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Fundamental Studies of Turbulent Boundary Layers with
Injection or Suction through Porous Wall*
(4th Report, Application of Fundamental Theoretical
Analysis to Cascaded Blades with Injection or
Suction and Cascade Tests)

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In the present paper, the authors intend to discuss the applicability of the analytical method for a turbulent boundary layer having pressure gradients with injection or suction treated in the previous papers to the boundary layer on the cascaded blade surfaces with injection or suction all over them and to investigate the influence of the injection or the suction on the cascaded blade characteristics by the cascade tests.

The following main results are obtained:

(1) The analytical method for a turbulent boundary layer having pressure gradients with injection or suction treated in the previous papers is applicable to the boundary layer on the blade surfaces with injection or suction all over them.

(2) Judging from the total pressure loss coefficient, though the optimum injection ratio must be determined after due consideration of the cooling characteristics of the blade, the available injection ratio in the fluid dynamic characteristics is $v_o/u_o \leq 0.004$. And the effect of the suction all over the blade surfaces is found little for a high performance cascade (such as NACA A₃K₇).

1. Introduction

The fundamental research for improving performances of a gas turbine has been undertaken by the authors, analyzing theoretically and experimentally the velocity distribution, the mean skin friction coefficient, the separation point of the turbulent boundary layer and so on for a flat plate with injection or suction^{(1)~(3)}. Based on these studies the present paper deals with the test results of the turbine blades in cascade with injection or suction through their whole surfaces and the effect of injection or suction on the blade in cascade performance, and discusses the applicability of the fundamental, theoretical analyses in the previous papers to the boundary layer on the blade surface.

In applications of the fundamental research of the turbulent boundary layer with injection or suction to the gas turbine, the former is mainly used for cooling the turbine blade and controlling⁽⁴⁾ the

boundary layer and these applications are reported in various references^{(7)~(17)}. The latter is used for controlling⁽⁶⁾ the boundary layer on the blade surface of the compressor and the turbine, resulting in the improvement of the blade performance, and many investigations^{(18)~(20)} have been reported over many years.

This investigation is conducted concerning the injection and the suction throughout the whole blade surface which are considered to be most effective for cooling the blade and controlling the boundary layer on the blade surface.

The major problems taken up here are: (1) Comparison between the velocity profile of the boundary layer on the blade surface and that of the boundary layer on the flat plate; (2) the applicability of the analysis of the turbulent boundary layer with injection or suction in the previous papers^{(1)~(5)} to that of the boundary layer on the blade surface; (3) the applicability of the analyzing method for estimating the separation point of the boundary layer on the blade surface with injection throughout the whole blade surface treated in the previous report⁽³⁾; (4) the investigation of the effect of the injection and the suction throughout the whole blade surface on the cascade performance, and the practical determination of the injection or suction ratio; and (5) the derivation of the boundary layer thick-

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ness on the blade surface with injection or suction by the analysis as described in the previous papers⁽²⁾⁽⁵⁾, the theoretical calculation of the total pressure loss coefficient of the blade in cascade with injection and suction, and their comparison with the experiments. Further, the derivation of the practical calculation method of the total pressure loss coefficient or the wake momentum loss coefficient from the injection flow rate or injection ratio through semi-experimental, theoretical calculation, and so on.

2. Nomenclature

- $H, \bar{H}$: shape factors, $= \delta^*/\theta$,
 $= \delta^{**}/\theta$
 p_{BS} : static pressure at the blade surface
 Δp_B : difference of static pressure across the cascade
 $p_{T1}, \bar{p}_{T1}$: total pressure upstream of cascade, average value of p_{T1}
 $p_{T2}, \bar{p}_{T2}$: total pressure downstream of cascade, average value of p_{T2}
 p_{D2th} : theoretical dynamic pressure at cascade downstream
 p_{Tj} : total pressure of injection or suction air
 p_{2t}, p_{T2t} : static and total pressures at blade trailing edge
 $p_{2\infty}, p_{T2\infty}$: static pressure and total pressure far downstream of cascade
 Re_L : Reynolds number, $= \bar{u}_o l_c / \nu$
 $\bar{u}_o$: average velocity of cascade upstream and downstream, $= (u_{o1} + u_{o2})/2$
 u_{o2t} : velocity of main stream at cascade trailing edge
 x : distance from leading edge along blade surface
 β_{c1}, β_{c2} : inlet angle, outlet angle to and from cascade
 ϵ, ϵ_B : experimental value and theoretical value of total pressure loss coefficient
 ρ_j : density of injection or suction air
 $\delta^*, \delta^{**}, \theta$: displacement thickness, energy thickness, momentum thickness
 θ_c : turning angle of camber line

