621-522 : 621-82 : 629-113

Analysis of Characteristic Performance of Hydrostatic Transmission for Vehicles*

By Yoshinori ITO** Toshiyuki SARUMARU**

Real comprehension of specific character of hydrostatic transmission is gained only from investigation into respective performances of hydraulic instruments and power flow originated from engine and transferred to load through hydraulic drive system; this flow is to be regarded as a power train system. This paper takes up our investigation into and experiment in distinctive feature on characteristic formula of hydraulic drive system gained from approximate characteristic formula of hydraulic instruments. Investigation and experiment have been performed on the characteristic of a power train system combined with a diesel engine. This investigation and experiment revealed the following: Impedance of hydrostatic transmission larger than that of a torque converter is rather similar to characteristic of gear transmission. This hydrostatic transmission provides a continuously variable speed ratio not available for gear transmission. The above features could bring forth a better power controllability different from other transmissions.

1. Introduction

Research on the use of hydrostatic transmissions as a power transmission for motor vehicles has been carried on for years⁽¹⁾⁽²⁾, and the increasing level of performance of hydraulic pumps and motors is also making it possible to apply such hydrostatic transmissions to some industrial vehicles.

There are two types of hydrostatic transmissions, the pure hydrostatic type and the hydro-mechanical type, and here attention shall be paid to the former of these two. In order to understand the performance of such a hydrostatic transmission, it is necessary to study not only the performance of the individual hydraulic mechanisms composing a system but also the flow of power from the prime mover through the hydrostatic transmission to the load as a total system.

In this paper, we will introduce the performance of its features⁽³⁾ and of a power train system combining a hydrostatic transmission with a diesel engine⁽⁴⁾, and report on the results of some experiments concerning the system efficiency control.

2. Terminology

The terminology and designations in this paper, other than those specifically defined elsewhere herein, are as follows.

C_d : drag coefficient of hydraulic pump and motor, C_f : friction loss coefficient of same, C_s : slip coefficient of same, D : maximum displacement of same cm^3/rev , D_i : internal drag coefficient of

engine kgcm/rps , D_L : drag coefficient of load (test bench) kgcm/rps , D_r : drag coefficient of reduction gear kgcm/rps , F : drive force kg , F_c : constant loss force kg , H : horse power PS , I : inertia of chassis and load kgs^2/cm , J : moment of inertia kgcms/rps , K : tangential compliance of drive wheel cm/kg , K_N : feedback gain of engine governor cm/rps , K_q : conversion gain of fuel into torque $\text{kgcm/cm}^3/\text{rev}$, K_r : input gain of engine governor cm/rad , K_s : gain of fuel injection pump $\text{cm}^3/\text{rev cm}$, K_v : gain of swash plate servo $1/\text{rad}$, N : revolution speed rps (if not specified rpm), P : pressure kg/cm^2 , Q : flow rate cm^3/s , R : rolling radius of drive wheel cm , T : torque kgcm (if not specified kgm), T_b : combustion torque of engine kgcm (if not specified kgm), T_c : constant loss torque kgcm , T_d : time constant of dynamometer s , T_e : time constant of engine combustion torque s , T_θ : time constant of swash plate servo s , V : vehicle speed cm/s , Z : impedance kgcm/rps , kg/cm/s , $a, b, c \& d$: elements of matrix, e_r : speed ratio, i : reduction ratio, k_c : bulk modulus of conduit (hydraulic oil and conduit wall) kg/cm^2 , l : length of conduit cm , q : flow rate of fuel cm^3/rev , r : input of engine governor rad (if not specified degree), r_c : radius of conduit cm , s : Laplace operator $1/\text{s}$, t : time s , v : volume of flow path cm^3 , x : displacement of rack of fuel injection pump cm , Γ : transfer matrix, α : displacement ratio of hydraulic pump and motor, ϵ : mechanical loss coefficient of hydrostatic transmission, ζ : drag coefficient of hydrostatic transmission kgcm/rps , θ : lever angle of swash plate servo rad (if not specified degree), κ : bulk modulus of hydraulic oil kg/cm^2 , μ : coefficient of viscosity kgs/cm^2 , ν : coefficient of kinematic viscosity cm^2/s , ξ : constant loss torque of hydrostatic transmission kgcm , ρ : density of hydro-

* Received on 3rd November, 1974

** Toyo Umpanki Co., Ltd. Technical Research Institute Ryugasaki City 3, Ibaragi Prefecture

lic oil $\text{kg s}^2/\text{cm}^4$, suffixes 1, 2, 3, 4, e , l and L indicate hydraulic pump, conduit, motor, reduction gear, prime mover, load and dynamometer load, respectively.

3. Performance of the hydrostatic transmission

First, we derived the characteristic equations of the hydrostatic transmission as a collection of structural elements such as hydraulic pumps and motors, and conducted studies and tests regarding their features.

