Dynamic Response of Bayonet-Type Heat Exchangers*

(Part 1, Response to Inlet Temperature Changes)

By Isao TODO**

Bayonet-type heat exchangers are classified into four cases by the flow directions of shell- and tube-side fluids. The transfer functions, relating outlet temperature responses to changes in inlet temperatures of both tube- and shell-side fluids, are obtained for bayonet-type heat exchangers. Numerical examples are presented, and the effects of flow directions on the frequency response are examined. Experiments of frequency response are carried out, and the data are shown to be in good agreement with the theoretical results.

1. Introduction

Bayonet-type heat exchangers have special merits as follows: (i) The maintenance is easy, because both the inner- and the outer-tubes can be detached from the shell; and (ii) it is unnecessary to take into account the thermal stress, because the thermal expansions of the walls in flow directions are free to take place independently of each other. Therefore, bayonet-type heat exchangers are fit for use, when the maintenance is frequently carried out, or when there is a large temperature difference between shell-side and tube-side fluids. Bayonet-type heat exchange methods are applied in a certain class of steam generators. In both design and control schemes of heat exchanger processes, the knowledge of dynamics gives effective information. The dynamic responses of many kinds of heat exchangers, such as multitube heat exchangers, are analyzed. However, the dynamics of bayonet-type heat exchangers has not been known to us. In the present paper, the dynamics of the bayonet-type heat exchangers is analyzed and discussed from the control engineering point of view. The transfer functions are obtained in which the outlet temperature responses are related to changes in inlet temperatures of both fluids. The theory is confirmed by experiments.

2. Nomenclature

\[
\begin{align*}
a_{11} &= \frac{K_t L_d}{w_1 \bar{e}_1}, & a_{12} &= \frac{K_t L_d}{w_1 \bar{e}_1}, & a_1 &= \frac{K_t L_d}{w_1 \bar{e}_1} \\
b_{11} &= \frac{h_t L_d}{C_s \bar{e}_1}, & b_{12} &= \frac{h_t L_d}{C_s \bar{e}_1}, & b_1 &= \frac{h_t L_d}{C_s \bar{e}_1} \\
b_t &= \frac{h_t L_d}{C_s \bar{e}_1}, & b_1 &= \frac{h_t L_d}{C_s \bar{e}_1} \\
C &= \text{heat capacity of wall per unit length along the flow} \quad \text{kcal/m}^\circ\text{C} \\
c_p &= \text{specific heat of fluid} \quad \text{kcal/kg}^\circ\text{C} \\
h &= \text{heat transfer coefficient} \quad \text{kcal/m}^2\text{h}^\circ\text{C} \\
K &= \text{overall heat transfer coefficient} \quad \text{kcal/m}^2\text{h}^\circ\text{C} \\
L &= \text{perimeter of the heat exchange surface} \quad \text{m} \\
l &= \text{length of shell} \quad \text{m} \\
P &= \text{Laplace transform operator for } \xi \quad \text{dl} \\
r = \frac{\xi}{\xi} \quad \text{dl} \\
r_2 = \frac{\xi}{\xi} \quad \text{S} \quad \text{dl} \\
S &= \text{sectional area} \quad \text{m}^2 \\
s &= \text{Laplace transform operator for } \tau \quad \text{dl} \\
T_{L1} &= l / \bar{e}_1 \quad \text{h} \\
t &= \text{time} \quad \text{h} \\
v &= \text{fluid velocity} \quad \text{m/h} \\
w &= \text{heat capacity of fluid per unit length, } w_1 = c_p T_b S_1, \quad w_2 = c_p T_b S_1, \quad w = c_p T_s \quad \text{kcal/m} \quad \text{C} \\
z &= \text{distance along the heat exchanger} \quad \text{m} \\
T &= \text{specific weight} \quad \text{kg/m}^3 \\
\Delta(\) &= \text{deviation of ( ) from steady state value} \\
\theta &= \text{temperature} \quad \text{C} \\
\xi &= \text{dimensionless distance} = z / l \quad \text{dl} \\
\tau &= \text{dimensionless time} = t / T_{L1} \quad \text{dl} \\
o &= \text{dimensionless circular frequency} \quad \text{dl}
\end{align*}
\]

