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Electrodeposition Stress Measurement of a Sphere by X-Ray in Depositing Method*

By Osamu DOI** and Atsumi OHTSUKI***

The authors classified measuring methods of internal stress in deposited layer into two kinds, i.e., depositing method and removing method, and proposed a series of the measuring theories for a plate, a cylinder and a sphere.

In the present paper, they propose a new method of applying X-ray to a multi-layered sphere in depositing process and show a compensating formula about a measured stress for the difference between base temperature and measuring one in the case of a two-layered sphere.

As application of their theory, they measured the internal stresses of two-layered spheres chromium deposited under two different conditions on steel balls and compared the results with those obtained by applying the removing method to the same specimen.

1. Introduction

The authors classified the methods of internal stress measurement of electrodeposited layer into a depositing method and a removing one and proposed a series of the measuring theories for a plate (Depositing Method⁽¹⁾, Removing Method^{(2)~(4)}), a cylinder (Depositing Method^{(5),(6)}, Removing Method^{(7),(8)}) and a sphere (Removing Method⁽⁹⁾).

The intent of this paper is to propose a new general measuring theory of the Depositing Method for a sphere by applying X-ray.

As application of this theory, the authors measured internal stresses of a chromium deposited layer and a base steel ball and confirmed the practicability of this method by comparing the results with those obtained by the X-ray in removing method⁽⁹⁾ for the same specimen.

In a general multi-layered sphere consisting of different materials, each element has a different elastic constant and a different linear expansion coefficient, so that internal stresses of deposited layers are influenced by temperature and consequently both temperatures of electrodeposition and stress measurement should be controlled sufficiently. It is necessary to specify the base temperature for denoting the internal stress value, and if there is a difference between a base temperature and a measuring one, a proper stress compensation is needed.

Assumptions for a theoretical analysis are as follows:

(1) Electrodeposition stresses are a function of radius r only.

(2) Each deposited layer and base metal element are all isotropic and their individual elastic constants are different from each other.

(3) Radial displacements of adjacent layer are equal at every boundary.

List of symbols

- r_i, r_{i+1} : inner and outer radius of i th spherical element
 ρ : depositing radius
 r : arbitrary radius within deposited layer ($r \leq \rho$)
 ρ_i : arbitrary radius within i th layer ($r_i \leq \rho_i \leq r_{i+1}$)
 $\sigma_{ir}(\rho), \sigma_{it}(\rho), \sigma_{it}(\rho)$: radial, tangential electrodeposition stresses and tangential X-ray measured stress of i th layer
 $\sigma'_{ir}(\rho_i, \rho), \sigma'_{it}(\rho_i, \rho)$: radial, tangential residual electrodeposition stresses at ρ_i ($r_i \leq \rho_i \leq r_{i+1}$) of i th layer when deposition reaches ρ ($r_{i+1} \leq \rho \leq r_{i+2}$)
 $\sigma''_{ir}(\rho_i, \rho), \sigma''_{it}(\rho_i, \rho)$: changes of radial and tangential stresses at ρ_i of i th layer when deposition reaches ρ
 $\epsilon_{ir}(\rho_i, \rho), \epsilon_{it}(\rho_i, \rho)$: changes of radial and tangential strains at ρ_i of i th layer when deposition reaches ρ
 $p_i(\rho)$: contact pressure ($r_{i+1} \leq \rho \leq r_{i+2}$) or change of contact pressure ($\rho \geq r_{i+2}$) at a boundary between i th layer and $(i+1)$ th layer when deposition reaches ρ

2. Analytical formula of residual electrodeposition stress measurement

The principle in the present method (Depositing Method) is based on the successive measurement of a surface stress by X-ray during deposition, and residual electrodeposition stresses within deposited layer and internal stresses caused within

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** Professor, Faculty of Engineering, Hokkaido University, Kita-ku Sapporo.

*** Research staff, Hokkaido Industrial Research Institute, Nishi-ku Sapporo.

base metal can be determined from theoretical formulas.

