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On the Aseismic Design of Liquid Storages*

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The present situation of the aseismic design of liquid storages and its points of issue are pointed out at first. The fundamental concepts of a systematization of the aseismic design of liquid storages are proposed based on these points of issue. The authors carried out a series of experiments and analyses to make clear the fundamental vibrational characteristics of liquid storages. An earthquake response observation of a cylindrical storage model of diameter 4.0 m and response analyses of sloshing of liquid in cylindrical and spherical storages are reported in this paper. The results are summed up into formula and nomogram which are convenient and available for the response calculation of sloshing of liquid in the aseismic design of liquid storages.

1. Introduction

The sloshing of liquid is a phenomenon which has been known since old times and has become to attract attention recently. Abramson edited the "Dynamic Behavior of Liquid in Moving Containers⁽¹⁾" in July, 1963. Reviewing about 70 references in this paper, we find that this phenomenon was analyzed first about 1950 for the space vehicle stability problem. It is very recently that the sloshing phenomenon has become a problem in the aseismic engineering. Newmark and Rosenblueth gave a comment on sloshing in their paper⁽²⁾ which is known as a textbook for aseismic engineering.

Housner analyzed the effect of sloshing of liquid in his paper of 1957⁽³⁾. This was summed up in TID-Report⁽⁴⁾ which was published by USAEC and is adopted in the aseismic design of liquid storages widely. Housner presented an excellent way of analysis. However, his model seems to be too simplified to analyze the effects of sloshing of liquid, because he used the ordinary acceleration response spectra in his analysis.

In connection with the earthquake damages (for example, Fig. 1), the sloshing phenomenon was reported first on the occasions of the Long Beach Earthquake of 1933⁽⁵⁾ in the United States and the Niiga-

ta Earthquake of 1964⁽⁶⁾ in Japan. However, the fact that a disastrous damage was caused in the Great Alaskan Earthquake⁽⁶⁾⁽⁷⁾ three months before the Niigata Earthquake is not well known. There were four yards of oil tanks within the 150 km area from the epicenter. The fire broke out in three yards out of these four yards. In Anchorage district, only 4 tanks had suffered no damages among 21 tanks. The damage ratio of these tanks comes up to 80%.

In Japan, there are more than 400 tanks in Kawasaki district alone. The design condition of the tanks larger than 50 ML is very severe even for the static stress. It is predictable that a disaster would happen, if a strong earthquake happened in these districts.

2. The Liquid Storage and the Sloshing Phenomenon

The earthquake ground motion contains a short period component and a long period component. The former is dominant in the range of acceleration and the latter in the range of displacement. It is the sloshing phenomenon in the aseismic engineering that the liquid surface of storage comes to resonant to the latter. This displacement-dominant range means the range of semi-long period^(*) between about

Fig. 1 An example of the earthquake damages of the liquid storage

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(*) We call this range "semi-long period" in the aseismic engineering because "long period" means more than 20 sec in seismology.

1 sec and 15 sec. The periods of sloshing of liquid in ordinary scale storages mostly fall in this range. For example, an approximate relation between the period of sloshing T_1 sec and the capacity C ML with respect to about 140 tanks which are located in Negishi and Ogijima area, Kanagawa Prefecture, is shown by the next equation.

$$T_1 = 3.93 C^{0.224} \dots \dots \dots (1)$$

This shows that the periods of sloshing of liquid range between 3 sec and 12 sec for the capacities between 0.2 ML and 200 ML. These periods are considerably longer than the periods of ordinary structures.

The displacement wave now in problem is estimated to be a substantial wave. This is thought to be proved by a report of the San Fernando Earthquake⁽⁸⁾ which shows that the periods of sloshing of damaged storages were around 4.5 sec and that the damages were caused in wide areas.

3. The Present Situation of the Aseismic Design of Liquid Storages and its Points of Issue

In the aseismic design of liquid storages up to this time, the stability of the storage was the only item checked by the law to consider the possibility of its overturn by a certain level of horizontal acceleration. After that the technique of dynamic analysis came to be gradually adopted in the aseismic design of structure, and also in the aseismic design of liquid storage (for example, Ref. (4)). The response analysis now in use for the aseismic design of liquid storage, however, is based on the acceleration records of strong earthquakes. The effect of the component of long period is almost lost in the acceleration records to analyze the sloshing of liquid whose period is considerably longer than that of a structure.