3.1 Characteristic equations of the hydrostatic power train

Hydraulic power, generated by the prime mover, is transmitted through the hydrostatic transmission to the load as shown in Fig. 1. The flow of power in such

Fig. 1 Flow of power
a case is not a unilateral flow of ideal signals as dealt with in control engineer-

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{D_1 \alpha_1} & \frac{1}{2\pi \mu \alpha_1} (C_{n1} + \frac{\pi \mu v_1 s}{\kappa D_1}) \\ \frac{1}{D_1 \alpha_1} (C_{d1} \mu D_1 + J_1 s) & \frac{1}{2\pi \mu \alpha_1} \{ (C_{d1} \mu D_1 + J_1 s) (C_{n1} + \frac{\pi \mu v_1 s}{\kappa D_1}) + \mu D_1 \alpha_1 (\alpha_1 + C_{f1}) \} \end{pmatrix} \begin{pmatrix} Q_1 \\ P_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c1} = \Gamma_1 \begin{pmatrix} Q_1 \\ P_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c1} \quad (1)$$

Here, T_{c1} is constant loss torque unrelated to Q_1 and P_1 .

3.1.2 Conduit Judging from the degree of approximation of the performance of the power train as a whole, a lumped parameter system approximation is sufficient, and this is expressed by Eq. (2).

$$\begin{pmatrix} Q_1 \\ P_1 \end{pmatrix} = \begin{pmatrix} 1 + \frac{\rho l^2 s (r_c^2 s + 8\nu)}{2\kappa r_c^2} & \frac{\pi r_c^2 l s}{\kappa} \\ \frac{\rho l (r_c^2 s + 8\nu)}{\pi r_c^4} + \frac{\rho^2 l^2 s (r_c^2 s + 8\nu)^2}{4\pi r_c^6 \kappa} & 1 + \frac{\rho l^2 s (r_c^2 s + 8\nu)}{2r_c^2 \kappa} \end{pmatrix} \begin{pmatrix} Q_2 \\ P_2 \end{pmatrix} = \Gamma_2 \begin{pmatrix} Q_2 \\ P_2 \end{pmatrix} \quad (2)$$

3.1.3 Motor The following equation is derived in the same manner as for the pump.

$$\begin{pmatrix} Q_2 \\ P_2 \end{pmatrix} = \begin{pmatrix} D_3 \alpha_3 + \frac{(\kappa C_{d3} D_3 + \pi \mu v_3 s)(C_{d3} \mu D_3 + J_3 s)}{\mu \kappa D_3 (\alpha_3 - C_{f3})} & \frac{\kappa C_{d3} D_3 + \pi \mu v_3 s}{\mu \kappa D_3 (\alpha_3 - C_{f3})} \\ \frac{2\pi (C_{d3} \mu D_3 + J_3 s)}{D_3 (\alpha_3 - C_{f3})} & \frac{2\pi}{D_3 (\alpha_3 - C_{f3})} \end{pmatrix} \begin{pmatrix} N_3 \\ T_3 \end{pmatrix} + \begin{pmatrix} C_{d3} \\ \frac{\mu (\alpha_3 - C_{f3})}{2\pi} \end{pmatrix} T_{c3} = \Gamma_3 \begin{pmatrix} N_3 \\ T_3 \end{pmatrix} + \begin{pmatrix} C_{d3} \\ \frac{\mu (\alpha_3 - C_{f3})}{2\pi} \end{pmatrix} T_{c3} \quad (3)$$

Here, T_{c3} is constant loss torque unrelated to N_3 and T_3 .

3.1.4 Reduction gear The input and output performance of the gear box is represented by the following equation. (J_4 is the equivalent value at the output end of the reduction gear.)

$$\begin{pmatrix} N_3 \\ T_3 \end{pmatrix} = \begin{pmatrix} i & 0 \\ D_4 + J_4 s & 1 + C_{f4} \end{pmatrix} \begin{pmatrix} N_4 \\ T_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c4} = \Gamma_4 \begin{pmatrix} N_4 \\ T_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c4} \quad (4)$$

Here, T_{c4} is constant loss torque unrelated to N_4 and T_4 .

3.1.5 Drive wheel The following equation is derived as the mechanical series circuit of the chassis, load, K , I , and Z_L .

$$\begin{pmatrix} N_4 \\ T_4 \end{pmatrix} = \begin{pmatrix} \frac{1 + K I s^2}{2\pi R} & \frac{K s}{2\pi R} \\ R I s & R \end{pmatrix} \begin{pmatrix} V \\ F \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F_c = \Gamma_5 \begin{pmatrix} V \\ F \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F_c \quad (5)$$

Here, F_c is constant loss of force unrelated to V and F .

3.1.6 Hydrostatic power train When the correlation of input of pump and output of drive wheel are sought from Eq. (1) through (5), the following equation may be derived to express the overall performance of the hydrostatic power train system.