3. Experimental apparatus and measuring methods

3.1 Experimental apparatus

(1) The cascade testing apparatus is shown in Fig. 1. The specific turbulence of the wind tunnel is less than 0.1%. (2) The cascade is composed of five blades and the three inner blades perform the function of injection or suction. The injection or suction is made in the range of 67% of the blade width at its central portion. It is revealed by the measurements of the velocity distributions and outlet angle distributions along the blade width

at the cascade downstream that a two-dimensional flow is kept at the central portion of the blade width. (3) The injection or suction from the porous blade is made by the roots-blower⁽²⁾⁽³⁾ (4) A fish-mouth type Pitot-tube is used for measuring the total pressure within the blade boundary layer, and

Fig. 1 Cascade tunnel

Section I	Suction side	$0 \leq x/l \leq 0.04$
Section II	Suction side	$0.04 \leq x/l \leq 0.28$
Section III	Suction side	$0.28 \leq x/l \leq 0.83$
Section IV	Pressure side	$0 \leq x/l \leq 0.83$

(a) Profile of the blade section tested

(b) Nomenclature for cascade

Fig. 2 Tested blade and cascade

a static pressure tube -- stainless steel pipe of 1 mm outside diameter with 0.2 mm hole in diameter for measuring the static pressure. (5) For the measurement of the wake behind the cascade, a Pitot-tube mounted with a total pressure tube, a static tube and a direction indicator is employed.

Table 1 Specification of blade and cascade tested and experimental conditions

Specification of blade and cascade tested (NACA A ₃ K ₇ blade)				
1	Camber angle $\theta_c=95^\circ$	9	Length of camber line $x_c=165\text{mm}$	
2	Lift coefficient $C_{L_0}=3.14$	10	Length along suction surface $x=185\text{mm}$	
3	Inlet angle 69.6°	11	Length along pressure surface $x=155\text{mm}$	
4	Chord length $l_c=150\text{mm}$	12	Blade number; five blades (Injection or suction is accomplished by center three blades.)	
5	Span $S=300\text{mm}$			
6	Position of maximum thickness $x_c/l_c=0.2$			
7	Maximum thickness $T/l_c=0.2$			
8	Aspect ratio $S/l_c=2.0$			

Experimental conditions of cascade test					
Theoretical outlet velocity from cascade		$u_{02th}=25\text{m/s}$			
Reynolds number		$Re = 1.9 \times 10^5 \sim 2.8 \times 10^5$			
Injection or suction ratio v_a/u_a		-0.002, -0.001, 0, 0.003, 0.004, 0.005, 0.006, 0.007			
Incidence angle i		$0^\circ, +5^\circ, \pm 10^\circ, \pm 15^\circ$			
		Solidity: $\sigma = l_c/t$ (chord length/pitch)			
		1.5	2.0	2.5	3.0
Stagger angle ξ^*	15°	o	o	o	o
	24°	o	o	o	o
	35°	o	o	o	o

(The mark o denotes the condition of cascade test)

(a) Pressure side

(b) Suction side

NACA A₃K₇ blade, $i=0^\circ$, $\sigma=1.5$, $\xi^*=24^\circ$, $u_{02th}=25\text{ m/s}$

Fig.3 Distributions of the injected or sucked air velocities

3.2 Tested blade and cascade

The aerofoil used in this experiment is NACA A₃K₇ blade, the inside of which is hollowed so that the injection or suction can be made through the blade shell. Aluminum powder of 120 mesh is mixed with plastic powder and the mixture is molded into a blade and baked at approx. 200°C . The sectional thickness of the blade is 3 mm throughout except in the vicinity of the trailing edge so that uniform ventilation can be obtained. The tested blade and cascade are shown in Fig.2 and their dimensions are listed in Table 1.

4. Measuring method and calculation of experimental value

4.1 Experimental conditions

The cascade test is conducted under the conditions given in Table 1.

4.2 Measuring of pressure distribution on blade

Pressure lead pipes of 0.5 mm outside diameter stainless steel are embedded into the holes drilled vertically on the surface of the porous blade.

4.3 Measurement of internal pressure of hollow blade

The pressure measurement of each section of the hollow blade is made at the central portion using a static pressure tube of 1 mm outside diameter (the static pressure hole is 0.2 mm).