There are a large number of studies on the vibrational characteristics and the response of a liquid storage. To list up the representative studies, Refs. (1), (9) and (10) are general investigations, Refs. (3), (4), (11), (12) and (13) are for a cylindrical storage and Refs. (14), (15) and (16) are for a spherical storage. There are, however, a small number of studies which are available for the practical design. It may be said that the aseismic design in this field is rather behind the times.

From these considerations, the authors pointed out that the aseismic design of liquid storage should be carried out from the following view points.

- 1) Long period ground motion: To make clear the characteristics such as the period, the amplitude and the duration of the semi-long period ground motion.
- 2) Vibrational characteristics of liquid storage: To make clear the vibration-

al characteristics of liquid storage by analytical and experimental studies.

- 3) Response analysis of liquid storage: To systematize the techniques of response analysis of liquid storage depending upon 1) and 2).
- 4) Stress analysis of liquid storage: To systematize the techniques of stress analysis of liquid storage for the input of 3).

4. Earthquake Damages of Liquid Storages

The earthquake damages of cylindrical liquid storages are reported in Refs. (5) and (6) by one of the authors. These show various modes which can be classified as follows.

- 1) Swinging and rotation of floating roof
- 2) Failure of top angle
- 3) Failure of cone roof
- 4) Torsional buckling of side wall
- 5) Oval deformation of side wall
- 6) Buckling of the lower part of side wall
- 7) Breaking of the welded portion of side wall and bottom plate
- 8) Failure of inner column
- 9) Horizontal sliding of tank
- 10) Inclination of tank
- 11) Subsiding of tank

Items 1) to 9) are thought to be the damages caused directly or indirectly by the sloshing of liquid and items 10) and 11) by the deformation of the base.

5. Mechanisms of Earthquake Damages of Liquid Storages

The mechanisms of earthquake damages of liquid storages shown in Chap. 4 can be classified into next three kinds, excluding the damages caused directly by the deformation of the base.

- 1) Failure resulting from the inertia force caused by the earthquake acceleration when the effective time of the earthquake loading is much shorter than the period of sloshing of liquid.
- 2) Failure resulting from the sloshing of liquid which is caused by the non-stationality of the acceleration input.
- 3) Failure resulting from the sloshing of liquid which is caused by resonance to the semi-long period component contained in the earthquake ground motion.

Reviewing design techniques in various papers, the techniques for 1) to 3) are found rather confused.

There is a well known design technique based on Housner's theory which treats a liquid separated into a fixed mass and a free mass. This relates to the above mentioned 1) to 3) as follows.

Phenomenon 1) is a case in which the effect of the fixed mass is dominant and can be treated in the same way as a re-

sponse analysis of the structure. The response of the storage is near to the acceleration input as in the case where the ground acceleration worked on a rigid system (Fig. 3). The free mass of liquid composes a system of period of the sloshing of liquid with a very much light damping. It was found out that there exist two mechanisms in the response of this system. One is a response caused by the nonstationarity of the acceleration input and corresponds to phenomenon 2). The other is a response caused by resonance to a semi-long period component contained in the earthquake ground motion. The characteristics of semi-long period earthquake ground motion are still not well known. However, it is observed that the semi-long period ground motion is almost near to a sinusoidal wave (Fig. 3). This suggests that this response analysis can be treated as a transient response analysis to a sinusoidal wave input (Sec. 7.5).

The estimation of the ground motion remains as a problem in future. Recent analysis of the San Fernando Earthquake shows that the amplitude of the ground motion was 42 cm⁽¹⁷⁾.

If the values such as the period, the amplitude and the duration are assumed for the design, the response of the storage can be calculated by the method proposed in Sec. 7.5.