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \Gamma_0 \begin{pmatrix} V \\ F \end{pmatrix} + \Gamma_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} F_c + \Gamma_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c4} + \Gamma_3 \begin{pmatrix} C_{d3} \\ \frac{\mu (\alpha_3 - C_{f3})}{2\pi} \end{pmatrix} T_{c3} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} T_{c1} \quad (6)$$

Here, $\Gamma_0 = \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\Gamma_1 = \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4$, $\Gamma_2 = \Gamma_1 \Gamma_2 \Gamma_3$, $\Gamma_3 = \Gamma_1 \Gamma_2$

ing; instead, it is a bilateral flow wherein the features of the transmission result in information on the state of the load being fed back to affect the operating conditions of the prime mover. The power is handled in terms potential and flow,⁽⁵⁾ with impedance defined as

$$\text{Impedance} = \frac{\text{potential}}{\text{flow}}$$

In this report, torque, pressure, and drive force are taken as potential, and revolution speed, flow rate, and vehicle speed are taken as flow. When such an approach is used, it is possible to derive the following type of transfer matrix from the approximation of each structural element.

3.1.1 Pump Various matters have been discussed concerning the performance of pumps,⁽³⁾ but since the degree of seal of each element and the lubrication conditions fluctuate with changes in the operating conditions, it is difficult to make highly precise statements of the performance. Based on a variety of hypotheses, however, such performance is given mathematical form and Eq. (1) is derived by using several of these simple approximations.

$$\frac{T_1}{N_1} = \frac{c' + d'Z_1}{a' + b'Z_1} \quad (7)$$

$$\frac{F}{V} = \frac{-c' + a'Z_1}{d' - b'Z_1} \quad (8)$$

Here, Z_1 is the input impedance of the load, Z_2 is the output impedance of the prime mover, and a' , b' , c' , and d' are as defined below.

$$a' = \frac{1}{2\pi R \alpha_1 (\alpha_3 - C_{fs})} \left[\left(\frac{1}{D_1} + \frac{4C_{n1}l}{\pi^2 r_c^4} \right) \left\{ iD_3(\alpha_3^2 - C_{fs}\alpha_3 + C_{ns}C_{ds}) + \frac{C_{ns}D_r}{i\mu} \right\} + \frac{C_{n1}}{D_3} \left(iC_{ds}D_3 + \frac{D_r}{i\mu} \right) \right]$$

$$b' = \frac{R(1+C_{fs})}{i\alpha_1(\alpha_3 - C_{fs})\mu} \left[C_{ns} \left(\frac{1}{D_1} + \frac{4C_{n1}l}{\pi^2 r_c^4} \right) + \frac{C_{n1}}{D_3} \right]$$

$$c' = \frac{1}{2\pi R \alpha_1 (\alpha_3 - C_{fs})} \left[\mu \left\{ C_{ds} + \frac{4D_1}{\pi^2 r_c^4} (\alpha_1^2 + C_{f1}\alpha_1 + C_{n1}C_{ds}) \right\} \left\{ iD_3(\alpha_3^2 - C_{fs}\alpha_3 + C_{ns}C_{ds}) + \frac{D_r C_{ns}}{i\mu} \right\} \right. \\ \left. + D_1(\alpha_1^2 + C_{f1}\alpha_1 + C_{n1}C_{ds}) \left(iC_{ds}\mu + \frac{D_r}{iD_3} \right) \right]$$

$$d' = \frac{R(1+C_{fs})}{i\alpha_1(\alpha_3 - C_{fs})} \left[C_{ns}C_{ds} + D_1(\alpha_1^2 + C_{f1}\alpha_1 + C_{n1}C_{ds}) \left(\frac{4C_{n1}l}{\pi^2 r_c^4} + \frac{1}{D_3} \right) \right]$$

In short, input impedance of load Z_1 is converted according to Eq. (7) and appears at the input of the hydrostatic transmission, and conversely, the output impedance of the prime mover Z_2 is converted according to Eq. (8) and appears at the output of the hydrostatic power train.

Now, taking as an example a variable displacement pump and a fixed displacement motor ($\alpha_3=1$) wherein $D_1=D_3=33.3\text{cm}^3/\text{rev}$, let us assume that the rolling radius R of the drive wheel equals 32cm, the reduction ratio i of the reduction gear equals 20, the efficiency of the reduction gear equals 100%, the coefficient of viscosity of the hydraulic oil μ equals $3.15 \times 10^{-7}\text{kg s/cm}^2$, and the loss coefficients of the hydraulic pump and motor are $C_{p1}=C_{ds}=10^{-8}$, $C_{d1}=C_{ds}=1.4 \times 10^{-5}$, $C_{f1}=C_{fs}=6 \times 10^{-3}$, $l=50\text{cm}$, $r_c=1\text{cm}$, and $\alpha_1=1$. When such values are assumed, input impedance becomes $T_1/N_1 = (0.00247 + 1.60Z_1)/(0.0995 + 0.00305Z_1)$. Taking a case where $Z_1 \approx 0$ with almost no variance in load (running resistance) when vehicle speed changes, this becomes $T_1/N_1 = 0.024 \text{ kg cm/rps}$. In contrast, if input impedance of the load is infinity (output stall), T_1/N_1 becomes 524 kg cm/rps and does not become infinite as in a gear transmission, but it does reach a high value compared to the torque converter. This is explained conceptually by Fig. 2. Compared to the torque converter, the hydrostatic transmission more faithfully reflects the input impedance of the load in its input. This feature thus

Fig. 2 Comparison of input impedance of various transmissions

makes the hydrostatic transmission closer in nature to a gear transmission than to a torque converter. In terms of matching with the prime mover as well, it displays the same type of nature as a gear transmission.