Subscripts

\[ \begin{align*}
f &= \text{tube-side fluid} \\
l &= \text{inner tube-side fluid} \\
o &= \text{outer tube-side fluid} \\
\text{Numerals are not used for shell-side fluid as subscripts.} \\
\end{align*} \]

* Received 29th August, 1974.
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† dl denotes dimensionless numbers.
hi : inner tube
ho : outer tube
s : shell
i : inlet when used with \( \theta \);
inner tube-side
o : outlet when used with \( \theta \);
outer tube-side
im : passage between inner- and outer-tubes

Superscripts
(\( T \)): matrix transpose of (\( T \))
(\( - \)): steady state value when used with
\( a', a, b, k, K \) and \( \theta \)

3. Fundamental equations

A bayonet-type heat exchanger is schematically shown in Fig.1(a). Fig.1(b) is a simplification of Fig.1(a). There are four cases in bayonet-type heat exchanger processes as shown in Fig.1.

The following basic assumptions are made in deriving the fundamental equations:

(i) The temperature is uniform over any cross section at right angles to the flow directions.
(ii) Heat losses are neglected.
(iii) The fluids are incompressible.
(iv) The thermal conductivities of the walls are infinite in the radial direction or assumed to be included in heat transfer coefficients, and zero in flow directions.
(v) Heat transfer coefficients do not change with the temperatures of fluids or solid walls.

Under the assumptions stated above, the following equations are derived:

\[
\begin{align*}
\frac{\partial \theta_i}{\partial t} + \nu \frac{\partial \theta_i}{\partial x} &= \frac{h_{11}}{w_1} (\theta_{1i} - \theta_i) \\
\frac{\partial \theta_o}{\partial t} + \nu \frac{\partial \theta_o}{\partial x} &= \frac{h_{12}}{w_2} (\theta_{1o} - \theta_o) \\
&+ \frac{h_{21}}{w_1} (\theta_{2i} - \theta_o) \\
\frac{\partial \theta_i}{\partial t} + \nu \frac{\partial \theta_i}{\partial x} &= \frac{h_{21}}{w_1} (\theta_{2i} - \theta_i) \\
&+ \frac{h_{21}}{w_1} (\theta_{2o} - \theta_i) \\
\frac{\partial \theta_o}{\partial t} + \nu \frac{\partial \theta_o}{\partial x} &= \frac{h_{22}}{w_2} (\theta_{2i} - \theta_o) \\
&+ \frac{h_{22}}{w_2} (\theta_{2o} - \theta_o) \\
\end{align*}
\]

(1)

In this paper, the upper part of double sign (i.e. \( \text{\( \bar{\gamma} \)}} \) or \( \text{\( \text{\( \bar{\gamma} \)}} \)) is for cases C*-P, and P-C*, and the lower part, for cases C-P*, and P*-C.

Introducing both the dimensionless time and the dimensionless distance

\[
\xi = \frac{x}{l_i}, \quad \tau = \frac{t}{T_{4i}}
\]

(2)

and using the relations given by

\[
\begin{align*}
\frac{\partial h_{11}}{\partial \xi} &= a_{1i}(\theta_{1i} - \theta_i) \\
\frac{\partial h_{12}}{\partial \xi} &= a_{1o}(\theta_{1o} - \theta_o) \\
\frac{\partial h_{21}}{\partial \xi} &= a_{2i}(\theta_{2i} - \theta_i) \\
\frac{\partial h_{22}}{\partial \xi} &= a_{2o}(\theta_{2o} - \theta_o) \\
\frac{\partial \theta_i}{\partial \tau} &= b_{i}(\theta_i - \theta_i) \\
\frac{\partial \theta_o}{\partial \tau} &= b_{o}(\theta_o - \theta_o)
\end{align*}
\]