6. Earthquake Response Observation of a Cylindrical Storage Model⁽¹⁸⁾

6.1 Outline of the Observation

The authors carried out an experimental observation to know the response of a cylindrical storage during natural earthquakes. A cylindrical liquid storage model of diameter 4.0 m and height 2.0 m (Fig. 2) was set on the plant model area, which is for the observation of earthquake response of various types of equipments and pipings, the Chiba Field Station, Institute of Industrial Science, University of Tokyo. This model was designed such that the period of sloshing of liquid might be equal to one of the dominant periods of the displacement of the ground motion. The observation was carried out for liquid surface displacements at two points on diameter, pressures at two

Fig. 2 The cylindrical storage model

points of side wall and strains at two points of the vessel.

In addition to these items, the earthquake ground motion was observed on the basement of the building nearby. The acceleration of the ground motion was measured by a strain gage type accelerometer. The displacement of the ground motion was picked up by a dynamic electromagnetic type seismograph, the signal of which was introduced into a double integration circuit to obtain the displacement. The gain characteristic of this circuit is flat between 0.1 sec and 10 sec.

The recording was controlled by a starter of level 0.5 gal. The operation of the oscillograph and the data recorder was continued for 6 min.

6.2 Results of Observation

About 50 records of observation were obtained from Jan. 1972 to Nov. 1974. These records were classified and analyzed as follows.

A. Classification of the Response of the Liquid Storage

It was found, according to the survey on these records, that the response of the liquid storage can be classified into next three types.

1) Acceleration type response (cf. earlier part of Fig. 3):

The response whose wave form is near to the wave form of the acceleration of the ground motion. In this response, the maximum value of the response appears at a time close to that of the acceleration of the ground motion. This response is called acceleration type response because it is thought that the liquid storage responds in a wide frequency range to the frequency component of the acceleration of the ground motion. The mechanism of this type response corresponds to the fixed mass in Chap. 5.

2) Displacement type response (cf. later part of Fig. 3):

The response whose wave form is near to the wave form of the displacement of the ground motion. In this response, the maximum value of the response appears rather later than that of the acceleration of the ground motion. This response is called displacement type response because it is thought that the liquid storage responds resonantly to the frequency component of the displacement of the ground

Fig. 3 An example of mixed type response

motion. The mechanism of this type response corresponds to the free mass in Chap. 5. It was found that the wave form of the ground displacement which excites the response of this type is near to a sinusoidal wave and that the wave form of the response is also near to a sinusoidal wave.

3) Mixed type response (Fig. 3):

The response which shows acceleration type response in its early stage but displacement type response in its later stage and gives the same order maximum value in both types of response.

B. Response of the Liquid Storage

Responses of the liquid storage were analyzed as the response value by 1 gal of the ground acceleration for the acceleration type response and as the response value by 1 mm of the ground displacement for the displacement type response.

Almost all the responses of the liquid surface displacement are the displacement type. Response values of the side wall pressure are shown in Fig. 4. This shows that there exist both types of responses for the side wall pressure but the acceleration type response appears more frequently than the displacement type response. This also shows that the response values at the upper point (PR 1) are smaller than those at the lower point (PR 2) for the acceleration type response, while the response values at the upper point (PR 1) are larger than those at the lower point (PR 2) for the displacement type response. This fact agrees with the analytical result in Sec. 7.3 well.

Tab. 1 shows the statistical values calculated for the response values in

Fig. 4 The response value of the side wall pressure

Tab. 1 Statistical values of the response values of the side wall pressure

type of response	item	mean value	coefficient of variation
acceleration type response	PR 1	1.36 mmAq/gal	0.833
	PR 2	1.98 mmAq/gal	1.04
displacement type response	PR 1	12.4 mmAq/mm	0.354
	PR 2	10.2 mmAq/mm	0.453

Fig. 4. It was found, from this table, that the dispersion of response is comparable to that of an ordinary structure for the displacement type response, while it is very large for the acceleration type response.

7. Response Analysis of Cylindrical Liquid Storage

A method to evaluate the displacement type response of a cylindrical liquid storage which is subjected to a earthquake ground motion is shown in this chapter.

In order to evaluate the displacement type response of a liquid storage, it is reasonable to assume that the storage is a rigid container, because the period of sloshing of liquid is so long compared to the period of vibration of the vessel that the coupling effect between the sloshing of liquid and the vibration of the vessel can be neglected. The analysis in this chapter is based on this assumption.