When internal and external pressure p_1 , p_2 act on a hollow thick-walled sphere with internal and external radii r_1 , r_2 , radial and tangential stresses σ_r , σ_t at r ($r_1 \leq r \leq r_2$) are as follows:

$$\left. \begin{aligned} \sigma_r &= \frac{p_1 r_1^3 - p_2 r_2^3}{r_2^3 - r_1^3} - \frac{p_1 - p_2}{r_2^3 - r_1^3} \left(\frac{r_1 r_2}{r} \right)^3 \\ \sigma_t &= \frac{p_1 r_1^3 - p_2 r_2^3}{r_2^3 - r_1^3} + \frac{p_1 - p_2}{2(r_2^3 - r_1^3)} \left(\frac{r_1 r_2}{r} \right)^3 \end{aligned} \right\} \dots (1)$$

The relations between radial displacement u and radial, tangential strains ϵ_r , ϵ_t are

$$\left. \begin{aligned} \epsilon_r &= \frac{du}{dr} = \frac{1-2\nu}{E} \frac{p_1 r_1^3 - p_2 r_2^3}{r_2^3 - r_1^3} - \frac{1+\nu}{E} \frac{p_1 - p_2}{r_2^3 - r_1^3} \left(\frac{r_1 r_2}{r} \right)^3 \\ \epsilon_t &= \frac{u}{r} = \frac{1-2\nu}{E} \frac{p_1 r_1^3 - p_2 r_2^3}{r_2^3 - r_1^3} + \frac{1+\nu}{E} \frac{p_1 - p_2}{2(r_2^3 - r_1^3)} \left(\frac{r_1 r_2}{r} \right)^3 \end{aligned} \right\} \dots (2)$$

where E : Young's modulus, ν : Poisson's ratio

The relation among σ_{mr} , σ_{mt} , $\sigma_{m't}$ in final m th layer deposition shown in Fig.1 is

$$\sigma_{mr}(\rho) = \sigma_{mi}(\rho) + \sigma_{m'i}(\rho, \rho) \dots (3)$$

Stress changes $\sigma'_{ir}(\rho_i, \rho)$, $\sigma'_{it}(\rho_i, \rho)$ at ρ_i ($r_i \leq \rho_i \leq r_{i+1}$) of i th layer when deposition reaches ρ are expressed in the following.

$$\left. \begin{aligned} \sigma'_{ir}(\rho_i, \rho) &= \frac{p_{i-1} r_{i-1}^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} - \frac{p_{i-1} - p_i}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \\ \sigma'_{it}(\rho_i, \rho) &= \frac{p_{i-1} r_{i-1}^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} + \frac{p_{i-1} - p_i}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \end{aligned} \right\} (i=1, \dots, m) \dots (4)$$

Similarly, strains $\epsilon'_{ir}(\rho_i, \rho)$, $\epsilon'_{it}(\rho_i, \rho)$ at ρ_i are

$$\left. \begin{aligned} \epsilon'_{ir}(\rho_i, \rho) &= \frac{1-2\nu_i}{E_i} \frac{p_{i-1} r_{i-1}^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} - \frac{1+\nu_i}{E_i} \frac{p_{i-1} - p_i}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \\ \epsilon'_{it}(\rho_i, \rho) &= \frac{1-2\nu_i}{E_i} \frac{p_{i-1} r_{i-1}^3 - p_i r_{i+1}^3}{r_{i+1}^3 - r_i^3} + \frac{1+\nu_i}{E_i} \frac{p_{i-1} - p_i}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \end{aligned} \right\} (i=1, \dots, m) \dots (5)$$

where $p_m = \sigma_{mr}$, $p_0 = 0$, $r_{m+1} = \rho$

Stress change $\sigma'_{mt}(\rho, \rho)$ of Eq.(3) is given from the relations of Eq.(4).

$$\sigma_{m'i}(\rho, \rho) = \frac{\sigma_{mr}(\rho)}{2(\rho^3 - r_m^3)} \{ 3r_m^3 \frac{p_{m-1}}{\sigma_{mr}(\rho)} - (2\rho^3 + r_m^3) \} \dots (6)$$

Considering the assumption (3) in Eq.(5), electrodeposition stress $\sigma_{mr}(\rho)$ of m th layer is shown as follows,

$$\sigma_{mr}(\rho) = \frac{[E_{m-1}(r_m^3 - r_{m-1}^3)\{2(1-2\nu_m)r_m^3 + (1+\nu_m)\rho^3\} + E_m(\rho^3 - r_m^3)\{2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3\}]}{3E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)\rho^3} p_{m-1} - \frac{E_m(1-\nu_{m-1})(\rho^3 - r_m^3)r_{m-1}^3}{E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)\rho^3} p_{m-2} \dots (7)$$