4.4 Determination of injection or suction velocity

The injection or suction velocity is determined using the calibration curve of ventilation of the blade materials which have been obtained beforehand (1)~(5). One example $i=0^\circ$ is shown in Fig.3 from the experimental results of the velocity ratio for various incidence angles. The velocity ratio is comparatively uniform on the whole blade surface.

4.5 Measurement of blade boundary layer

4.5.1 Recognition of turbulent boundary layer

The boundary layer velocity distribution on the blade surface without injection or suction is measured and compared with the velocity distribution by Doenhoff et al.⁽²⁾ It is revealed that a turbulent boundary layer is fully developed at $x=50\text{ mm}$. Therefore, the measuring range in the direction of x is set for $x \geq 50\text{ mm}$.

4.5.2 Measurement of velocity distribution

The velocity distribution within the boundary layer is measured at the following six positions: $x=50, 80, 100, 120, 144, 166\text{ mm}$.

4.5.3 Calculation of boundary layer characteristic values

From the velocity distribution within the boundary layer are calculated δ^* , δ^{**} , θ , H and $\bar{H}$. The shearing stress at the wall and the friction work are calculated using Eqs. (8) and (9) presented in the reference (2).

4.6 Measurement of wake behind cascade

The measurement position is at 1/5 of the chord length in the direction of flowing out -- 30 mm downstream from the trailing edge. The measurements are made of the total pressure, the static pressure and the outlet angle of the wake along one pitch of the cascade at the central blade with injection or suction.

5. Definition of coefficients representing cascade performance⁽²²⁾⁻⁽²⁴⁾

5.1 Total pressure loss coefficient ϵ

The difference between the total pressures of flows in and out the control volume for one pitch of the cascade is considered to be the total pressure loss of the cascade with injection or suction.

The total pressure loss $(\bar{p}_{T1} - \bar{p}_{T2})$ is defined to be nondimensional using the theoretical flowing out dynamic pressure and ϵ is defined as follows, considering that, as stated hereinafter in this paper, the principal aim is to establish the practical calculation method of ϵ and the wake momentum loss coefficient c_{w1} .

$$\epsilon = (\bar{p}_{T1} - \bar{p}_{T2}) / p_{D2th} \quad (1)$$

where,

$$\bar{p}_{T1} = (p_{T1} \rho_1 g u_{o1} t \cos \beta_{c1} + \int_B p_{Tj} \rho_j g v_o dx) / (\rho_1 g u_{o1} t \cos \beta_{c1} + \int_B \rho_j g v_o dx) \quad (2)$$

$$\bar{p}_{T2} = \int_0^t p_{T2} \rho_2 g u_2 \cos \beta_{c2} dt / \int_0^t \rho_2 g u_2 \cos \beta_{c2} dt \quad (3)$$

$\int_B dx$: integration along the blade surface

$\int_0^t dt$: integration for one pitch

5.2 Wake momentum loss coefficient c_{w1}

c_{w1} is defined for the unit length in the span direction as follows:

$$c_{w1} = \int_0^t \rho_2 u_2 (u_{o2} - u_2) \cos \beta_{c2} dt / \{ (1/2) \rho_1 u_{o1}^2 l_c \} \quad (4)$$

5.3 Turning angle θ_a

$$\theta_a \text{ is defined as follows:} \quad \theta_a = \beta_{c1} + \bar{\beta}_{c2} \quad (5)$$

where $\bar{\beta}_{c2}$ is the average value of the outlet angle of the main stream in the wake behind the cascade.

5.4 Pressure coefficient c_p

The dynamic pressure at the upstream of the cascade varies significantly with the cascade conditions (σ , ξ^* , v_o/u_o , i). Therefore, the pressure coefficient c_p is defined by the following equation,

$$c_p = (p_{s1} - p_{s2}) / p_{D2th} \quad (6)$$

5.5 Lift coefficient c_L , drag coefficient c_D and lift-drag ratio F_L/F_D ⁽²⁴⁾

The lift F_L and the drag F_D are the normal component and the parallel component respectively of the average vector u_{om} of u_{o1} and u_{o2} . Here, β_{cm} is in the direction of u_{om} . c_L and c_D are shown as follows⁽²⁴⁾:

$$c_L \equiv F_L / \{ (1/2) \rho u_{o1}^2 l_c \} = (1/\sigma) [\{ \sin 2\beta_{c1} + (\sin 2\beta_{c2}/t) \int_0^t (p_{D2}/p_{D1}) dt \} \cos \beta_{cm} - \{ (\Delta p_B/p_{D1}) + 2 \cos^2 \beta_{c1} - (2 \cos^2 \beta_{c2}/t) \times \int_0^t (p_{D2}/p_{D1}) dt \} \sin \beta_{cm}] \quad (7)$$