In addition to the displacement type response, the acceleration type response should be evaluated to design a liquid storage. This is evaluated by Housner's method which was shown in Chap. 5.

7.1 Basic Equations for Response Analysis

It is the object in this section to induce the basic equations to evaluate the sloshing effects of liquid in a cylindrical storage during a ground motion $d_g(t)$.

An analytical model of the cylindrical storage is shown in Fig. 5. The storage is assumed to be a rigid cylindrical container. R denotes the radius of the container and H the depth of liquid. If it is assumed that the liquid is an incompressible ideal fluid and is irrotational, the velocity potential ϕ can be introduced. Then the motion of liquid in the container is described by next four equations.

Fig. 5 Analytical model of the cylindrical liquid storage

1) Continuity condition

$$\nabla^2 \phi = 0 \quad (2)$$

2) Boundary condition at the side wall

$$-\frac{\partial \phi}{\partial r} \Big|_{r=R} = v_g(t) \cos \theta, \quad v_g = \dot{d}_g \quad (3)$$

3) Boundary condition at the bottom

$$-\frac{\partial \phi}{\partial z} \Big|_{z=0} = 0 \quad (4)$$

4) Boundary condition on the free surface

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0; \quad z = H \quad (5)$$

Solving Eqs. (2) to (5), the velocity potential ϕ is obtained as follows⁽¹¹⁾⁽¹⁹⁾

$$\phi = R \sum_{i=1}^{\infty} \frac{2}{\varepsilon_i^2 - 1} \frac{J_1(\varepsilon_i(r/R))}{J_1(\varepsilon_i)} \times \left[\frac{\cosh \{\varepsilon_i(z/R)\}}{\cosh \{\varepsilon_i(H/R)\}} v_i - v_g \right] \cos \theta \quad (6)$$

where $J_1(r)$ denotes a Bessel function of the first order and ε_i the i -th root of the equation $J_1(\varepsilon) = 0$ (Tab. 2).

The nominal velocity response of liquid of each mode v_i is given as a solution of the next velocity-base equation

$$\ddot{v}_i + \omega_i^2 v_i = \dot{a}_g \quad (7)$$

where ω_i denotes the eigen circular frequency of sloshing of liquid of each mode and is given as follows

$$\omega_i = \sqrt{\frac{g}{R} \varepsilon_i \tanh \left(\varepsilon_i \frac{H}{R} \right)} \quad (8)$$

and $a_g (= \dot{v}_g = \dot{d}_g)$ the acceleration of the ground motion.

If the existence of damping is assumed, Eq. (7) can be modified into next form by introducing the damping ratio ζ_i .

$$\ddot{v}_i + 2\zeta_i \omega_i \dot{v}_i + \omega_i^2 v_i = \dot{a}_g \quad (9)$$

It is Eq. (7) or Eq. (9) that represents the feature of the response of sloshing of liquid and its characteristic is equal to the ordinary acceleration response equation.

The response of the liquid surface displacement is given as follows by using Eq. (6)

$$\frac{\eta}{R} = \frac{1}{R} \left\{ \frac{1}{g} \frac{\partial \phi}{\partial t} \Big|_{z=H} \right\} = \frac{1}{g} \sum_{i=1}^{\infty} \frac{2}{\varepsilon_i^2 - 1} \frac{J_1(\varepsilon_i(r/R))}{J_1(\varepsilon_i)} (\alpha_i - \alpha_g) \cos \theta \quad (10)$$

where $\alpha_i (= \dot{v}_i)$ denotes the nominal acceleration response of liquid of each mode and is obtained from Eq. (7) or Eq. (9).