Contact pressure p_i between i th layer and $(i+1)$ th one is described by considering the assumption (3).

$$p_i(\rho) = k_{i-1} p_{i-1}(\rho) - l_{i-1} p_{i-2}(\rho) \quad (i=2, \dots, m-1) \dots (8)$$

Fig.1 Multi-layered sphere of m layers

Coefficients k , l of Eq.(8) are

$$k_i = \frac{[E_i(r_{i+1}^3 - r_i^3)\{2(1-2\nu_{i+1})r_{i+1}^3 + (1+\nu_{i+1})r_{i+2}^3\} + E_{i+1}(r_{i+2}^3 - r_{i+1}^3)\{2(1-2\nu_i)r_{i+1}^3 + (1+\nu_i)r_i^3\}]}{3E_i(1-\nu_{i+1})(r_{i+1}^3 - r_i^3)r_{i+2}^3}$$

$$l_i = \frac{E_{i+1}(1-\nu_i)(r_{i+2}^3 - r_{i+1}^3)r_i^3}{E_i(1-\nu_{i+1})(r_{i+1}^3 - r_i^3)r_{i+2}^3}$$

The following matrix representation is obtained from Eq.(8).

$$\begin{bmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & -k_2 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & -k_m \\ 1 & -k_{m-1} & l_{m-2} & 0 \end{bmatrix} \begin{bmatrix} p_2/p_1 \\ p_3/p_1 \\ \vdots \\ p_{m-1}/p_1 \end{bmatrix} = \begin{bmatrix} k_1 \\ -l_2 \\ 0 \\ 0 \end{bmatrix} \dots (9)$$

Putting the coefficient matrix with the $(m-2)$ rows and columns as $D^{(m-2)}$,

$$D^{(m-2)} = j^{(m+1)(m-2)} \quad (j = \sqrt{-1})$$

Putting as $D_i^{(m-2)}$ the determinant obtained by substituting the column vector of the right side of Eq.(9) in the i th column of the coefficient matrix $D^{(m-2)}$, contact pressure p_i is shown in the following.

$$p_i = (D_{m-i}^{(m-2)} / D^{(m-2)}) p_1 = D_{m-i}^{(m-2)} j^{(m+1)(m-2)} p_1 \quad (i=2, \dots, m-1) \dots (10)$$

Namely, variables $p_2 \sim p_{m-1}$ are all represented by variable p_1 only. In Eq.(7), putting $K(\rho)$, $L(\rho)$ as

$$K(\rho) = \frac{[E_{m-1}(r_m^3 - r_{m-1}^3)\{2(1-2\nu_m)r_m^3 + (1+\nu_m)\rho^3\} + E_m(\rho^3 - r_m^3)\{2(1-2\nu_{m-1})r_m^3 + (1+\nu_{m-1})r_{m-1}^3\}]}{3E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)\rho^3}$$

$$L(\rho) = \frac{E_m(1-\nu_{m-1})(\rho^3 - r_m^3)r_{m-1}^3}{E_{m-1}(1-\nu_m)(r_m^3 - r_{m-1}^3)\rho^3}$$

Considering the relation of Eq. (10) with respect to p ,

$$\begin{aligned}\sigma_{nr}(\rho) &= p_{n-1} \left\{ K(\rho) - \frac{p_{n-2}}{p_{n-1}} L(\rho) \right\} \\ &= p_{n-1} \left\{ K(\rho) - \frac{D_2^{(n-2)}}{D_1^{(n-2)}} L(\rho) \right\} \quad \dots\dots\dots(11)\end{aligned}$$

Putting $K(\rho) = \frac{D_2^{(n-2)}}{D_1^{(n-2)}} L(\rho) = \frac{3}{F(\rho)}$ in above Eq. (11), equation (12) is obtained.

$$\frac{p_{n-1}}{\sigma_{nr}(\rho)} = \frac{F(\rho)}{3} \quad \dots\dots\dots(12)$$

Substituting Eq. (12) into Eq. (6) and putting

$$H_n(\rho) = \frac{r_n^3(F(\rho)-1)-2\rho^3}{\rho^3-r_n^3},$$

$$\sigma_{ni}'(\rho, \rho) = \frac{H_n(\rho)}{2} \sigma_{nr}(\rho) \quad \dots\dots\dots(13)$$