$$c_D \equiv F_D / \{ (1/2) \rho u_{o1}^2 l_c \} = (1/\sigma) [\{ \sin 2\beta_{c1} + (\sin 2\beta_{c2}/t) \int_0^t (p_{D2}/p_{D1}) dt \} \sin \beta_{cm} + \{ (\Delta p_B/p_{D1}) + 2 \cos^2 \beta_{c1} - (2 \cos^2 \beta_{c2}/t) \times \int_0^t (p_{D2}/p_{D1}) dt \} \cos \beta_{cm}] \quad (8)$$

where,

$$\Delta p_B = p_{s1} - \bar{p}_{s2} \quad (9)$$

$$\bar{p}_{s2} = \left(\int_0^t p_{s2} \rho_2 u_2 \cos \beta_{c2} dt \right) / \left(\int_0^t \rho_2 u_2 \cos \beta_{c2} dt \right) \quad (10)$$

$$\text{The lift-drag ratio } F_L/F_D \text{ is} \quad F_L/F_D = c_L/c_D \quad (11)$$

6. Test results of injection or suction cascade and consideration

The test range of the cascade in this paper is $1.9 \times 10^5 < R_{lc} < 2.8 \times 10^5$. The representation of the magnitude of the incidence angle includes the sign to the central line of the blade.

6.1 Velocity distribution in the wake

Figure 4 is one example of the test results of the wake behind the cascade. The region with a velocity defect becomes wider and deeper for a larger injection ratio v_o/u_o and a boundary layer on the blade surface develops with an increased v_o/u_o . And the region with a velocity defect decreases in the case with suction and the effect of the boundary layer control due to suction is recognized.

Fig. 4 Velocity distribution in a wake

Fig. 5 ϵ - i curves

6.2 Total pressure loss coefficient ϵ

Figure 5 shows ϵ taking a parameter v_o/u_o . In the case of $\xi^*=24^\circ$, $\sigma=1.5$, ϵ at

$v_o/u_o=0.007$ increases slightly for $-15^\circ < i < 0^\circ$ and increases significantly for $i \geq 10^\circ$. In the case of $\xi^*=24^\circ$, $\sigma=2.0$ (the optimum solidity for $\xi^*=24^\circ$ as mentioned hereinafter), ϵ is smaller on the whole than in the case of $\sigma=1.5$.

The optimum injection ratio should be considered taking into account the blade cooling characteristics as well as the total pressure loss coefficient. In the case of injection the allowable range of ϵ at $v_o/u_o=0$ is assumed to be no more than 10 % increase. Then, in the case of the optimum solidity ($\sigma=2.0$) the practical injection ratio is $v_o/u_o \leq 0.004$ for $-15^\circ \leq i \leq +8^\circ$. The effect of suction for $\sigma=1.5$ and $i=15^\circ$ is proved by the fact that the value of ϵ at $v_o/u_o=0$ decreases due to suction. From the experimental results for the case near the optimum solidity ($\sigma=2.0 \sim 2.5$) the effect of suction is considered to be less, because the ratio of the friction loss to the cascade loss is large for a blade having good performance. The optimum suction ratio in these experimental results is $v_o/u_o \approx -0.001$. Therefore, it is appropriate to make the suction at the blade surface cause the separation of the boundary layer for the purpose of getting the suction effect sufficiently.

6.3 Wake momentum loss coefficient c_{w1}

Fig. 6 shows c_{w1} , derived from the measurement of the velocity distribution of the wake behind the cascade. The value of c_{w1} for the case of $\xi^*=24^\circ$ and $\sigma=2.0$ is approximately a half of that for the case of $\xi^*=24^\circ$ and $\sigma=1.5$.

6.4 Turning angle θ_q

The test results of θ_q for the case of $\xi^*=24^\circ$ and $\sigma=1.5$ are shown in Fig. 7. In the range of $0 \leq v_o/u_o \leq 0.007$ for the case of $-15^\circ \leq i \leq 0^\circ$, θ_q decreases for a larger injection ratio.