Let us consider the output performance in the same way. Assuming the same conditions as for the input impedance in the previous example and solving for the output performance, $F/V = (-0.00247 + 0.0995Z_2)/(1.60 - 0.00305Z_2)$. Solving for the output impedance of operations driven by an ideal fixed-speed prime mover ($Z_2=\infty$), F/V equals 32.6 kg/cm/s (negative) and does not become infinite as in a gear transmission, but it does reach a higher value than a torque converter. Output impedance in the case of a fixed-torque prime mover ($Z_2 \neq 0$) becomes 0.00154 kg/cm/s (negative). This is explained conceptually in Fig. 3 and is compared in Fig. 4 with the output performance of various transmissions using infinite output impedance prime movers.

A comparison of the features of the hydrostatic transmission as described above with those of other forms of transmission is made in Table 1.

Fig. 3 Comparison of output impedance of various transmissions

Fig. 4 Comparison of output performance of various transmissions

3.3 Bench test of hydrostatic transmission static performance

Static performance taking the pump as input and the reduction gear as output was sought in a bench test. As the equipment to be tested, the pump, conduit, motor, reduction gear, and charging pump were combined in a single unit, with each of these elements described in Table 2. As shown in Fig. 5, the drive was provided by a three-phase shunt commutator motor, and the power was absorbed by a direct current electric dynamometer. The primary measurement made in this bench test was the assessment of impedance, and measurement was performed of the input and output, temperature, and pressure by holding the input revolutions and lever angle of the swash plate servo constant and varying the load torque.

Fig. 6 gives test results on output performance. When these results are coordinated, N_1 and T_1 when the power flow is in the positive direction (drive provided by an electric motor) are shown by the following equations.

$$N_1 = \frac{\alpha_1}{i} N_2 - \frac{1}{Z_4} T_4, \quad T_1 = (1+\varepsilon) \frac{\alpha_1}{i} T_4 + \zeta N_4 + \xi \quad (9)$$

Here, N_{40} represents N_4 when $T_4=0$, and Z_4 represents the output impedance (electric motor output impedance equals ∞) at the reduction gear output, ε , ζ and ξ respectively represent the mechanical loss coefficient.

Transmission	Features
Hydrostatic transmission	<ol style="list-style-type: none"> (1) High impedance, making it closer to the gear transmission than to the torque converter (large stiffness). (2) Continuously variable speed ratio. (3) Clutch not necessary. (4) Simple reversal of direction of revolution. (5) There is a large degree of freedom in control of power, and control can be applied not only to the output but to the operational conditions of the prime mover.
Torque convertor	<ol style="list-style-type: none"> (1) Low impedance (small stiffness, and changes in load bring great fluctuations in speed even when running with a prime mover with good speed regulation performance). (2) Suited for automatic exchange of speed and torque. (3) Reversing device and clutch are necessary.
Gear transmission	<ol style="list-style-type: none"> (1) Impedance is infinite. (2) Continuously variable speed ratio impossible. (3) Clutch necessary. (4) Offers better efficiency and lower price than above two types.

efficient, drag coefficient, and constant loss torque sought in this test, and these are presented in Figs. 7 and 8. The value of i is 2.8, which is the same as the reduction ratio of the tested equipment.

The following equation can be derived from Eq. (9) for a given point of motion and vicinity.

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} \frac{i}{\alpha_1} & \frac{i}{\alpha_1 Z_4} \\ \zeta & (1+\varepsilon) \frac{\alpha_1}{i} + \frac{\xi}{Z_4} \end{pmatrix} \begin{pmatrix} N_4 \\ T_4 \end{pmatrix} \quad (10)$$

In order to make a comparative study of these test results and calculated estimated values, Eq. (6) is taken at steady state and the matrix elements between the pump and the reduction gear are taken as a' , b' , c' and d' . When it is assumed that $i=2.8$, $D_1=D_2=51.6 \text{ cm}^3/\text{rev.}$, $\alpha_3=1$, $\mu=3.15 \times 10^{-7} \text{ kg s/cm}^2$, $C_{11}=C_{33}=10^{-8}$, $C_{41}=C_{35}=1.4 \times 10^{-5}$, $C_{11}=C_{33}=6 \times 10^{-3}$ (6), $C_{14}=0$, $l=60 \text{ cm}$, and $r_c=1 \text{ cm}$, the following values may be derived: $a' \approx 2.8/\alpha_1$, $b' \approx 4.40 \times 10^{-4}/\alpha_1$, $c' \approx (0.16 + D_1) \times \alpha_1/2.8$, and $d' \approx \alpha_1/2.8$. The output impedance when using a prime mover with output impedance ∞ becomes $Z_4=6.37 \times 10^3 \text{ kg cm/rps}$ (negative). A comparison of these calculated values with the matrix elements of Eq. (10) and the test results in Figs. 7 and 8 shows that while they do not match precisely in terms of quantity, they do match in the sense that in both the calculated and actual test values, a' and d' , the basic matrix elements for the exchange of input and output torque and speed of revo-