From Eqs. (3), (13), the relation of σ_{nr} , σ_{nt} and σ_{nt} is shown in Eq. (14).

$$\sigma_{nr}(\rho) = \frac{2}{H_n(\rho)} (\sigma_{nr}(\rho) - \sigma_{ni}(\rho)) \quad \dots\dots\dots(14)$$

Equilibrium condition of σ_{nr} and σ_{nt} yields

$$\sigma_{nr}(\rho) = \frac{2}{\rho^2} \int_{r_n}^{\rho} r \sigma_{ni}(r) dr \quad \dots\dots\dots(15)$$

Equating Eq. (14) to Eq. (15)

$$\frac{1}{\rho^2} \int_{r_n}^{\rho} r \sigma_{nr}(r) dr = \frac{1}{H_n(\rho)} (\sigma_{nr}(\rho) - \sigma_{ni}(\rho)) \quad \dots\dots\dots(16)$$

Putting $\phi(\rho)$ as follows,

$$\int_{r_n}^{\rho} r \sigma_{ni}(r) dr = \phi(\rho) \quad \dots\dots\dots(17)$$

Hence the following differential equation is derived.

$$\frac{d\phi(\rho)}{d\rho} + \frac{H_n(\rho)}{\rho} \phi(\rho) = \rho \sigma_{nr}(\rho) \quad \dots\dots\dots(18)$$

Under consideration of condition

$\phi(r_n) = 0$, $e^{-U_n(\rho)} \neq 0$, the solution of Eq. (18) is given as

$$\phi(\rho) = e^{-U_n(\rho)} \int_{r_n}^{\rho} r \sigma_{nr}(r) e^{U_n(r)} dr \quad \dots\dots\dots(19)$$

where $\int_{r_n}^{\rho} \frac{H_n(\xi)}{\xi} d\xi = U_n(\rho) - U_n(r_n)$

In Eq. (18) or (19),

$$\begin{aligned}\frac{H_n(\rho)}{\rho} &= \frac{\left[2E_{n-1}(r_n^3 - r_{n-1}^3) \{ (1-2\nu_n)r_n^3 - (1+\nu_n)\rho^3 \} \right. \\ &\quad \left. - E_n(2\rho^3 + r_n^3) \{ 2(1-2\nu_{n-1})r_n^3 \right. \\ &\quad \left. + \left\{ \left(1 - \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) + \nu_{n-1} \left(1 + \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right\} r_{n-1}^3 \right]}{\rho \left[E_{n-1}(r_n^3 - r_{n-1}^3) \{ 2(1-2\nu_n)r_n^3 + (1+\nu_n)\rho^3 \} \right. \\ &\quad \left. + E_n(\rho^3 - r_n^3) \{ 2(1-2\nu_{n-1})r_n^3 \right. \\ &\quad \left. + \left\{ \left(1 - \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) + \nu_{n-1} \left(1 + \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right\} r_{n-1}^3 \right]} \\ &= \frac{1}{\rho} \frac{\left[E_{n-1}(1+\nu_n)(r_n^3 - r_{n-1}^3) \right. \\ &\quad \left. + E_n \left\{ 2(1-2\nu_{n-1})r_n^3 + \left\{ \left(1 - \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right. \right. \right. \\ &\quad \left. \left. \left. + \nu_{n-1} \left(1 + \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right\} r_{n-1}^3 \right\} \right]}{\left[E_{n-1}(r_n^3 - r_{n-1}^3) \{ 2(1-2\nu_n)r_n^3 + (1+\nu_n)\rho^3 \} \right. \\ &\quad \left. + E_n(\rho^3 - r_n^3) \{ 2(1-2\nu_{n-1})r_n^3 \right. \\ &\quad \left. + \left\{ \left(1 - \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) + \nu_{n-1} \left(1 + \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right\} r_{n-1}^3 \right]} \quad \dots\dots\dots(20)\end{aligned}$$

Putting

$$\begin{aligned}Y(\rho) &= E_{n-1}(r_n^3 - r_{n-1}^3) \{ 2(1-2\nu_n)r_n^3 + (1+\nu_n)\rho^3 \} \\ &\quad + E_n(\rho^3 - r_n^3) \{ 2(1-2\nu_{n-1})r_n^3 + \left\{ \left(1 - \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right. \right. \\ &\quad \left. \left. + \nu_{n-1} \left(1 + \frac{3D_2^{(n-2)}}{D_1^{(n-2)}} \right) \right\} r_{n-1}^3 \right\}\end{aligned}$$

equation (20) is reduced to the following equation.