Tab. 2 The value of ε_i

i	ε_i	i	ε_i
1	1.841	6	18.015
2	5.331	7	21.165
3	8.536	8	24.311
4	11.706	9	27.457
5	14.863	10	30.602

The vibrational pressure of liquid (referred to as merely pressure hereafter) p is given as follows

$$\frac{p}{\gamma_f H} = \frac{1}{\gamma_f H} \left\{ \frac{\gamma_f}{g} \frac{\partial \phi}{\partial t} \right\} = \frac{1}{g(H/R)} \sum_{i=1}^{\infty} \frac{2}{\varepsilon_i^2 - 1} \frac{J_1(\varepsilon_i(r/R))}{J_1(\varepsilon_i)} \times \left[\frac{\cosh \{\varepsilon_i(z/R)\}}{\cosh \{\varepsilon_i(H/R)\}} \alpha_i - \alpha_g \right] \cos \theta \quad (11)$$

where γ_f denotes the weight of liquid per unit volume and $\gamma_f H$ the static pressure at the bottom. The side wall pressure p_w and the bottom pressure p_b are obtained by putting $r=R$ or $z=0$ in Eq. (11) respectively.

The resultant horizontal force F_x is given as follows by using the side wall pressure p_w .

$$\frac{F_x}{W_f} = \frac{1}{g} \sum_{i=1}^{\infty} \frac{2}{\varepsilon_i^2 - 1} \times \left[\frac{\tanh \{\varepsilon_i(H/R)\}}{\varepsilon_i(H/R)} \alpha_i - \alpha_g \right] \cos \theta \quad (12)$$

where $W_f (= \pi R^2 H \gamma_f)$ denotes the total weight of liquid in the container.

The overturning moments about y -axis due to the side wall pressure p_w and the bottom pressure p_b are given respectively as follows.

$$\frac{M_w}{W_f(H/2)} = \frac{1}{g} \sum_{i=1}^{\infty} \frac{4}{\varepsilon_i^2 - 1} \left[\frac{\tanh \{\varepsilon_i(H/R)\}}{\varepsilon_i(H/R)} \alpha_i - \frac{1}{2} \alpha_g \right] \quad (13)$$

$$\frac{M_b}{W_f(H/2)} = \frac{1}{g(H/R)^2} \sum_{i=1}^{\infty} \frac{4}{\varepsilon_i^2(\varepsilon_i^2 - 1)} \times \left[\frac{1}{\cosh \{\varepsilon_i(H/R)\}} \alpha_i - \alpha_g \right] \quad (14)$$

Actual overturning moment is given as the sum of M_w and M_b .

7.2 Period of Sloshing of Liquid

The period of sloshing of liquid of each mode can be calculated by using Eq. (8). As an example, the period of sloshing of liquid of about 100 Ml cylindrical oil storage (Diameter: 80 m, Height: 20 m) was calculated: the fundamental period is 10.97 sec and the 10th order 2.29 sec. This shows that the period of sloshing of liquid is rather long even in the higher order.

Fig. 6 The nomogram to calculate the period of sloshing of liquid in cylindrical and spherical storages

The fundamental period of sloshing of liquid is so significant of all and so frequently calculated that it is converted into a convenient nomogram (Fig. 6). The curve to calculate the fundamental period of sloshing of liquid in a spherical storage (Sec. 8.1) is also shown in this figure.

7.3 Steady Sinusoidal Response

The response of liquid during steady sinusoidal input was calculated. It was found, from this result, that the steady sinusoidal response of liquid is classified into three cases which are shown in Fig. 7. Case i) has significance in the aseismic engineering because the side wall is exposed to a pressure near to the input acceleration. This corresponds to the acceleration type response in Sec. 6.2. Case ii) also has significance in the aseismic engineering because the sloshing phenomenon appears. This corresponds to the displacement type response in Sec. 6.2. Case iii) has little significance in the aseismic engineering because the response is very small. The most interesting point is the difference in the side wall pressure between case i) and case ii). This is thought to explain well the difference of experimental results between the acceleration type response and the displacement type response in Sec. 6.2.

7.4 The Response of the Fundamental Mode

We will pay attention to the response of the fundamental mode to evaluate the displacement type response. As the response of the fundamental mode is thought to be similar to case ii), the maximum amplitude of the liquid surface displacement h is obtained as follows

$$\frac{h}{R} = \frac{\gamma_1}{R} \Big|_{\frac{h}{R} = 0} = 0.837 \frac{1}{g} (\alpha_1 - \alpha_g) \quad (15)$$

The nominal acceleration response of the fundamental mode $\alpha_1 (= \ddot{h}_1)$ is obtained as a solution of Eq. (9) for $i=1$.

