are made for this solution. (1) Absorption coefficient, reflectivity, and refractive index depend on the wavelength only, and they are independent of temperature. (2) Heater and cooler walls have diffuse surfaces. (3) Semitransparent medium' surfaces are specular, and there is no scattering in the medium. (4) The local thermodynamic equilibrium is valid. The heat transfer processes in semitransparent medium ($\gamma > 1$) are shown in Fig.3, and expressed by equation (4) to (12). In order to analyze the combined radiation and conduction problem by finite difference equation, the temperature field is divided into infinite plate of same thickness Δx ($n=2$ to $N-1$), $\Delta x/2$ ($n=1, N$) and T_n^p denotes the temperature at $x=n\Delta x$, $t=p\Delta t$. An explicit type finite differential equation is obtained as expressed in equation(3) by considering the conservation of energy in the n-th factor, using every heat transfer process, equations(4) to (12), as described previously.

$$T_n^{p+1} = T_n^p + \frac{\Delta t}{c\rho_n \Delta x} \Delta q_n \quad \text{----- (3)}$$

where

$$\Delta q_n = q_n - q_{n+1}, \quad q_n = q_n^R + q_n^c,$$

$$q_n^c = k_{n-1,n} (T_{n-1}^p - T_n^p) / \Delta x$$

$$q_n^R = q_n^{g1} - q_n^{g2} + q_n^{s1} - q_n^{s2} + q_n^{R1} - q_n^{R2} + q_n^{TR1} - q_n^{TR2} \quad \text{---- (4)}$$

$$q_n^{g1} = \sum_{\lambda=1}^l 2r_{\lambda}^2 \sum_{m=1}^{n-1} E b_{\lambda m} \{E_s(\tau_{m+1, n-1}) - E_s(\tau_{m, n-1})\} \quad \text{----- (5)}$$

$$q_n^{g2} = \sum_{\lambda=1}^l 2r_{\lambda}^2 \sum_{m=n+1}^N E b_{\lambda m} \{E_s(\tau_{n, m-1}) - E_s(\tau_{n, m})\} \quad \text{----- (6)}$$

$$q_n^{s1} = \sum_{\lambda=1}^l 2r_{\lambda}^2 (1 - \rho_{\lambda}) B_{1\lambda} \{E_s(\tau_{1, n-1}) - a^2 E_s(\tau_{1, n-1}/a)\} \quad \text{----- (7)}$$

$$q_n^{s2} = \sum_{\lambda=1}^l 2r_{\lambda}^2 (1 - \rho_{\lambda}) B_{2\lambda} \{E_s(\tau_{n, N}) - a^2 E_s(\tau_{n, N}/a)\} \quad \text{----- (8)}$$

$$q_n^{TR1} = \sum_{\lambda=1}^l 2r_{\lambda}^2 a^2 \sum_{m=1}^N E b_{\lambda m} \{E_s(\tau_{1, m-1}/a + \tau_{1, n-1}/a) - E_s(\tau_{1, m}/a + \tau_{1, n-1}/a)\} \quad \text{---- (9)}$$

$$q_n^{TR2} = \sum_{\lambda=1}^l 2r_{\lambda}^2 a^2 \sum_{m=1}^N E b_{\lambda m} \{E_s(\tau_{m+1, N}/a + \tau_{n, N}/a) - E_s(\tau_{1, m}/a + \tau_{1, n-1}/a)\} \quad \text{---- (10)}$$

$$q_n^{R1} = \sum_{\lambda=1}^l 2r_{\lambda}^2 \rho_{\lambda} \left[\sum_{m=1}^N Eb_{\lambda m} \{E_3(\tau_{1,m-1} + \tau_{1,n-1}) - a^2 E_3(\tau_{1,m-1}/a + \tau_{1,n-1}/a) - E_3(\tau_{1,m} + \tau_{1,n-1}) + a^2 E_3(\tau_{1,m}/a + \tau_{1,n-1}/a)\} + (1-\rho_{\lambda}) B_{2\lambda} \{E_3(\tau_{1,N} + \tau_{1,n}) - a^2 E_3(\tau_{1,N}/a + \tau_{1,n}/a)\} \right] \quad (11)$$