Fig. 5 Block diagram of bench test apparatus

Table 2. Specifications of Hydrostatic Transmission

Displacement	Pump: $D_1 = 51.6 \text{ cm}^3/\text{rev}$ $\alpha_1 = 0 \sim 1$ (variation possible within this range) Motor: $D_2=51.6 \text{ cm}^3/\text{rev.}$ $\alpha_2=1$ (fixed)
Continuous rated pressure	350 kg/cm ²
Max. permissible pressure	700 kg/cm ²
Max. permissible revolution	3100 rpm
Structure	Pump & motor (axial plunger with swash plate), reduction gear, charging pump were combined in a single unit. Swash plate was controlled by hydraulic servo mechanism.
Hydraulic fluid	Mobile #300
Reduction Gear	Spur gear, oil-bath lubrication, $i=2.8$

lution, represent approximations of the simple geometric relationship between i and α_1 . In the case of the values for b_r and Z_1 , the calculated values and the actual test results yielded values of the same magnitude, but in order to raise the level of accuracy, it will be necessary to determine the values of C_1 and the apparent value of μ more accurately. In the case of C_1 , the magnitude of α_1 or non-pump drag generated by pump output flow generally has a large bearing on the calculated results, but in the test results, the pump itself also exerted a strong influence. Such influence probably consists of the accuracy of the value of C_1 and the influence of the torque of the charging pump and the drag torque of the cylinder block, which were not taken into account in the calculations.

When seeking quantitative precision, it is necessary to conduct further studies of C_1 , C_2 , or the performance equations, but when looking in general at the flow of power, the steady state values of Eq. (6) express the hydrostatic transmission performance as described in the preceding section very well. To provide a test example of torque converter performance as given in Fig. 4 above, a test was conducted on a torque converter (with gear box) having about the same level of capacity as the test hydrostatic transmission, and this test example is presented in Fig. 9.

Fig. 10 displays the efficiency of the hydrostatic transmission with charging pump and reduction gear. This efficiency varies depending on the operating conditions, but it is better at higher speed

ratios and greater torques. Fig. 11 gives the efficiency of the torque converter (with gear box) presented in Fig. 9. A comparison of Fig. 10 and Fig. 11 shows that in terms of efficiency, the hydrostatic transmission is not inferior to a torque converter (3 elements, 1 stage, 1 phase, with gear box).

4. Performance in combination with diesel engine

Internal combustion engine speed regulation and performance have traditionally been dealt with in terms of control engineering.⁽¹⁾ In this section we will present a study and test results on a diesel engine with all-speed governor in combination with the hydrostatic transmission of the preceding sections and a dynamometer.

4.1 Performance of the combination

The influence of the inertia, compressibility, and other factors of the oil within the hydrostatic transmission can, it is assumed, be ignored because of such factors as the transient response of the engine and the inertia of the load. The inertias of the moving parts of the pump and motor are considered to be combined with the inertia of the engine and load, respectively. From Eq. (10), we can deal with the vicinities of operational points (N_{40} , T_{40} , α_{10}) by seeking a linear approximation of the values of N_1 and T_1 for variables N_4 , T_4 , and α_1 , thereby deriving the following equation.

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} \frac{i}{\alpha_{10}} & \frac{i}{\alpha_{10}Z_4} \\ \zeta & (1+\varepsilon)\frac{\alpha_{10}}{i} + \frac{\varepsilon}{Z_4} \end{pmatrix} \begin{pmatrix} N_4 \\ T_4 \end{pmatrix} + \begin{pmatrix} -(N_{40} + \frac{T_{40}}{Z_4})\frac{i}{\alpha_{10}^2} \\ (1+\varepsilon)\frac{T_{40}}{i} \end{pmatrix} \alpha_1 = F_{40} \begin{pmatrix} N_4 \\ T_4 \end{pmatrix} + F_{a1} \alpha_1 \dots\dots\dots(11)$$

Next, seeking the relationship between the dynamometer load and the hydrostatic transmission output produces Eq. (12).

Fig. 6 Test results of output performance

Fig. 7 Values of ε , ζ and ξ

Fig. 8 Values of Z_4

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (J_L s + D_L) & 1 \end{pmatrix} \begin{pmatrix} N_L \\ T_L \end{pmatrix} = \Gamma_L \begin{pmatrix} N_L \\ T_L \end{pmatrix} \quad (12)$$

From Eqs. (11) and (12), the relationship between the hydrostatic transmission input and the load can be represented by the following equation.

$$\begin{pmatrix} N_1 \\ T_1 \end{pmatrix} = \Gamma_{a0} \Gamma_L \begin{pmatrix} N_L \\ T_L \end{pmatrix} + \Gamma_a \alpha_1 \quad (13)$$

Meanwhile, the performance of the diesel engine is complicated because the variations in fuel-torque conversion are intermittent, but the simplified process of governor input, fuel supply, torque, and feedback of speed in the vicinity of the operating points is dealt with as shown in Fig. 12.