(3)

Eq. (1) can be normalized as follows:

\[
\begin{align*}
\frac{\partial \theta_{1i}}{\partial \xi} + \nu \frac{\partial \theta_{1i}}{\partial \xi} &= a_{1i}(\theta_{1i} - \theta_i) \\
\frac{\partial \theta_{1o}}{\partial \xi} + \nu \frac{\partial \theta_{1o}}{\partial \xi} &= a_{1o}(\theta_{1o} - \theta_o) \\
\frac{\partial \theta_{2i}}{\partial \xi} + \nu \frac{\partial \theta_{2i}}{\partial \xi} &= a_{2i}(\theta_{2i} - \theta_i) \\
\frac{\partial \theta_{2o}}{\partial \xi} + \nu \frac{\partial \theta_{2o}}{\partial \xi} &= a_{2o}(\theta_{2o} - \theta_o)
\end{align*}
\]

(4)

Let the temperature be expressed as the sum of a steady-state term and a time-dependent term as follows:

\[
\begin{align*}
\theta_i &= \theta_{i0} + \theta_{i1}(\xi) + \Delta \theta_i \\
\theta_o &= \theta_{o0} + \theta_{o1}(\xi) + \Delta \theta_o \\
\theta_{i0} &= \theta_{i0}(\xi) + \Delta \theta_{i0} \\
\theta_{o0} &= \theta_{o0}(\xi) + \Delta \theta_{o0}
\end{align*}
\]

Fig.1 Bayonet-type heat exchanger
(Case C*-P)

(a) Schematic representation

(b) Notations

Fig.2 Classification by flow-directions

(a) C*-P

(b) P*-C

(c) C-P*

(d) P*-C
Substituting Eq. (4) into Eq. (3) and using the steady state relations yields the equations in terms of the variables designated by $J$.

In this paper, the velocities are assumed to be constant. Therefore, $h$, $a'$ and $b$ are equal to $h$, $a'$ and $b$, respectively.

The Laplace transform of the resulting equations can be taken with respect to $\tau$, by putting all the initial conditions at $\tau=0$ as zero. Then, eliminating $\dot{\theta}_h$, $\dot{\theta}_a$, and $\dot{\theta}_b$, and using the relationship
\[
\ddot{a}_1' = \dot{a}_1' = \dot{a}_1 
\]
yields
\[
\frac{d}{d\tau} \theta(\xi, s) = A \theta(\xi, s) \quad \text{------------------(6)}
\]
where
\[
\theta(\xi, s) = [\theta_1(\xi, s) \ \theta_2(\xi, s) \ \theta_3(\xi, s)]^T \quad \text{------------------(7)}
\]
\[
A = \begin{bmatrix}
\pm f_1 & \mp g_1 & 0 \\
\pm g_1 & \mp f_1 & \pm g_1 \\
0 & g & -f
\end{bmatrix} \quad \text{------------------(8)}
\]

In Table 1, the symbols ($f_1$, $g_1$, etc.) used in Eq. (8) are tabulated for the following four cases: (1) With heat capacities of inner-tube, outer-tube and shell; (2) with heat capacities of inner-tube and outer-tube; (3) with heat capacity of shell; and (4) without heat capacities of inner-tube, outer-tube and shell.

Using the law of heat transmission, the following relations are obtained.
\[
\dot{a}_1 = \frac{a_1 b_1}{(s + b_2)} \quad \dot{a}_2 = \frac{a_2 b_2}{(s + b_3)} \quad \dot{a}_3 = \frac{a_3 b_3}{(s + b_1)} 
\]

4. Transfer functions

Taking the Laplace transform of Eq. (6) with respect to $\xi$ yields
\[
\theta(p, s) = (pI - A)^{-1} \theta(0, s) \quad \text{------------------(10)}
\]
where $I$ is a unity matrix.