$$\frac{H_n(\rho)}{\rho} = \frac{1}{\rho} - \frac{Y'(\rho)}{Y(\rho)} \quad \dots\dots\dots(21)$$

From Eq. (21),

$$U_n(\rho) = \ln \left\{ \frac{\rho}{Y(\rho)} \right\}, \quad U_n(r) = \ln \left\{ \frac{r}{Y(r)} \right\}$$

Substituting above relations into Eq. (19),

$$\phi(\rho) = \frac{Y(\rho)}{\rho} \int_{r_n}^{\rho} \frac{r^2}{Y(r)} \sigma_{nr}(r) dr \quad \dots\dots\dots(22)$$

From Eqs. (15), (17) and (22), electro-deposition stresses σ_{nr} , σ_{nt} of m th layer are

$$\sigma_{nr}(\rho) = \frac{2}{\rho^2} \phi(\rho) = \frac{2}{\rho^3} Y(\rho) \int_{r_n}^{\rho} \frac{r^2}{Y(r)} \sigma_{nr}(r) dr \quad \dots\dots\dots(23)$$

$$\sigma_{ni}(\rho) = \frac{1}{\rho} \frac{d\phi(\rho)}{d\rho} = \sigma_{nr}(\rho) - \frac{H_n(\rho)}{\rho^3} Y(\rho) \int_{r_n}^{\rho} \frac{r^2}{Y(r)} \sigma_{nr}(r) dr \quad \dots\dots\dots(24)$$

When deposition reaches r_{m+1} , residual electrodeposition stresses $\sigma_{nr}(\rho_m, r_{m+1})$, $\sigma_{nt}(\rho_m, r_{m+1})$ are

$$\left. \begin{aligned}\sigma_{nr}(\rho_m, r_{m+1}) &= \sigma_{nr}(\rho_m) + \sigma_{nr}'(\rho_m, r_{m+1}) \\ \sigma_{ni}(\rho_m, r_{m+1}) &= \sigma_{ni}(\rho_m) + \sigma_{ni}'(\rho_m, r_{m+1})\end{aligned} \right\} \quad \dots\dots\dots(25)$$

where

$$\left. \begin{aligned}\sigma_{nr}'(\rho_m, r_{m+1}) &= \frac{p_{m-1}(r_{m+1})r_m^3 - \sigma_{nr}(r_{m+1})r_{m+1}^3}{r_{m+1}^3 - r_m^3} \\ &\quad - \frac{p_{m-1}(r_{m+1}) - \sigma_{nr}(r_{m+1})}{r_{m+1}^3 - r_m^3} \left(\frac{r_m r_{m+1}}{\rho_m} \right)^3 \\ \sigma_{ni}'(\rho_m, r_{m+1}) &= \frac{p_{m-1}(r_{m+1})r_m^3 - \sigma_{nr}(r_{m+1})r_{m+1}^3}{r_{m+1}^3 - r_m^3} \\ &\quad + \frac{p_{m-1}(r_{m+1}) - \sigma_{nr}(r_{m+1})}{2(r_{m+1}^3 - r_m^3)} \left(\frac{r_m r_{m+1}}{\rho_m} \right)^3\end{aligned} \right\} \quad (26)$$

and residual electrodeposition stresses $\sigma_{ir}(\rho_i, r_{m+1})$, $\sigma_{it}(\rho_i, r_{m+1})$ of an arbitrary i th layer $\{ 2 \leq i \leq (m-1) \}$ are