Fig. 6 c_{w1} - i curves

Fig. 7 θ_q - i , v_o/u_o curves

(a)

(b)

Fig. 8 Pressure coefficients

6.5 Pressure coefficient c_p

Fig. 8 is one example of the test results of the pressure distribution on the blade ($\xi^*=24^\circ$, $\sigma=1.5$, $i=0^\circ, 15^\circ$). For $\xi^*=15^\circ$ the pressure distribution on the suction surface of the blade has a strong adverse pressure gradient at the downstream of the minimum pressure point, and a constant pressure region at the further downstream. It seems that a laminar separation (bubble) accompanying the reattachment happens there⁽²⁹⁾ ($x_c/l_c \approx 0.2 \sim 0.4$ for $v/u_o \approx 0 \sim 0.007$). For $i=0^\circ$ there does not happen any laminar separation and the laminar boundary layer turns into a turbulent boundary layer.

6.6 Lift coefficient c_L , drag coefficient c_D and lift-drag ratio F_L/F_D

One example of the test results of c_L , c_D and F_L/F_D is shown in Fig. 9. The lift-drag ratio is not affected significantly by the change of the injection ratio, but it is increased greatly if a suction takes place.

6.7 Effects of solidity σ and stagger angle ξ^*

For investigating the cascade performance for various solidities and stagger

Fig. 9 c_L , c_D , F_L/F_D , $c_{w1}-v_o/u_o$ curves

Fig. 10 $c_{w1}-\sigma$, ξ^* curves

angles the $c_{w1}-\sigma$ curves and the $c_{w1}-\xi^*$ curves are given in Fig. 10. From the $c_{w1}-\sigma$ curves the minimum c_{w1} is obtained at approx. $\sigma=2.0 \sim 2.5$ and then the optimum value of σ is $\sigma=2.0 \sim 2.5$. The fact seems not to be affected by the presence of injection or suction.

From the $c_{w1}-\xi^*$ curves the value of c_{w1} decreases and the cascade loss diminishes for a greater stagger angle. However, the quantity of gas passing through the cascade is reduced for a greater stagger angle and then the turbine output decreases. The values of c_{w1} do not show any noticeable difference for $\xi^*=24^\circ$ and for $\xi^*=35^\circ$. Therefore, it is considered that $\xi^*=24^\circ$ is appropriate for setting the cascade.

6.8 Comparison of the cascade test with the one at NACA

Prior to the comparison of the test results⁽²⁹⁾ at NACA with the present experiments, the differences of the experimental conditions are as follows: (1) The maximum thickness ratio of the blade tested at NACA is $T/l_c=0.15$, and that in this experiment is $T/l_c=0.20$. (2) The tested

Comparison of cascade test results ($\sigma=1.5$, $\xi^*=24^\circ$, $\theta_c=95^\circ$) of NACA A3K7 blade by authors with those by NACA

Fig. 11 θ , $c_{w1}-i$ curves

Fig. 12 Velocity distribution

blade is a porous one and its surface roughness (approx. 20μ R_{max} $L_{2.5}$ mm) is greater than that of NACA. (3) Reynolds numbers are $3.2 \times 10^5 \leq R_{Lc} \leq 5.0 \times 10^5$ for NACA, and $2.3 \times 10^5 \leq R_{Lc} \leq 2.6 \times 10^5$ for this experiment.

The experimental values of C_{w1} and θ_g obtained by NACA and by the authors are shown respectively in Fig. 11 (the values of NACA are the estimated experimental ones). The turning angles in this experiment are smaller by 1° to 2° than that of NACA. It is considered that the surface roughness of the tested blade is greater than that of NACA and the boundary layer on the blade surface grows thicker and then the stream is hard to flow along the blade. This is proved by the fact that the tested values of C_{w1} are almost twice the values of NACA. Further, the suction of the boundary layer through the side walls is not made and Reynolds number is about a half of that of NACA, and then the value of C_{w1} seems to differ from NACA's test results.

7. Applicability of the fundamental theoretical analysis to the boundary layer of the blade with injection or suction, theoretical analysis of the total pressure loss coefficient and consideration of the test results

Fig. 13 $\log\left(\frac{\tau_w}{\rho u_o^2}\right) - \log R_\theta$ curves

Fig. 14 $\log\left\{\frac{(d+t)}{\rho u_o^2} + \frac{1}{2}\left(\frac{v_o}{u_o}\right)\right\} - \log R_\theta$ curves

7.1 Application of the theoretical analysis to the blade surface boundary layer of the cascade with injection or suction and consideration of the test results

The analytical method (2)(3)(5) of the turbulent boundary layer with injection or suction having an adverse pressure gradient is examined about whether or not it can apply to the boundary layer on the porous blade surface (refer to Fig. 8(a) for the pressure gradient of the main stream).