$$N_L = \frac{K_r K_s K_q}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} r + \frac{B_2 s^2 + B_1 s + B_0}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} \alpha_1 - \frac{C_2 s^2 + C_1 s + C_0}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} T_L \quad (15)$$

Here,

$$A_3 = \frac{i J_L J_s T_q}{\alpha_{10} Z_4}, \quad A_2 = \frac{i}{\alpha_{10} Z_4} \{ J_L (J_s + T_q D_s) + (Z_4 + D_L) J_s T_q \} + \left\{ (1 + \varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} J_L T_q$$

$$A_1 = \frac{i}{\alpha_{10} Z_4} \{ J_L (D_s + K_q K_s K_N) + (Z_4 + D_L) (J_s + T_q D_s) \} + \left\{ (1 + \varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} (T_q D_L + J_L) + \zeta T_q$$

$$A_0 = \frac{i}{\alpha_{10} Z_4} (Z_4 + D_L) (D_s + K_q K_s K_N) + \left\{ (1 + \varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} D_L + \zeta$$

$$B_2 = \frac{i J_s T_q}{\alpha_{10}^2} \left(N_{40} + \frac{T_{40}}{Z_4} \right), \quad B_1 = \frac{i (J_s + T_q D_s)}{\alpha_{10}^2} \left(N_{40} - \frac{T_{40}}{Z_4} \right) - \frac{T_q}{i} (1 + \varepsilon) T_{40}$$

$$B_0 = \frac{i (D_s + K_q K_s K_N)}{\alpha_{10}^2} \left(N_{40} + \frac{T_{40}}{Z_4} \right) - \frac{1}{i} (1 + \varepsilon) T_{40}, \quad C_2 = \frac{i J_s T_q}{\alpha_{10} Z_4}$$

$$C_1 = \frac{i}{\alpha_{10} Z_4} (J_s + T_q D_s) + T_q \left\{ (1 + \varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\}, \quad C_0 = \frac{i}{\alpha_{10} Z_4} (D_s + K_q K_s K_N) + \left\{ (1 + \varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\}$$

The displacement ratio α_1 of the hydrostatic transmission is controlled by the lever angle θ of the swash plate through use of a hydraulic servomechanism. Eq. (15) yields the following equation, when the relationship between α_1 and θ is approximated by first order lag.

$$N_L = \frac{K_r K_s K_q}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} r + \frac{K_q (B_2 s^2 + B_1 s + B_0)}{(A_3 s^3 + A_2 s^2 + A_1 s + A_0) (T_\theta s + 1)} \theta - \frac{C_2 s^2 + C_1 s + C_0}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} T_L \quad (16)$$

Fig. 9 Output performance of torque converter (with gear mission)

Fig. 10 Efficiency of hydrostatic transmission

Based on such a hypothesis, seeking the engine output performance within the range wherein torque does not reach saturation levels produces the following equation.

$$N_s = \frac{K_r K_s K_q}{\Delta} r - \frac{T_q s + 1}{\Delta} T_s \quad (14)$$

Here,

$$\Delta = J_s T_q s^2 + (J_s + T_q D_s) s + (D_s + K_q K_s K_N)$$

Since the engine and the hydrostatic transmission are directly coupled, N_1 equals N_s and T_1 equals T_s , and since the hydrostatic transmission output axle is directly coupled to the load, N_2 equals N_L (with axle stiffness infinite). Solving for the relationship of r , α_1 , T_L , and N_L in Eqs. (13) and (14) produces the following equation.

4.2 Bench test of combination

An outline of the bench test is given in Fig. 13. The diesel engine has a displacement of 2207 cm³ and is a swirl chamber type, having four cylinders and four cycles with mechanical all-speed governor. The hydrostatic transmission is the same type as used in earlier sections, and its output is speeded up and absorbed by an eddy current electric dynamometer. The equivalent moment of inertia at the out-

Fig. 11 Efficiency of torque converter (with gear mission)

Fig. 12 Operating process of diesel engine

put end of the hydrostatic transmission is roughly the same as if the hydrostatic transmission were actually mounted on a vehicle.

4.2.1 Static performance test

Let us seek the output impedance of the hydrostatic transmission when it is used in combination with a diesel engine. First, seeking the static performance of the engine from Eq. (14) produces the following equation.

$$N_e = \frac{K_e K_s K_g}{D_e + K_e K_s K_N} r - \frac{1}{D_e + K_e K_s K_N} T_e \quad (17)$$

Thus, the output impedance of the engine is $Z_e = D_e + K_e K_s K_N$, and solving this using the test results presented in Fig. 14 with the governor set constant yields a value for Z_e of 480 kg/rps.

Next, Fig. 15 presents one example of the results of measurements of the output of the hydrostatic transmission in combination with the diesel engine, keeping r and θ constant.

From these results, we obtain the hydrostatic transmission output impedance values shown in Fig. 16 for pump displacement ratio α_1 .