$$q_n^{R2} = \sum_{\lambda=1}^l 2r_{\lambda}^2 \rho_{\lambda} \left[\sum_{1}^N Eb_{\lambda m} \{E_3(\tau_{m+1,N} + \tau_{n,N}) - a^2 E_3(\tau_{m+1,N}/a + \tau_{n,N}/a) - E_3(\tau_{m,N} + \tau_{n,N}) + a^2 E_3(\tau_{m,N}/a + \tau_{n,N}/a)\} + (1-\rho_{\lambda}) B_{1\lambda} \{E_3(\tau_{1,N} + \tau_{n,N}) - a^2 E_3(\tau_{1,N}/a + \tau_{n,N}/a)\} \right] \quad (12)$$

$$\tau_{m,n} = \sum_{i=m}^n \Delta x_i \kappa_i$$

$$Eb_{\lambda m} = C_1 / \left[\lambda^5 \left\{ \exp\left(\frac{C_2}{\lambda T_m} - 1\right) \right\} \right]$$

$$C_1 = 3.2179 \times 10^8 \text{ kcal } \mu^4 / \text{m}^2 \text{h}$$

$$C_2 = 14388 \text{ } ^\circ \text{K}$$

$$E_3(t) = \int_0^1 \mu e^{-t/\mu} d\mu$$

$$a = \sqrt{1 - 1/r_{\lambda}^2}$$

$$\mu = \cos \theta$$

Fig.3 Heat transfer processes in semitransparent medium.

3. Examination of the superposition approximation of radiation and conduction.

The applicability of the superposition approximation of radiation and conduction which is assumed when the thermal conductivity of semitransparent medium is calculated is examined. The superposition approximation assumes that the total heat flux in the medium can be presented by a simple sum of pure radiation and pure conduction, and the applicability this approximation is examined for the parameters of the surface emissivity ϵ , radiation conduction parameter N_R and optical thickness τ_0 . Fig.4 shows the relation between N_R and the proportion of radiation heat flux $Q_R (=q_R/q_c)$ as the parameter of optical thickness. Solid line denotes the result of combined radiation and conduction, and dashed line denotes the result of superposition approximation, $Q_R' (=q_R'/(q_R' - q_c'))$. In the range of the optical thicknesses below $\tau_0 = 0.01$, or above $\tau_0 = 10$, these two curves

become coincident, but in the range of $\tau_0 = 0.1$ to 1.0 , these two disagree.

Considering the dependency of surface emissivity, the discrepancy of these curves is larger when the emissivity becomes smaller. But in the range of the optical thicknesses above $\tau_0 = 10$, the dependency of emissivity is not found, and this discrepancy become small. In order to discuss quantitatively this superposition approximation, the region where this approximation can be applied within an error of 5% is clarified; there by the error is defined as

$$[(q_t - q_R) - (q_t' - q_R')] / (q_t - q_R)$$

Fig.5 shows the region as the parameters of N_R, τ_0 and ϵ . In this figure, the range of right hand side is a region where this approximation can be applied, and the opposite side is one where this cannot be applied. The applicable region becomes the narrowest about $\tau_0 = 3$, and in this case, this approximation can be applied above $N_R = 5$ only. But, in the range of the optical thicknesses above $\tau_0 = 5$ or below $\tau_0 = 1$, this applicable region becomes wider, and also this is wider when the surface emissivity has a large value.

Lastly, when the superposition approximation in the measurement of thermal conductivity of semitransparent medium can not be applied, it is necessary to use the iterative calculation method or to change the sample thickness.

Fig.4 The proportion of radiation heat flux by the combined radiation and conduction, and the superposition approximation.

4. Examination about the effect of radiation heat transfer in the measurement of thermal conductivity for the semitransparent medium

In order to examine the effect of the

Fig.5 The region where the superposition approximation of radiation and conduction can be applied.

radiation heat transfer generally, the gray medium approximation is used. As the boundary condition on the numerical calculation, the prescribed surface temperature condition which correspond to the measuring condition of thermal conductivity, is used. The examination about the effect of radiation in the medium is performed as to both temperature distribution and heat flux, under steady state condition.