As an example of response calculation, the response spectrum of the maximum amplitude of the liquid surface displacement was calculated (Fig. 8). The input wave is a normalized El Centro NS, 1940. This shows that the response spectrum of h/R is similar to the relative acceleration spectrum of a single-degree-of-freedom system. The response curve becomes extremely flat with the increase of the period of sloshing of liquid because of lack of long period components in the record.

7.5 Convenient Calculation Method for the Displacement Type Response

It was pointed out in Sec. 6.2 that the wave form of the semi-long period dominant ground motion which excites the sloshing of liquid is near to a sinusoidal

Fig. 7 Steady sinusoidal response

Fig. 8 An example of response spectrum of liquid surface displacement

wave and the wave form of the response is also near to a sinusoidal wave. From this fact, it is reasonable to assume that the ground motion can be simulated by a sinusoidal wave whose period is equal to the dominant period of the ground motion and amplitude equal to the maximum displacement of the ground motion. However, the component of the same period does not continue so long during an earthquake that the response does not grow up to the level of steady sinusoidal response. Then to what level of transient response during a sinusoidal wave input, the response during an earthquake grows up? If this is known, the response during an earthquake can be estimated by the transient response during a sinusoidal wave input. This concept was already reported in Ref. (20) by one of the authors.

Assume that the ground motion is simulated by

$$\alpha_g(t) = D_g \omega_g^2 \sin\left(2\pi \frac{t}{T_g}\right) \quad (t \geq 0) \quad (16)$$

then, the nominal acceleration response of liquid $a_1(t)$ is given in the next form as a solution of Eq. (9)

$$a_1(t) = D_0 \omega_0^2 a_1(t) \quad (17)$$

where $a_1(t)$ is a non-dimensional acceleration response.

Therefore, the maximum responses to the input of n cycles of sinusoidal wave can be estimated as follows:

Liquid surface displacement

$$\frac{h(n)}{R} = 0.837 \Psi_{st} \{A_a(n) + 1\} \quad (18)$$

Side wall pressure

$$\frac{P_w(n)}{\gamma_f H} = 0.837 \frac{1}{H/R} \Psi_{st} \{A_a(n) + 1\} \quad (19)$$

Bottom pressure

$$\frac{P_b(n)}{\gamma_f H} = 0.837 \frac{1}{H/R} \Psi_{st} \times \left[\frac{1}{\cosh\{1.84(H/R)\}} A_a(n) + 1 \right] \quad (20)$$

where

$$A_a(n) = \max_{0 \leq t \leq nT_0} |a_1(t)| \quad (21)$$

$$\Psi_{st} = \frac{D_0 \omega_0^2}{g} : \text{seismic coefficient of the ground motion} \quad (22)$$

The relation between $A_a(n)$ and the damping ratio ζ_1 is shown in Fig. 9 for the reso-

Fig. 9 Relation between $A_a(n)$ and the damping ratio ζ_1

Fig. 10 Relation between $A_a(n)$ and the period ratio λ_1

nant case⁽²¹⁾. This shows that the damping ratio ζ_1 gives a little influence on $A_a(n)$ in the range where the damping ratio ζ_1 is small ($\zeta_1 < 1\%$ for ordinary liquid storage). Setting $\zeta_1 = 0.1\%$ for example, $A_a(n)$ is given as a function of the period ratio

$$\lambda_1 = \frac{T_1}{T_0} = \frac{\text{period of sloshing}}{\text{dominant period of the ground motion}} \quad (23)$$

This is shown in Fig. 10.

Summing up these results, the maximum response of a liquid storage which is subjected to the input of n cycles of sinusoidal wave can be calculated by using Fig. 10 and Eqs. (18) to (20).

These results are converted into a convenient nomogram in Fig. 11. The example in this nomogram shows that the maximum response of the liquid surface displacement of about 100 Ml cylindrical storage (Diameter: 80 m, Height: 20 m) is 80 cm to the input of 3 cycles of sinusoidal wave whose period is 12 sec and amplitude 10 cm.