Taking the inverse Laplace transform of Eq. (10) with respect to $p$, and putting $\xi=1$, yields
\[
\theta(1, s) = \begin{bmatrix}
\phi_{11}(p) & \phi_{12}(p) & \phi_{13}(p) \\
\phi_{21}(p) & \phi_{22}(p) & \phi_{23}(p) \\
\phi_{31}(p) & \phi_{32}(p) & \phi_{33}(p)
\end{bmatrix} \theta(0, s) \quad \text{------------------(11)}
\]
where
\[
\phi_{mn} = \sum_{l=1}^{\infty} \frac{\phi_{ma}(p) \epsilon^{s}}{s^{n+m-2}} \quad \text{------------------(12)}
\]

\[
\phi_{11}(p) \ \phi_{12}(p) \ \phi_{13}(p) \\
\phi_{21}(p) \ \phi_{22}(p) \ \phi_{23}(p) \\
\phi_{31}(p) \ \phi_{32}(p) \ \phi_{33}(p)
\]

\[
\begin{bmatrix}
p \mp f_1 & (s + f_1) & 0 \\
0 & g + g_1 & \mp g_1 \\
0 & -f & \mp f_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
p \pm f_1 & \mp g_1 & 0 \\
0 & g & -f
\end{bmatrix} \quad \text{------------------(13)}
\]

$p_1$, $p_2$ and $p_3$ are the three roots of $|pI - A| = 0 \quad \text{------------------(14)}$

From Eqs. (12) and (13),
\[
\Phi_{11} = \phi_{11}(p) \quad \Phi_{12} = \phi_{12}(p) \quad \Phi_{13} = \phi_{13}(p) \quad \text{------------------(15)}
\]

\[
\Phi_{21} = \pm (\dot{a}_1 \delta_3) \Phi_{31} \quad \Phi_{22} = \pm (\dot{a}_1 \delta_3) \Phi_{32} \quad \Phi_{23} = \pm (\dot{a}_1 \delta_3) \Phi_{33}
\]

The input-output relations can be expressed in the matrix form
\[
[\dot{\theta}_h(\xi, s)] = G(s) \begin{bmatrix}
\dot{\theta}_h(\xi, s) \\
\dot{\theta}_h(\xi, s)
\end{bmatrix} \quad \text{------------------(16)}
\]

where $G(s)$ is a matrix transfer function which relates outlet temperature responses to changes in inlet temperatures. $G_{CR}(s)$, $G_{CT}(s)$ and $G_{DR}(s)$ represent for cases $C$-$P$, $P$-$C$, $C$-$P$ and $P$-$C$, respectively. By putting $s = j\omega$ in $G(s)$, a matrix

<table>
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<th>Table 1</th>
<th>Comparison of expression formulae</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
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<tr>
<td>With heat capacities of inner-tube, outer-tube and shell</td>
<td>With heat capacities of inner-tube and outer-tube</td>
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<tr>
<td>$f_1$</td>
<td>$s + \frac{a_1(\xi + b_2)}{s + b_1 + b_2}$</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$r_{12} + \frac{a_2(\xi + b_3)}{s + b_1 + b_2}$</td>
</tr>
<tr>
<td>$f$</td>
<td>$r_{12} + \frac{a_3(\xi + b_3)}{s + b_1 + b_2}$</td>
</tr>
<tr>
<td>$g_1$</td>
<td>$\frac{a_1}{s + b_1 + b_2}$</td>
</tr>
<tr>
<td>$g_2$</td>
<td>$\frac{a_2}{s + b_2}$</td>
</tr>
<tr>
<td>$g$</td>
<td>$\frac{a_3}{s + b_3}$</td>
</tr>
<tr>
<td>$\frac{a_4}{s + b_3}$</td>
<td>$\dot{a}_4$</td>
</tr>
</tbody>
</table>
frequency transfer function $G(j\omega)$ is obtained, where $j=\sqrt{-1}$