7.1.1 Velocity distribution in the boundary layer

A tested example of the velocity distribution in the boundary layer on the porous blade surface is shown in Fig. 12. The real line in the figure is the velocity distribution in the turbulent flow presented by Doenhoff et al. (21) From the figure the fact of One-Parameter Family is satisfied under the conditions of $H=1.8$ at $v_o/u_o \geq 0$ and $0 \leq v_o/u_o \leq -0.002$.

7.1.2 Shearing stress at the wall

The test results of the shearing

Fig. 15 $H-H$ curves

Fig. 16 $H-L$ curves

Fig. 17 $\theta-x$ curves

stress at the wall of the porous blade with injection or suction in the turbulent boundary layer are shown in Fig. 13. In the figure the equations presented by Ludwig et al. (25) and Maeda (3) are shown by a solid line and a dotted line respectively. In the range where the velocity distribution agrees with that of Doenhoff et al., the experimental values of the shearing stress coincide well with the equation by Ludwig et al. and in the other range the experimental values correspond well to the equation by Maeda.

7.1.3 Friction work

The experimental values of the friction work in the turbulent boundary layer on the porous blade surface with injection or suction are shown in Fig. 14. The equations by Truckenbrodt (26) and Maeda (2)(3) are shown in the same figure by the solid lines which just coincide with the experimental values. Here, the value of β_t for the case of suction is shown by the following equation:

$$\beta_t = 4.17 \times 10^4 (v_o/u_o)^3 + 5.75 \times 10^5 (v_o/u_o)^2 + 2.89 (v_o/u_o) + 0.56 \times 10^{-2} \dots (12)$$

7.1.4 Relations of $H-\bar{H}$ and $H-L$

The following equation is presented in the preceding papers (2)(3)(5) which is applicable to both regions far from and neighbouring to the separation point in the turbulent boundary layer with injection or suction.

$$\bar{H} = (1.201H + 0.472)/(H - 0.188) \dots (13)$$

Fig. 18 $H-x$ curves

Fig. 19 $\alpha-q$ curve

The experimental values of $H-\bar{H}$ of the cascade with injection or suction are plotted in Fig. 15. They show comparatively close agreement with Eq. (13) in comparison with the equations presented by others (27)(28) and therefore, Eq. (13) is used. Figure 16 shows the experimental values of $H-L$. They coincide well with the $H-L$ curve experimentally derived for the flat plate. Then, Maeda's curve shown in Fig. 16 is used for the $H-L$ curve.

7.1.5 Comparison of semi-theoretical curves of θ_q and H , and experimental values

The values of θ and H calculated by the analytical method in the preceding papers (2)(3)(5), and their experimental values are shown in Figs. 17 and 18. In Fig. 17 the experimental values agree well with the calculated values in the region I--the region far from the separation point. In the region II--the region neighbouring to the separation point--, the experimental values are greater than the calculated ones for a greater injection ratio. In Fig. 18 the calculated values of H coincide with the experimental ones in the region I, and the difference between the calculated values and the experimental ones becomes significant for a greater injection ratio and the experimental values decrease greatly in comparison with the calculated ones in the region II. The values of H in the region II are obtained by the method derived in the preceding paper and are shown by a dotted line in Fig. 18 (this is called the second semi-theoretical $H-x$ curve). Thus, the experimental values of H in the region II coincide comparatively well with the second semi-theoretical $H-x$ curve.

7.1.6 Comparison of semi-theoretical values of $R\theta_o'$ and H_o , and experimental values

The experimentally derived correlations (3) used for calculating $R\theta_o'$ and H_o in the preceding paper (3), and the experimental values of $\alpha-q$ of the porous blades with injection are shown by a solid line

Fig. 20 $K-L$ curves

and the mark o in Fig. 19. Their coincidence is comparatively well. Therefore, in the analysis of the separation point of the cascade with injection, α is derived and then u_0' is calculated using the correlation, and $R_{\theta 0}'$ and H_0 can be determined using the theoretical equations of θ and H .

7.1.7 Comparison of the second K-L calculation curves and experimental values, and comparison of H_{s2th} with H_{s2exp} and $H_{spho}^{(3)}$.