Now the most significant point is the estimation of the equivalent input cycle number. Assuming that the storage is subjected to the input of n cycles of sinusoidal wave at resonant period, the response of the displacement of the center of gravity $x_g(t)$ can be obtained by the use of Eq. (18), and the transient response factor of $x_g(t)$ to $d_0(t)$ is given as follows

$$A_g(n) = 1.21 \frac{\tanh\{1.84(H/R)\}}{H/R} n \quad (24)$$

The relation between the transient response factor $A_g(n)$ and the response factor A_a obtained as the result of experiments during earthquake motions is shown in Fig. 12. It is found from this figure and oth-

Fig. 11 The nomogram to calculate the response of the liquid surface displacement $h(n)$ and the side wall pressure $P_w(n)$ (to the acceleration input of semi-infinite sinusoidal wave)

er results of response analysis that the equivalent input cycle number is 3 or 4 except the case of extreme resonance.

This suggests that the responses during most of earthquakes can be estimated by setting

$$n=3\sim 4 \quad (25)$$

in the formulation above except the case of extreme resonance.

Fig. 12 Relation between $A_g(n)$ and A_r

8. Response Analysis of Spherical Liquid Storage

A method to evaluate the response of a spherical liquid storage which is subjected to a earthquake ground motion is shown in this chapter.

As already known, the system of a spherical storage can be regarded as a connection of a rigid system and a flexible system as shown in Fig. 13. The rigid system, whose period is T_f , is composed of the container W_0 , the fixed weight of liquid W_f and the supporting structure.

Fig. 13 Vibrational model of spherical storage

The flexible system, whose period is equal to the period of sloshing of liquid T_s , is composed of the free weight of liquid W_s . The other notations in use are as follows.

$$S(h) = W_s/W : \text{free liquid ratio} \quad (26)$$

$$W = W_{\max} \eta(h) : \text{weight of liquid} \quad (27)$$

$$W_{\max} : \text{full weight of liquid}$$

$$\eta(h) = h^2(3-2h) : \text{liquid fullness ratio}$$

$$h = H/D : \text{liquid depth ratio}$$

The response of the rigid system can be analyzed by the same method as is used for the ordinary structure. The response of the flexible system can be analyzed by the same method as is used for the cylindrical storage in Sec. 7.5.

8.1 Vibrational Characteristics of the Spherical Storage

The significant vibrational characteristics of the spherical storage are next three points.

- 1) Sloshing patterns
- 2) Period of sloshing of liquid: T_s
- 3) Free liquid ratio: $S(h)$

The authors carried out an experiment⁽²²⁾ by which a model of spherical storage of diameter 50 cm was shaken on a shaking table to evaluate these characteristics. The results of the experiment are as follows.

1) Sloshing patterns:

In the case of $h < 0.3$ and $h > 0.8$, the liquid moves along the wall of the container keeping almost the same shape at its rest. This is a typical pattern which appears in spherical storage. In the case of $0.3 < h < 0.8$, the liquid shows a complex pattern as a superposition of the fundamental mode which moves as plane over the higher order mode which moves as ripple. This tendency is most remarkable around $h = 0.6$. In addition to these sloshing patterns, some rotating patterns of liquid were observed. They are thought to be caused by the disturbance for the wave.

2) Period of sloshing of liquid:

The correspondence between the experimental results and the analytical results is shown in Fig. 14. The authors formulated a semi-experimental formula for the period of sloshing of liquid in a spherical container through the consideration of this figure.

It was found, from the result of experiment, that the liquid moves along the wall of the container keeping almost the same shape at its rest when the liquid is very little. So the motion of liquid can be modeled by a rigid pendulum model in this case. The period of this model T is plotted by the chain line in Fig. 14, which approximates well the result of experiment when the liquid is very little. This curve crosses the ordinate at point P.

Fig. 14 The period of sloshing of liquid in the spherical storage model

Next, the spherical container was approximated by a cylindrical container which circumscribed the surface of liquid. The period of sloshing of liquid in this container T_c is plotted by the dotted line in Fig. 14. This shows that the feature of this curve is similar to the result of experiment and both cross each other at point Q where h is around 0.65.