$$\begin{aligned}\sigma_{ir}(\rho_i, r_{m+1}) &= \sigma_{ir}(\rho_i, r_{i+1}) + \sum_{k=i+1}^m \sigma_{ir}'(\rho_i, r_{k+1}) \\ &= \sigma_{ir}(\rho_i, r_{i+1}) + \frac{r_i^3 \sum_{k=i+1}^m \{ p_{i-1}(r_{k+1}) - r_{i+1}^3 \sum_{k=i+1}^m \{ p_i(r_{k+1}) \} \}}{r_{i+1}^3 - r_i^3} \\ &\quad - \frac{\sum_{k=i+1}^m \{ p_{i-1}(r_{k+1}) \} - \sum_{k=i+1}^m \{ p_i(r_{k+1}) \}}{r_{i+1}^3 - r_i^3} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3 \\ \sigma_{it}(\rho_i, r_{m+1}) &= \sigma_{it}(\rho_i, r_{i+1}) + \sum_{k=i+1}^m \sigma_{it}'(\rho_i, r_{k+1}) \\ &= \sigma_{it}(\rho_i, r_{i+1}) + \frac{r_i^3 \sum_{k=i+1}^m \{ p_{i-1}(r_{k+1}) \} - r_{i+1}^3 \sum_{k=i+1}^m \{ p_i(r_{k+1}) \}}{r_{i+1}^3 - r_i^3} \\ &\quad + \frac{\sum_{k=i+1}^m \{ p_{i-1}(r_{k+1}) \} - \sum_{k=i+1}^m \{ p_i(r_{k+1}) \}}{2(r_{i+1}^3 - r_i^3)} \left(\frac{r_i r_{i+1}}{\rho_i} \right)^3\end{aligned} \quad \dots\dots\dots(27)$$

Internal stresses $\sigma_{ir}(\rho_i, r_{m+1})$, $\sigma_{it}(\rho_i, r_{m+1})$ caused within a base hollow sphere are represented in the following.

$$\left. \begin{aligned} \sigma_{1r}(\rho_1, r_{m+1}) &= \sum_{k=2}^m \sigma_{1r}'(\rho_1, r_{k+1}) = - \frac{r_2^3 \sum_{k=2}^m \{p_1(r_{k+1})\}}{r_2^3 - r_1^3} \left(1 - \frac{r_1^3}{\rho_1^3}\right) \\ \sigma_{1t}(\rho_1, r_{m+1}) &= \sum_{k=2}^m \sigma_{1t}'(\rho_1, r_{k+1}) = - \frac{r_2^3 \sum_{k=2}^m \{p_1(r_{k+1})\}}{r_2^3 - r_1^3} \left(1 + \frac{r_1^3}{2\rho_1^3}\right) \end{aligned} \right\} (28)$$

The calculating formulas for a solid two-layered sphere are described as below.

$$\begin{aligned} \sigma_{2r}(\rho_2, r_3) &= \frac{2[E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)\rho_2^3\} + 2E_2(1-2\nu_1)(\rho_2^3 - r_2^3)]}{\rho_2^3} \\ &\times \int_{r_2}^{\rho_2} \frac{r^2 \sigma_{2r}(r)}{[E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)r^3\} + 2E_2(1-2\nu_1)(r^3 - r_2^3)]} dr \\ &+ \frac{p_1(r_3)r_2^3 - \sigma_{2r}(r_3)r_2^3}{r_2^3 - r_1^3} - \frac{p_1(r_3) - \sigma_{2r}(r_3)}{r_2^3 - r_1^3} \left(\frac{r_2 r_3}{\rho_2}\right)^3 \dots\dots\dots (29) \end{aligned}$$

$$\begin{aligned} \sigma_{2t}(\rho_2, r_3) &= \sigma_{2r}(\rho_2) - \frac{2[E_1\{(1-2\nu_2)r_2^3 - (1+\nu_2)\rho_2^3\} - E_2(1-2\nu_1)(2\rho_2^3 + r_2^3)]}{\rho_2^3} \\ &\times \int_{r_2}^{\rho_2} \frac{r^2 \sigma_{2r}(r)}{[E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)r^3\} + 2E_2(1-2\nu_1)(r^3 - r_2^3)]} dr \\ &+ \frac{p_1(r_3)r_2^3 - \sigma_{2r}(r_3)r_2^3}{r_2^3 - r_1^3} + \frac{p_1(r_3) - \sigma_{2r}(r_3)}{2(r_2^3 - r_1^3)} \left(\frac{r_2 r_3}{\rho_2}\right)^3 \dots\dots\dots (30) \end{aligned}$$

$$\sigma_{1r}(\rho_1, r_3) = \sigma_{1t}(\rho_1, r_3) = -p_1(r_3) \dots\dots\dots (31)$$

where

$$\begin{aligned} \sigma_{2r}(r_3) &= \frac{2[E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)r_3^3\} + 2E_2(1-2\nu_1)(r_3^3 - r_2^3)]}{r_3^3} \\ &\times \int_{r_2}^{r_3} \frac{r^2 \sigma_{2r}(r)}{[E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)r^3\} + 2E_2(1-2\nu_1)(r^3 - r_2^3)]} dr \\ p_1(r_3) &= \frac{3E_1(1-\nu_2)r_2^3 \sigma_{2r}(r_3)}{E_1\{2(1-2\nu_2)r_2^3 + (1+\nu_2)r_3^3\} + 2E_2(1-2\nu_1)(r_3^3 - r_2^3)} \end{aligned}$$