The first and the second K-L calculation curves for the turbulent boundary layer of the porous blade with injection, the approximate straight lines and the experimental values are shown in Fig. 20. The intersection of the second K-L curve and the second approximate straight line, that is, the minimum point of the second K-L curve is the estimated separation point. The value of H corresponding to the value of L for the estimated separation point -- L_{s2th} -- is denoted by H_{s2th} . Then, the estimated $H_{s2th}^{(3)}$ from the velocity distribution and the calculation of τ_0 , and the shape parameters of H_{spho} and H_{s2t} at the separation point observed by putting a string in the stream are compared, their agreement being good (refer to Fig. 18).

7.1.8 comparison of semi-theoretical value of shape parameter and experimental value, and comparison of theoretically estimated separation point x_{s2th} and experimentally estimated one

In Fig. 18 the experimental values of H and the second semi-theoretical H - x curve for $H > H_0$ are shown and their good coincidence is observed. The separation point x_{s2th} estimated from the second K-L curve and the separation points x_{sprof} and $x_{spho}^{(3)}$ observed by putting a string in the stream and using the velocity distribution show comparatively good agreement (refer to Fig. 18).

7.2 Derivation of theoretical value of total pressure loss coefficient ϵ_B and consideration of experimental value ϵ

The boundary layer thickness at the blade trailing edge is calculated using the analytical method in the preceding papers^{(2) (5)} and the total pressure p_{T20} after completing the mixture of wake is obtained by Traupel's method⁽³⁰⁾. Then,

$$p_{T20} = p_{2\infty} + (\rho u_{2\infty}^2 / 2) = p_{2t} + (1/2) \rho u_{2t}^2 \{ [2(t \times \cos \beta_{2t} - t^* - \delta_t^* - \theta_t) \cos \beta_{2t} / t] - [(t \times \cos \beta_{2t} - t^* - \delta_t^*)^2 / t^2] + [(t \cos \beta_{2t} - t^* - \delta_t^* - \theta_t)^2 \sin^2 \beta_{2t} / (t \cos \beta_{2t} - t^* - \delta_t^*)^2] \} \quad (14)$$

Here, the suffix t represents the blade

Fig. 21 Comparison of theoretical total pressure loss coefficients with experimental values

trailing edge and t^* the thickness of the blade trailing edge.

The theoretical total pressure loss coefficient ϵ_B for this case is designated as follows:

$$\epsilon_B = (\bar{p}_{T1} - p_{T20}) / p_{02th} \quad (15)$$

The comparison of ϵ and ϵ_B is shown in Fig. 21. Comparatively close agreement is observed between ϵ and ϵ_B for $-0.002 \leq v_0/u_0 \leq 0.004$ and then the analytical method in the preceding papers^{(2) (5)} can be applied for estimating the total pressure loss coefficient of the cascade with injection or suction.

8. Consideration of practical, approximate calculation method based on semi-theoretical analysis for total pressure loss coefficient and wake momentum loss coefficient

It will be convenient for designing the cascade if the values of ϵ and ϵ_{w1} can be obtained from the injection flow rate or the injection ratio of the blade.

In order to get ϵ and ϵ_{w1} , the displacement thickness δ_w^* and the momentum thickness θ_w of the wake (the suffix w is attached to the designation concerning the wake hereinafter) must be obtained. The available analytical methods for getting δ_w^* and θ_w are: (1) A general correlation among the factors -- for example, δ_t^* , θ_t , δ_t^* , θ_w etc. -- is found experimentally, and then ϵ' and ϵ_{w1}' (the practical calculated values are attached with ' mark) are estimated.

(2) The wake functions by Coles or Schlichting are extended so as to be applicable to the case with injection or suction, the velocity distribution of the wake is obtained and then ϵ' and ϵ_{w1}' are estimated. Here, the above two methods are sufficiently considered taking into account the fact that the main stream exists between the blades and in the wake. In the case of (1) generality of the correlation among the factors (assuming that various conditions of the cascade are included in the thickness of the boundary layer of the blade trailing edge) is considered to be insufficient from the present range of experiments. In the case of (2), for example, Schlichting's equation⁽³¹⁾ of the velocity distribution of the wake behind the circular cylinder includes c_p and then the value of c_p must be given in any way for determining the velocity distribution of the wake.

From the above considerations it seems to be preferable for the practical calculation of ϵ_{w1}' and ϵ' to take the method of (1) on account of the practical applicability and the simplicity of the calculation at the present. The reason is that δ_w^* is obtained from δ_t^* and θ_t , and θ_w is calculated from the correlation equation among the factors, and so even if δ_t^* and θ_t are calculated based on a rough assumption which does not agree with the experimental value, the calculated values of δ_w^* and θ_w comparatively agree with the experimental

values using such correlation equation between their values and the experimental values of δ_w^* and θ_w with generality. Therefore, the outline of introducing ϵ' and C_{wi} from the injection flow rate G_i is dealt with hereinafter only for the case where the injection exists as a key to establishing the practical calculation method of ϵ' and C_{wi} .