From Eq. (16), we may seek the output impedance when r and θ are fixed, thereby deriving the following equation.

$$\frac{T_L}{N_L} = - \frac{\zeta + \left\{ (1+\varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} D_L + i(Z_4 + D_L) \frac{Z_e}{\alpha_{10} Z_4}}{\left\{ (1+\varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} + \frac{i Z_e}{\alpha_{10} Z_4}} \quad (18)$$

Further, T_L/N_L , the output impedance of the hydrostatic transmission, may similarly be represented by Eq. (19). When D_L is so small that it may be ignored, $T_L/N_L \approx T_4/N_4$.

$$\frac{T_4}{N_4} = - \frac{\zeta + \frac{i Z_e}{\alpha_{10}}}{\left\{ (1+\varepsilon) \frac{\alpha_{10}}{i} + \frac{\zeta}{Z_4} \right\} + \frac{i Z_e}{\alpha_{10} Z_4}} \quad (19)$$

The values calculated by substituting the values obtained in the experiments of preceding sections in Eq. (19) are given

Fig. 13 Outline of bench test of power train

Fig. 14 Output performance of engine

in Fig. 16 above. There is a good match between the calculated values and the observed values.

4.2.2 Dynamic performance test

We measured the response of output revolution speed $N_L (=N_4)$ for step signals for governor input r and swash plate lever angle θ and for input of first order lag of load torque T_L .

Meanwhile, we calculated in Eq. (16) the response when substituting the various numerical values obtained in the earlier experiments and in other supplementary experiments, the values substituted being shown in Table 3.

(1) Indicial response of revolution speed N_L for governor input r

When the governor input becomes too great, the engine torque saturates, so we took care to perform our experiments within the limits of the governor control. Examples of the results of these experiments are shown in Figs. 17 and 18.

Fig. 15 Output performance in combination with hydrostatic transmission and engine

Fig. 16 Output impedance in combination with hydrostatic transmission and engine

Table 3. Values used in dynamic performance calculation

J_e	kgcms/rps	16.1
J_L	kgcms/rps	371
i		2.8
T_e	s	0.09
D_e	kgcm/rps	0
D_L	kgcm/rps	0.0036
$(1+\varepsilon)$		1.1
ζ	kgcm/rps	11
$Z_e = D_e + K_e K_s K_N$	kgcm/rps	480
K_g	1/rad	2.36
T_g	s	0.15
$K_e K_s K_g$	kgcm/rad	23846
		($N_e = 1600$ rpm)
		19060 ($N_e = 2200$ rpm)
		2200 rpm)

As seen from these results, there is a one-to-one response between engine torque T_e and hydrostatic transmission output torque T_o with almost no time lag, testifying to the high degree of stiffness of the hydrostatic transmission. The figure shows the response calculated by applying inverse Laplace transformation to Eq. (16). The transient response of output revolution speed is faster in the calculated value. In the case of time on the order of 0.1 second, it is necessary to apply a correction of about 0.1 second dead time $e^{-0.1s}$ to the calculation equations. Since this paper is dealing not with a hydrostatic transmission as a servo system but with such a hydrostatic transmission as a means of transmitting power in combination with a diesel engine, an approximation of this scale presents no problems.

(2) Indicial response of revolution speed N_L for input of swash plate lever angle θ

Figs. 19 and 20 show the results of the tests. Just as in the case of response for the governor input, these results show that the impedance is great dynamically as well. These figures also display the values calculated from Eq. (16).

(3) Response of revolution speed N_L for load torque T_L with first order lag

As part of the performance of the dynamometer, it is impossible to generate step load torque T_L even when input of step signal is applied to the dynamometer controller. Since this performance of the dynamometer can be approximated in the form of a first order lag, the load torque when input of step signal $T_{in/s}$ is applied to the dynamometer controller is represented by $T_L = T_{in}/(Ts + 1)s$. The response when this T_L is in operation was sought from Eq. (16). Figs. 21 and 22 present the observed results and the calculated results. Since fluctuations are gentle, the results are close to those of the static performance and exhibit a good match between observed values and calculated values.

5. Control of Fuel Consumption

In the preceding sections, we have studied and tested the performance of the hydrostatic transmission and the performance of the power train system when this transmission is combined with a diesel engine. The power train system including the diesel engine is a dual-input system consisting of the input of the governor r and the lever angle θ of the swash plate, and its re-

Fig. 17 Indicial response for governor input ($N_e=2200$ rpm, $\alpha_{10}=0.51$)

Fig. 18 Indicial response for governor input ($N_e=2200$ rpm, $\alpha_{10}=0.96$)

Fig. 19 Indicial response for swash plate angle input ($N_e=1600$ rpm, $\alpha_{10}=0.51$)

lationship to output in the vicinity of the operating points is represented by Eq. (16). When this governor input r and the lever angle θ of the swash plate are related based on a given control target, it is possible to control not only the output but also the operating conditions within the power train system.

In the past, a variety of tests have been conducted in combination with an internal combustion engine⁽¹⁾ and on efficiency control of electric motor⁽²⁾, but in this paper we are studying overall efficiency control at steady state in a power train system combining a diesel engine and a hydrostatic transmission. Efficiency is represented by the fuel consumption per horsepower of output from the hydrostatic transmission, and below, this will be referred to as "overall specific fuel consumption."