4.1 Cases C*-P and P-C*

4.1.1 Case C*-P

Applying the boundary conditions

$\mathbf{A}(0, s) = [\mathbf{A}_{0}(s) \quad \mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s)]\mathbf{T}$

$\mathbf{A}(1, s) = [\mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s) \quad \mathbf{A}_{3}(s)]\mathbf{T}$

(17) to Eq. (11) yields

$G_{CP}(s) = \begin{bmatrix} G_{CP-0}(s) & G_{CP-1}(s) \\ G_{CP+1}(s) & G_{CP+2}(s) \end{bmatrix}$

$= \begin{bmatrix} \phi_{11} + \phi_{12} & -\phi_{12} + \phi_{11} + \phi_{13} \\ \phi_{12} + \phi_{11} & -\phi_{11} + \phi_{12} + \phi_{13} \end{bmatrix}$

(18) to Eq. (11) yields

$G_{PC}(s) = \begin{bmatrix} G_{CP-0}(s) & G_{CP-1}(s) \\ G_{CP-1}(s) & G_{CP-0}(s) \end{bmatrix}$

$= \begin{bmatrix} (\dot{\alpha}_{13} \dot{\alpha}_{21}) G_{CP-1}(s) & G_{CP-0}(s) \\ G_{CP-0}(s) & (\dot{\alpha}_{13} \dot{\alpha}_{21}) G_{CP-1}(s) \end{bmatrix}$

(19)

From Eq. (20), it is found that $G_{PC}(s)$ can be expressed in terms of the elements of $G_{CP}(s)$.

4.2 Cases C*-P and P*-C

4.2.1 Case C*-P

Applying the boundary conditions

$\mathbf{A}(0, s) = [\mathbf{A}_{0}(s) \quad \mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s)]\mathbf{T}$

$\mathbf{A}(1, s) = [\mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s) \quad \mathbf{A}_{3}(s)]\mathbf{T}$

(21) to Eq. (11) yields

$G_{CP}(s) = \begin{bmatrix} G_{CP-0}(s) & G_{CP-1}(s) \\ G_{CP+1}(s) & G_{CP+2}(s) \end{bmatrix}$

$= \begin{bmatrix} \phi_{11} + \phi_{12} & -\phi_{12} + \phi_{11} + \phi_{13} \\ \phi_{12} + \phi_{11} & -\phi_{11} + \phi_{12} + \phi_{13} \end{bmatrix}$

(22) to Eq. (11) yields

$G_{PC}(s) = \begin{bmatrix} G_{CP-0}(s) & G_{CP-1}(s) \\ G_{CP-1}(s) & G_{CP-0}(s) \end{bmatrix}$

$= \begin{bmatrix} (\dot{\alpha}_{13} \dot{\alpha}_{21}) G_{CP-1}(s) & G_{CP-0}(s) \\ G_{CP-0}(s) & (\dot{\alpha}_{13} \dot{\alpha}_{21}) G_{CP-1}(s) \end{bmatrix}$

(23)

4.2.2 Case P*-C

Applying the boundary conditions

$\mathbf{A}(0, s) = [\mathbf{A}_{0}(s) \quad \mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s)]\mathbf{T}$

$\mathbf{A}(1, s) = [\mathbf{A}_{1}(s) \quad \mathbf{A}_{2}(s) \quad \mathbf{A}_{3}(s)]\mathbf{T}$

(24) to Eq. (11) yields

$G_{CP}(s) = \begin{bmatrix} G_{CP-0}(s) & G_{CP-1}(s) \\ G_{CP+1}(s) & G_{CP+2}(s) \end{bmatrix}$

$= \begin{bmatrix} \phi_{11} + \phi_{12} & -\phi_{12} + \phi_{11} + \phi_{13} \\ \phi_{12} + \phi_{11} & -\phi_{11} + \phi_{12} + \phi_{13} \end{bmatrix}$

From Eq. (24), it is also found that $G_{PC}(s)$ can be expressed in terms of the elements of $G_{CP}(s)$.