Equation (16) is obtained based on the following assumptions: (i) G_i flowing out of the blade surface is equivalent to the flow quantity within the boundary layer of the blade trailing edge. (ii) The velocity distribution within the boundary layer follows a 1/7th power law. Here, the velocity of main stream is u_{∞} .

$$\delta_t = (8/7) (G_i / \rho g u_{\infty} S), \quad \delta_t^* = (1/7) (G_i / \rho g u_{\infty} S), \\ \theta_t = (1/9) (G_i / \rho g u_{\infty} S) \quad \dots \dots \dots (16)$$

As to the correlation between δ_t^* and θ_t obtained from Eq. (16), and δ_w^* and θ_w , the relation between $(\delta_t^* + t^*)$ and δ_w^* , and the relation between θ_t and θ_w are plotted as one example using experimental value, and then the following correlation equation is derived.

$$\delta_w^* = (180.53 \sigma^2 - 821.45 \sigma + 966.16) (\delta_t^* + t^*)^2 \\ - (1079.80 \sigma^2 - 4918.30 \sigma + 5796.32) (\delta_t^* + t^*) \\ + 2015.43 \sigma^2 - 8766.55 \sigma + 990.72 \quad \dots (17) \\ \theta_w = (77.28 \sigma^2 - 371.20 \sigma + 468.56) \theta_t^2 + (-7.38 \sigma^2 \\ + 33.22 \sigma - 37.35) \theta_t + (1.18 \sigma^2 - 6.18 \sigma \\ + 8.91) \quad \dots \dots \dots (18)$$

Here, δ_w^* and θ_w are calculated using Eqs. (17) and (18), $p_{T2\infty}$ is obtained by Traupel's method, ϵ' is obtained from Eq. (15) and C_{wi} is obtained from θ_w . (In this paper, the length in the span direction is unit length.) The calculated values are plotted in Figs. 21 and 6.

Attention is paid to the practical estimation of ϵ' and C_{wi} as above mentioned. And the correlation among all factors must be obtained experimentally for the application of this practical calculation method and for the generality in the practical design usage. However, the correlation among these factors can not be discussed theoretically and can not be clarified even experimentally at present as well as the generality owing to the limitations with respect to the cascade and the experimental conditions. Therefore, the present paper is conducted under the limited experimental conditions (including the technical problems of the injection and the experimental apparatus) using a specific aerofoil to find out the correlation among the factors concerned and to introduce a guide for the practical calculation method. The remaining problem will be discussed in the future.

9. Conclusions

Main conclusions are as follows:

(1) The calculated values of θ and H at the turbulent boundary layer on the blade surface well agree with the measured value and the analytical method in the preceding papers⁽²⁾⁽³⁾⁽⁵⁾ can be applied to the boundary layers on the blade surface with injection or suction.

(2) The estimated position of the separation point comparatively well agrees with that obtained by the experiments.

(3) In the case of large i ($i \geq 5^\circ$), the cascade performance is greatly affected by the injection.

(4) The optimum injection ratio must be determined taking into account the blade cooling characteristics also. If it is limited to within 10 % judging from the total pressure loss coefficient and considering the allowable range of the loss increment due to the injection, $\epsilon \approx 0.04$ is obtained around the designed incidence angle for the optimum solidity. If this value is considered the limit, the injection ratio of $v_o/u_o > 0.004$ is hardly possible on account of the hydrodynamic performance of the blade.

(5) Judging from the total pressure loss coefficients, it seems that, as to a blade having good performance such as NACA A3K7 blade, the effect of suction is small because the portion of the friction loss in the cascade loss is large, while the optimum suction ratio obtained by this experiment is $v_o/u_o \approx -0.001$. Therefore, it is considered more effective for improving the suction effect to cause a suction around the separation point on the blade surface.

(6) In the case of $\epsilon^* = 24^\circ$ and $i = 0^\circ$, ϵ_B obtained from the calculated value of the boundary layer thickness of the blade with injection or suction comparatively well agrees with the experimental value for $-0.002 \leq v_o/u_o \leq 0.004$.

(7) A practical calculation method of the total pressure loss coefficient and the wake momentum loss coefficient is suggested.

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