3. Stress compensation for a temperature change

Usually an electrodeposition stress varies due to electrodeposition bath temperature, therefore this temperature must be kept at a constant base temperature and the base temperature has to be expressed clearly.

In addition, an electrodeposited sphere is composed in general of multi-layers with different elastic constants and linear expansion coefficients, therefore the thermal stress would be added if the ambient temperature deviated from the base temperature after electrodeposition at a constant temperature. If the X-ray measuring temperature were different from the electrodeposition bath temperature (base temperature), a proper stress compensation would be needed.

Calculating formulas of a stress compensation for a two-layered sphere shown in the following will be offered.

Radial displacement Δu at r of a sphere under the temperature distribution of T_i is

$$\Delta u_i = A_i r + \frac{B_i}{r^2} + \frac{1+\nu_i}{1-\nu_i} \alpha_i \frac{1}{r^2} \int_{r_i}^r T_i \rho^2 d\rho \quad (i=1, 2) \dots\dots\dots (32)$$

From Eq.(32), compensating stresses for a temperature rise from the base one are shown as follows.

$$\left. \begin{aligned} \Delta \sigma_{1r} &= 2G_1 \left\{ \frac{1+\nu_1}{1-2\nu_1} A_1 - 2 \frac{B_1}{r^3} - \frac{1+\nu_1}{1-\nu_1} \alpha_1 \frac{2}{r^3} \int_{r_1}^r T_1 \rho^2 d\rho \right\} \\ \Delta \sigma_{1t} &= 2G_1 \left\{ \frac{1+\nu_1}{1-2\nu_1} A_1 + \frac{B_1}{r^3} + \frac{1+\nu_1}{1-\nu_1} \alpha_1 \left(\frac{1}{r^3} \int_{r_1}^r T_1 \rho^2 d\rho - T_i \right) \right\} \end{aligned} \right\} (33)$$

where $r_1 \leq \rho \leq r \leq r_2$ for $i=1$, $r \leq \rho \leq r \leq r_3$ for $i=2$,

Fig.2 Compensation of X-ray measured stress

Fig.3 Schematic view of electrodeposition apparatus

G_i : shear modulus, α_i : linear expansion coefficient, ν : Poisson's ratio.

Constants A_i , B_i in Eqs.(32),(33) are determined from the following boundary conditions.

- (1) $\Delta \sigma_{1r}(r_1) = 0$ at the internal surface $r=r_1$ of 1st layer for a hollow sphere, or $B_1=0$, $r_1=0$ for a solid sphere.
- (2) $\Delta \sigma_{1r}(r_2) = \Delta \sigma_{2r}(r_2)$ at the boundary $r=r_2$ of 1st and 2nd layers.
- (3) $\Delta u_1(r_2) = \Delta u_2(r_2)$ at the boundary $r=r_2$ of 1st and 2nd layers.
- (4) $\Delta \sigma_{2r}(r_3) = 0$ at the external surface $r=r_3$ of 2nd layer.

As shown in Eq.(33), compensating stresses are a function of depositing radius and radial distribution of temperatures.

Table 1 Chemical component and condition of electropolishing

Chemical components % (vol)	H ₃ PO ₄ 50 H ₂ SO ₄ 50
Temperature °C	30
Voltage V	DC 100
Current density A/dm ²	10
Electrolysis time s	60

Table 2 Chemical composition and conditions of electrodeposition and electrolyte for chromium

		Electrodeposition bath	Electrolyte
Chemical components g		CrO ₃ 250	NaOH 40
		H ₂ SO ₄ 2.5	Na ₂ CO ₃ 50
		H ₂ O 1000	H ₂ O 1000
Temperature °C		30	30
Current density A/dm ²	Specimen 1	30	15
	Specimen 2	20	5

For the two-layered solid sphere shown in the later examples, the compensation of X-ray measured stress for unit temperature rise from the base temperature is illustrated in Fig. 2 as a function of depositing radius.