Based on these results, The authors assumed that the period of sloshing of liquid in a spherical storage T_s is given as $T_s = T_c(\alpha + \beta h)$. The coefficients α and β were determined such that the curve of T_s passed the points P and Q. Finally, a semi-experimental formula was given as

$$T_s = T_c(1.30 - 0.462h) \quad (28)$$

The period T_s is plotted by the solid line in Fig. 14. This shows that T_s approximates the result of experiment well. The detail of this analysis is shown in Ref. (23).

One of the authors developed a simple method by which the period of sloshing of liquid in an axisymmetric container can be easily calculated⁽²⁴⁾. The result by this method is plotted by the double chain line in Fig. 14. It is found that T_s also approximates the result of this analysis well.

The nomogram for the period of sloshing of liquid in spherical storage in Fig. 6 is plotted based on this semi-experimental formula. It is found from Fig. 6 that the period of sloshing of liquid in spherical storage changes more strongly than that of cylindrical storage in accordance with a change in the liquid depth ratio h .

3) Free liquid ratio:

Assuming that the system in Fig. 13 is subjected to the input of n cycles of

sinusoidal wave whose period is T_s and maximum acceleration a_n , the maximum response of the base shear force $Q(n)$ is given by Eq. (29) which will be proposed in next section. Therefore, if a_n and $Q(n)$ are measured in an experiment, the free liquid ratio $S(h) = W_s/W$ can be calculated. The free liquid ratio $S(h)$ which was evaluated experimentally in this way is shown in Fig. 15.

8.2 Base Shear Force

The results obtained in the previous sections were applied to the model of spherical storage which was used for the experiment. The base shear force of this model to the input of a normalized El Centro wave was simulated. The result is shown in Fig. 16. The liquid depth ratio h or the liquid fullness ratio $\eta(h)$ is taken as abscissa and the maximum shearing coefficient q_{max} as ordinate. This shows that the response of the rigid system is dominant for small or large h , while the response of the flexible system is dominant around $h=0.5$.

Fig. 15 Free liquid ratio and base shear coefficient

Fig. 16 Base shear coefficient (result of a simulation)

The evaluation of the response of the flexible system is proposed in this section. The response of the rigid system can be evaluated by the same method as is used for the structure.

It was shown in Sec. 7.5 that the response of sloshing of liquid in cylindrical storage during an earthquake is estimated by the transient response to the input of n cycles of sinusoidal wave whose period is equal to the period of sloshing of liquid and amplitude to the maximum displacement of the ground motion. It is assumed in this section that this is also applicable for the spherical storage.

In this case, the maximum response of base shear force $Q(n)$ is given by

$$Q(n) = n\pi \frac{W_s}{g} a_g \quad (29)$$

By the use of Eqs. (26) and (27), Eq. (29) is modified into next form

$$Q(n) = n\pi K_g S(h) \eta(h) W_{\max} \quad (30)$$

where $K_g (= a_g/g)$ denotes the seismic coefficient of the input ground motion.

Next, we will define the base shear coefficient $q(h)$ as follows

$$q(h) = \frac{Q(n)}{nK_g W_{\max}} = \pi S(h) \eta(h) \quad (31)$$

This denotes the maximum response of base shear force per unit volume of liquid when the system is subjected to the input of one cycle of sinusoidal wave whose period is T and maximum acceleration 1.0 G. This was calculated by using the free liquid ratio $S(h)$ in Fig. 15. The result is also shown in Fig. 15. It is found from this figure that $q(h)$ gives its maximum value at $h=0.6$.

By using $q(h)$ in Fig. 15, the maximum base shear force $Q(n)$ can be evaluated by Eq. (31). Setting $n=3\sim 4$ in this result, we can estimate the maximum base shear force for design during a certain earthquake $Q_{\max}$.

9. Conclusions

The authors pointed out the present situation of the aseismic design of liquid storages and its points of issue at first and showed the fundamental concepts of a systematization of the aseismic design of liquid storages. Some experiments and analyses were carried out according to these concepts and the results were summed up into formula and nomogram which are convenient in practical use. It is expected that these results will be available for the response analysis of sloshing of liquid in the aseismic design of liquid storages.

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