4-1 Temperature distribution

The steady state temperature distribution in the semitransparent medium as the parameter of N_R at $\tau_0 = 1.0$ are shown in Fig.6 in dimensionless expression. At $N_R = 3.0$, the temperature distribution is almost linear so that conduction becomes dominant, but, this turns to a curve which has a point of inflection at $x/L = 0.5$ as N_R becomes small. Fig.7 shows the distribution as parameter of optical thickness at $N_R = 0.1$, and this curve is almost linear at $\tau_0 = 0.01$ to 0.1 , but at $\tau_0 = 1.0$, it turns to a large winding curve. Consequently, the radiation effect becomes large with a decreasing N_R or with increasing τ_0 .

Although it is not shown in the figure, the change of temperature distribution with the change of the surface emissivity is negligibly small.

Fig.6 Temperature distribution in semitransparent medium.

4-2 Radiation heat flux

Here, it is examined how the change of radiation heat flux depends on various

parameters, that is, radiation conduction parameter, N_R , optical thickness, τ_0 , and surface emissivity ϵ . Since the heat flux distribution in the medium at the steady state presents an up-convex curve for radiation heat flux, q_R , and a down-convex one for conduction heat flux, q_C , and the summation of these two fluxes becomes constant as shown in Fig.8 as a model, the heat flux is examined at the center of the medium, $x/L = 0.5$.

Fig.7 Temperature distribution in semitransparent medium.

Fig.8 Heat flux distribution model in semitransparent medium.

Fig.9 shows the proportion of radiation heat flux Q_R to N_R as the parameter of τ_0 . Q_R changes from 0.1 to 0.9 while N_R changes two orders, and in the range of $Q_R = 0.2$ to 0.8 changes rapidly with changing of N_R but in the range of the proportions of radiation below 0.1 or above 0.9, these curves become asymptote to $Q_R = 0$ or 1. Dashed line denotes the result of optically thick approximation. This approximation becomes almost coincident for the value above $\tau_0 = 5$, but it cannot be applied below $\tau_0 = 1$.

Next, the dependency of surface emissivity is considered. Fig.10 shows Q_R to N_R as parameters of τ_0 and ϵ . The smaller N_R is, the smaller Q_R becomes, and this tendency becomes larger with smaller under the effect of the surface radiosity which contributes largely to radiation heat flux because of larger photon mean free path. On the other hand, in case of large values of optical thickness above 5, there is little effect of surface emissivity, because

the internal radiation heat flux contributes more than surface radiosity.

Figs.11 and 12 show the change of Q_R due to the change of emissivity at $N_R=0.1$ and 1.0. These changes are based on the radiation transfer at $\epsilon=0.9$, and the difference between this value and value at $\epsilon=0.3$ and 0.6 is presented. At $N_R=0.1$, the difference has a maximum value about $\tau_0=0.1$, and at $N_R=1.0$, it has a maximum value about $\tau_0=1.0$, both for $\epsilon=0.3$ and 0.6.

Now, referring to the previous Fig.9,

Fig.9 The proportion of radiation heat flux.

Fig.10 The proportion of radiation heat flux.

Fig.11 The change of Q_R due to the changing emissivity.

Fig.12 The change of Q_R due to the changing emissivity.

the region where the radiation heat transfer can be neglected within an error of 5% and the region where the optically thick approximation can be applied within an error of 5% are shown in Fig.13 with the parameters of N_R and τ_0 . The range under the curve 1 is the region where radiation can be neglected, and the range above the curve 2 is the region where the optically thick approximation can be applied. In the range except for the previous two, the general solution of combined radiation and conduction must be used.

Fig.13 The region where the radiation heat transfer can be neglected, and the region where the optically thick approximation can be applied.

5. The effect of radiation heat transfer on the measurement of thermal conductivity for soda-lime glass.

The general examination described in the previous sections is applied to soda-lime glass which is one of the semitransparent media.

5-1 The measurement of thermal conductivity for soda-lime glass.