4. Examples of application

Applying the Depositing Method by X-ray to the depositing process of chromium on an annealed solid sphere, the authors measured residual electrodeposition stresses in deposited layer and internal stresses in base sphere and confirmed the practicability of this method by comparing the results with those obtained by applying the Removing Method⁽⁹⁾.

The base steel sphere is a bearing ball of SUJ-2 (JIS G 4805) with 1 inch diameter (JIS B 1501) and brightly annealed at 800 °C for 90 minutes.

Before deposition, a hole of 2.5 mm ϕ diameter and about 6 mm depth is bored and plugged with a leading wire. The sphere is dipped into an electrodeposition bath of glass as shown in Fig. 3.

After the sphere is polished electrolytically under the conditions of Table 1, electrodeposition is done in a general Sargent bath whose composition and depositing conditions are shown in Table 2. For the Removing Method, electrolysis shown in Table 2 is adopted.

Measurement of X-ray stress is done by Geiger Counter Method⁽¹⁰⁾ (0° - 45° method)

with an apparatus of parallel beam type and a peak point is determined by half width value method. Conditions of X-ray measurement are shown in Table 3 and the incident part of X-ray beam is the equator of a sphere whose pole is the plugged point of the leading wire. The elastic constants and linear expansion coefficients are shown in Table 4.

The temperature variation of electrodeposition bath is kept within less than ± 0.5 °C by utilizing a double vessel and an air stirring as shown in Fig. 3.

Because the electrodeposition bath temperature (base temperature) is set at 30 °C and the measuring temperature of X-ray stress at 20 °C, a compensation of X-ray measured stress is done for temperature difference 10 °C from Eq. (33).

Table 3 Conditions of X-ray stress measurement

Characteristic X-ray		Cr-K α
Tube voltage	kV	30
Tube current	mA	8
Count range	c/s	400
Time constant	s	16
Slit divergent angle	°	0.35
Irradiated area	mm ²	2 $\times$ 7

Table 4 Elastic constant and linear expansion coefficient

		Steel (SUJ-2)	Chromium (deposited layer)
E	kg/mm ²	2.1×10^4	1.85×10^4 (11)
ν		0.28	0.23 (12)
α	1/°C	1.20×10^{-5}	0.65×10^{-5} (13)

(a) Specimen 1 (electrodeposition bath temperature 30 °C, current density of electrodeposition 30 A/dm²)

(b) Specimen 2 (electrodeposition bath temperature 30 °C, current density of electrodeposition 20 A/dm²)

Fig.4 Distributions of residual electrodeposition stresses in deposited layer and internal stresses in base metal

The X-ray measured stresses, the residual electrodeposition stresses in deposited layer and the internal stresses in base sphere are shown in Figs.4(a),(b) for Specimens 1 and 2 respectively, and the results obtained by applying X-ray in Removing Method⁽⁹⁾ after deposition is together shown in the same figures.

In both specimens, tangential stresses of deposited layer are tensile and have similar tendencies to decrease in the exterior, but the stress in Specimen 1 of higher current density is revealed to be lower. On the other hand radial stresses are compressive, but their values are negligibly small compared with the tangential ones.

In a base sphere, both radial and tangential stresses are constant in all regions and the radial stresses of both deposited layer and base sphere coincide at the boundary.

The results by the present method agree considerably well with those by the Removing Method and only a slight difference of maximum 2 Kg/mm² appears.

5. Conclusions

(1) The authors have classified the measuring methods of residual electrodeposition stress into a Depositing Method and a Removing Method. They propose in this report a general theory of the Depositing Method by applying X-ray for an electrodeposited multi-layered sphere.

(2) In measuring the internal stresses of deposited members, a sufficient temperature control is needed and the base temperature (electrodeposition bath temperature) must be specifically expressed to show residual stresses. The authors point

out the necessity of a proper stress compensation for a difference between the base temperature and the measuring temperature and show the compensating formulas for a two-layered sphere.

(3) Applying the Depositing Method by X-ray, the authors measured the internal stress distributions of chromium deposited two-layered spheres and confirmed the practicability of this method by comparing with the results obtained by applying the Removing Method by X-ray.

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