5.1 Overall specific fuel consumption of power train system including diesel engine

Figs. 24 and 25 present test results for lines of constant overall specific fuel consumption. The diesel engine used in the tests and the hydrostatic transmission are those used in the preceding sections, and output of the engine has been left uncorrected.

Judging from Figs. 6, 24, and 25 and the engine's specific fuel consumption performance, the best approach for improving the overall specific fuel consumption of the system would seem to be holding down the engine revolution speed and holding the swash plate to the widest possible angle during use.

5.2 Test using overall specific fuel consumption controller

From the results of the preceding section, in order to improve the overall specific fuel consumption of the power train system, thought must be given to the relationship between r and θ for the necessary load conditions, as in the following explanation. (1) Make the displacement ratio α_1 as great as possible. (2) Run the engine at as low a revolution speed as possible. (3) Maximum value of displacement ratio α_1 is 1. (Motor has fixed displacement.) (4) There is a saturation limit value for the engine's output torque.

As one example of a controller that can satisfy the above, we conducted tests on a controller having a division mechanism, a multiplication mechanism, and an input torque inspection mechanism. When a given output revolution speed is set into this controller, within a narrow range of load torques, r and θ are fixed at values that can satisfy conditions (1) and (2) above. When the load torque increases and the input torque to the hydrostatic transmission rises and reaches a predetermined value (engine maximum saturation torque), r automatically increases and θ conversely falls; the controller is thus designed to continue to fulfill the above conditions as well as to keep the output revolution speed at a fixed level as far as possible.

Figs. 26 and 27 present examples of test results using this overall specific

Fig. 20 Indicial response for swash plate angle input ($N_e=1600$ rpm, $\alpha_{10}=0.96$)

Fig. 21 Response for load torque input ($N_e=1600$ rpm, $\alpha_{10}=0.51$)

Fig. 22 Response for load torque input ($N_e=1600$ rpm, $\alpha_{10}=0.96$)

Fig. 23 Control of power train system

fuel consumption controller. To make a comparison of overall specific fuel consumption when overall specific fuel consumption control is not applied, we measured overall specific fuel consumption by fixing governor input r for conditions of increasing engine revolution speed (Fig. 26. point a) and controlling lever angle θ of the swash plate such that output is the same as when overall specific fuel con-

sumption is being controlled. Comparing the two makes it clear that overall specific fuel consumption is better when overall specific fuel consumption control is conducted.

6. Conclusions

(1) Compared to a torque converter, a hydrostatic transmission has higher impedance (larger stiffness), and in this respect is close to a gear transmission in

Fig. 24 Test result of overall specific fuel consumption

Fig. 25 Test result of overall specific fuel consumption

Fig. 26 Test result of controller of overall specific fuel consumption

Fig. 27 Test result of controller of overall specific fuel consumption

performance. This is a major feature of the hydrostatic transmission.

(2) When the hydrostatic transmission is combined with a prime mover, it is possible to vary the displacement ratio continuously as a system parameter, and the degree of freedom of power control increases. In this respect, it is easier to control not only the output but also the operational state of the power train system including the prime mover than in the case of the torque converter or gear transmission.

(3) Using the characteristic equations of this paper, it is possible to make an estimate of its overall performance as a power train system.

(4) The hydrostatic transmission efficiency is not as high as in the case of a gear transmission, but it is not inferior to a torque converter with a power shift gear transmission (Three elements, one stage, one phase). The overall efficiency of the hydrostatic transmission as a power train system in combination with a prime mover can be improved when the prime mover control input and the swash plate angle are controlled with a suitable controller.

In conclusion, we wish to express our deep appreciation to Prof. Nakagawa of Osaka University for his valuable guidance and encouragement in this research.

References

- (1) Ishihara, T., et al. Preprint of Soc. Automotive Engrs. Japan (in Japanese), (1966-5), p1
 - (2) Thoma, J., Hydrostatische Getriebe, (1964), p128, Carl Hanser
 - (3) Ito, Y. and Sarumaru, T., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No.734-5, (1973-3), p18
 - (4) Ito, Y. and Sarumaru, T., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No.744-5, (1974-3), p92
 - (5) Takahashi, T., System to Seigyo (in Japanese), (1968), p117, Iwanami
 - (6) Ishihara, T., et al. Yuatu Kogaku (in Japanese), (1968), p179, Asakura
 - (7) Blackburn, J., et al. Fluid power Control, (1960), p133, John Wiley
 - (8) Machiyama, T., et al. Bull. of the Science and Engineering Research Laboratory, Waseda Univ. (in Japanese), Vol.37 (1966), p16
 - (9) Wilson, W., et al. Hydraulics and Pneumatics, Vol.23, No.2(1970-2), p60
 - (10) Takenaka, T. and Urata, E., Yuatu Seigyo (in Japanese), (1967), p129, Maruzen
 - (11) Takahashi, T., Jour, Japan Soc. Mech. Engrs. (in Japanese), Vol.62, No.483 (1959-4), p565; Vol.62, No.484(1959-5), p755
 - (12) Ikebe, Y. and Elaraby, M., Jour. of the Soc. of Instrument and Control Engrs. (in Japanese), Vol.8, No.3 (1969-3), p159
-