As a specimen, soda-lime glass of 1 cm thickness is used, and the absorption coefficient, reflectivity and refractive index of this glass reported[3] previously are presented in table 1. As a standard sample, the zirconate brick (Asahi Glass Co., Ltd) is used, and its thermal conductivity is measured by the steady state method, and the result is shown in Fig.14, and its temperature dependency equation is expressed in Table 1. The emissivity of this standard sample surface is shown in Table 1 which is obtained experimentally, and this value is the total emissivity which is independent of wavelength and temperature. The experimental apparatus of thermal conductivity is shown in Fig.15. In order to produce a one dimensional heat flow in the glass, the measurement is done in the vacuum furnace of which temperature coincides with the sample temperature. The error of the superposition approximation, for soda-lime glass of 1 cm thickness, is 7% at $T=600^\circ\text{C}$ and $\epsilon=0.8$. In this measurement, since the measuring temperature range is under 600°C , this error is neglected.

The result of the true thermal conductivity of soda-lime glass is shown in Fig.16, and in this figure, for reference, the apparent thermal conductivity is shown

Table 1. Thermal and optical properties of sample.

Soda-lime glass	
Absorption coefficient(1/m)	45.0 (0 to 2.7 m)
	450.0 (2.7 to 4.5 m)
	∞ (4.5 to ∞ m)
reflectivity	0.04 (independent of wavelength)
refractive index	1.5 (independent of wavelength)
Standard sample(Zirconate brick)	
Thermal conductivity (Kcal/mh $^{\circ}$ C)	$k=4.31-0.00268 \cdot T$
Surface emissivity	0.8 (independent of wavelength)

with dashed line. Though the apparent thermal conductivity increases rapidly at the temperature above 300 $^{\circ}$ C because of radiation heat flux, the true one is almost linear. The temperature dependency equation of true thermal conductivity is presented as equation(13) which is used in the next section for analyzing the temperature field.

$$k(T)=0.82 + 0.00088 \cdot T \quad \text{-----}(13)$$

Fig.14 The thermal conductivity of standard sample(Zirconate brick).

Fig.15 The measuring apparatus of thermal conductivity.

Fig.16 The thermal conductivity for soda-lime glass.

5-2 Examination of the radiation effect in the soda-lime glass.

Using the optical properties and the true thermal conductivity which was obtained in the previous section for the soda-lime glass, the effect of radiation heat transfer in this glass is examined. As the boundary condition, the prescribed surface temperature condition is used. Fig.17 shows the proportion of radiation heat transfer to the surface temperature at $\epsilon=0.9$ as the parameter the thickness, L . Since Q_R slightly depends on the temperature difference $T_h - T_c$, but largely depends on the higher surface temperature, the temperature of horizontal axis is the higher surface temperature T_h . The effect of radiation depends on the temperature and thickness, and at low temperature the radiation heat flux can be neglected, and this depends on the thickness. In this figure, the result of optically thick approximation is also presented. The applicability region of this approximation depends on the temperature and the thickness.

Now, using Fig.17, the regions shown in Fig.13 previously are applied to soda-lime glass, and consequently, Fig.18 presented with the parameters of temperature and thickness is obtained. The range under the curve 1 is the region where radiation heat transfer can be neglected within an error of 5 %, and the range under the curve 2 and above curve 1 is the region where the optically thick approximation can be applied within an error of 5 %, and in the range except for the previous ones, the general solution of radiation and conduction must be used.

Fig.17 The proportion of radiation heat flux for soda-lime glass

Fig.18 The region where the radiation heat transfer can be neglected, and the region where the optically thick approximation can be applied(for soda-lime glass).

6. Conclusions

The effect of radiation heat transfer in the measurement of thermal conductivity for the semitransparent medium was examined for general case, and then the method in this examination was applied to soda-lime glass, with the following results:

- (1) The region where the superposition approximation of radiation and conduction can be applied within an error of 5 % was clarified as the parameters of radiation conduction parameter, optical thick and surface emissivity.
- (2) The effect of surface emissivity on the radiation heat flux is small when the optical thickness increases, and when the optical thickness is above 5, the effect of emissivity can be almost neglected.
- (3) The region where the radiation heat transfer can be neglected within an error of 5 % and the region where the optically

thick approximation can be applied within an error of 5 % were clarified using the parameters of the radiation conduction parameter and optical thickness.

(4) The true thermal conductivity of soda-lime glass was obtained.

(5) For soda-lime glass, the region where radiation can be neglected and the region where the optically thick approximation can be applied were clarified using the parameters of temperature and the thickness.

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