5. Static characteristics

5.1 Cases C*-P and P-C*

Putting $s=0$, Eqs. (18) and (20) become

$G_{CP}(0) = G_{CP}(0) = \frac{1}{(p_{1} + \alpha_{23})} \begin{bmatrix} (p_{1} + \alpha_{23})e^{\alpha_{23}} & -\alpha_{23}e^{\alpha_{23}} \\ -(p_{1} + \alpha_{23})e^{\alpha_{23}} & (p_{1} + \alpha_{23})e^{\alpha_{23}} \end{bmatrix}$

(25)

where $p_{1}$ and $p_{2}$ are the two roots of $p^{2} + (\dot{\alpha}_{13} + \dot{\alpha}_{21})p - \dot{\alpha}_{13}\dot{\alpha}_{21} = 0$.

5.2 Cases C*-P and P*-C

Putting $s=0$, Eqs. (22) and (24) become

$G_{CP}(0) = G_{CP}(0)$

$= \frac{1}{(p_{1} - \alpha_{23})} \begin{bmatrix} (p_{1} - \alpha_{23})e^{\alpha_{23}} & -\alpha_{23}e^{\alpha_{23}} \\ -(p_{1} - \alpha_{23})e^{\alpha_{23}} & (p_{1} - \alpha_{23})e^{\alpha_{23}} \end{bmatrix}$

(27)

where $p_{1}$ and $p_{2}$ are the two roots of $p^{2} + (\dot{\alpha}_{13} - \dot{\alpha}_{21})p - \dot{\alpha}_{13}\dot{\alpha}_{21} = 0$.

6. Numerical examples

The effects of flow directions on the frequency responses are examined when the heat capacities of inner, outer-tubes and shell are neglected. The system parameters used as numerical examples are:

$\dot{\alpha}_{11} = 0.628$, $\dot{\alpha}_{13} = \dot{\alpha}_{21} = 1$, $r_{1} = r_{2} = 1$

The foregoing values are obtained by applying the Dittus-Boelter formula to a heat exchanger with $S_{1} = S_{2} = S$.

Frequency responses of $\mathbf{A}(s)/\mathbf{A}(s)$ and $\mathbf{A}(s)/\mathbf{A}(s)$ are shown in Fig. 3. Frequency responses of $\mathbf{A}(s)/\mathbf{A}(s)$ and $\mathbf{A}(s)/\mathbf{A}(s)$ are shown in Fig. 4. Fig. 3 shows that curve C is smaller than the others in the phase lag. This is due to
the fact that the output location is close to the inlet of input-side fluid.

7. Experimental investigation

Experiments of frequency response are carried out to confirm the theoretical results.

The heat exchanger used in the experiment is one having the inner tube made of copper, with outer diameter 14.0 mm and inner diameter 12.0 mm. The water tube is also made of copper, with outer diameter 22.2 mm and inner diameter 20.2 mm. The shell is made of stainless steel (SUS 304), with outer diameter 32.0 mm, inner diameter 28.0 mm, and length 4.044 meters.

The baffles are located at about 60 cm intervals in the tube-side and the shell-side. The shell wall is isolated from the outside using both glass wool (about 25 mm thick) and asbestos. The experimental apparatus is shown in Fig.5. Cold water flows through the shell.

Periodic changes in inlet temperature of tube-side fluid are introduced to the tube-side inlet by opening or closing the two solenoid operated valves as illustrated in Fig.5.

The experimental frequency response was determined from both the input and the output data recorded, using a Fourier analysis technique on a digital computer. As a temperature sensing element, a copper-constantan thermocouple with 0.3 mm diameter is used.

Frequency responses are shown in Fig.6 for case C-P* and in Fig.7 for case P*-C. Fig.6(a) and Fig.7(a) show the shell-side outlet temperature response $\Delta \theta_m$ to changes in tube-side inlet temperature $\Delta \theta_i$. Figs. 6(b) and 7(b) show the tube-side temperature response $\Delta \theta_i$ to changes in $\Delta \theta_m$. The theoretical responses in Fig. 6 and 7 are calculated from the transfer functions for the bayonet-type heat exchangers with heat capacities of inner-, outer-tubes and shell.

The system parameters shown in Table 2 are obtained by using the measured steady state values of inlet, outlet temperatures.

![Figure 4](image-url)  
**Fig.4** Frequency responses of $\Delta \theta_m(s)/\Delta \theta_i(s)$ and $\Delta \theta_i(s)/\Delta \theta_m(s)$

![Figure 5](image-url)  
**Fig.5** Schematic diagram of experimental apparatus for case P*-C

![Figure 6](image-url)  
(a) Response of $\Delta \theta_m(s)/\Delta \theta_i(s)$

![Figure 7](image-url)  
(b) Response of $\Delta \theta_i(s)/\Delta \theta_m(s)$

**Fig.6** Frequency response for case C-P*. See Table 2

In Fig.7(a), the phase lag is larger than 360° for higher range of $\omega$. Therefore, the lines of -360° and -720° coincide with 0°.

In Fig.7(a), the phase lag is larger than 360° for higher range of $\omega$. Therefore, the lines of -360° and -720° coincide with 0°.
Table 2 System parameters

<table>
<thead>
<tr>
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<th>2</th>
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<tr>
<td>$d_{11}'$</td>
<td>1.65</td>
<td>1.77</td>
</tr>
<tr>
<td>$d_{12}'$</td>
<td>2.20</td>
<td>1.77</td>
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<td>$d_{21}'$</td>
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<td>$d_{r}$</td>
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<tr>
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<td>1.38</td>
</tr>
<tr>
<td>$r_{2}$</td>
<td>1.47</td>
<td>1.49</td>
</tr>
<tr>
<td>$T_{L1}$</td>
<td>$2.14\times10^{-3}$ h</td>
<td>$2.12\times10^{-3}$ h</td>
</tr>
</tbody>
</table>

and velocities of both fluids.

Figs. 6 and 7 show that the experimental data are in good agreement with the theoretical results.

Both the gain (or amplitude ratio) curve and the phase one are oscillatory in Figs. 6(b) and 7(b). The reason is due to the fact that there are two paths by which the inlet temperature change has an effect on the outlet temperature response. The first path is related to the heat-transmission through the inner tubes. The second path is related to the transportation of the tube-side fluid. The response due to the second path appears in the outlet of tube-side after the dead time $(\tau/\phi_{b})+(\tau/\phi_{c})$. In Fig. 4, the gain (or amplitude ratio) curves A and B are also oscillatory. The reason is the same as stated above.

8. Conclusions

The dynamics of bayonett-type heat exchanger is analyzed and discussed from the control engineering point of view. The main results obtained are summarized as follows:

(1) The transfer functions, relating to the outlet temperature responses to changes in inlet temperatures of the fluids, are obtained in a simplified form available for computer programming. Experiments of frequency response are carried out, and the data are shown to be in good agreement with the theoretical results.

(2) From the transfer functions derived, it is shown that there exists a close relationship between cases C*-P and P-C*, and also between cases C-P* and P*-C. The static relations given by $G_{C*P}(0)=G_{PC*}(0)$, $G_{C*P}(\omega)=G_{PC*}(\omega)$ are obtained.

The author wishes to express his gratitude to Professor Y. Takahashi, University of California, for his valuable suggestions, to Professor S. Fuji, University of Tokyo, and Professor M. Masubuchi, Osaka University, for their encouragement. The author is also grateful to Mr. H. Ushiogi, Yokohama National University, Mr. N. Utsumi and Mr. K. Kanno, students at Yokohama National University, for their assistance in the experiments.

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