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A Study on Non-Steady Combustion*

(2nd Report, On Pressure Mode Fluctuation)

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This paper deals theoretically with the converting processes of energy from heat release into pressure mode generation. The converting rate is calculated about several representative types of heat release distribution. The main obtained results are as follows:

(1) When heat is given to a gaseous medium, $1/\kappa$ parts of this energy are transformed into the entropy mode energy, and the remaining $(\kappa-1)/\kappa$ parts are transformed into the pressure mode energy. The latter's ratio increases for the case that the gas is flowing.

(2) When the axial heat release distributing form is a unit rectangle pulse type, the converting rate is large in low frequency regions and diminishes variously in higher frequency regions. On the other hand, for sinusoidal and cosinusoidal types, the converting rate curves have different patterns. The one is similar to the unit rectangle pulse type, the other takes a curve such that the rate has low value in low frequency regions and high value in middle- or high- frequency regions.

(3) From the obtained results, it was inferred that the oscillations of middle- and high-frequencies in combustion instability depend upon the length of heat release region and the magnitude of heat release oscillations and upon their distributing states.

1. Introduction

Two types of propagating wave exist in a gaseous medium: one is an entropy mode fluctuation propagating with a flow velocity; and the other is a pressure mode fluctuation which travels with a sound speed relative to the flow velocity. With respect to the entropy mode fluctuations such as concentration, heat release, and temperature waves, a preceding paper deals with them in a one-dimensional type.^[1] As a continued report, this one intends to formulate the physical mechanism on the occurrence of pressure mode. The relationship between the strength of heat source and that of pressure wave generated is determined for the case that a plane heat source of finite strength is inserted into a flowing gas in a pipe of infinite length. At the same time, linear approximate solutions, which present the pressure wave patterns caused by several fluctuating sources different in space extension and in its distributing form, are derived. Furthermore, by the use of the obtained solutions, frequently characteristics are studied for the generated pressure waves.

These analyses are applied basically for the idealized and fundamental problems,

but they provide a general method for obtaining a more precise expression with respect to the propriety condition in a flame than the expressions such as Takeno,^[2]^[3] and Merk^[4] have already presented in the analysis of heat driven combustion oscillation. The analyses of the spatial distribution of heat release profile^[5] and of the distribution pattern are carried out from a microscopic view point in order to obtain a more detailed result with respect to the effect of spatial distribution of heat release fluctuation on the acoustic resonant oscillation. These analyses can only present a qualitative explanation of oscillation mechanism; however, they present the materials for quantitatively studying the oscillatory combustion in combination with the mass addition supplied oscillatively, the flame fluctuation and the acoustic boundary conditions of pipe system.

2. Notations

A : cross-sectional area of combustion chamber
 c : speed of sound
 C_h : converting rate from heat release into pressure wave generation
 c_p, c_v : specific heats at constant pressure and constant volume respectively
 D : equivalent diameter
 E : energy
 f : heat release rate per unit length and per unit time
 g : gravitational acceleration
 h_f : calorific value of fuel
 J : Joule's equivalent of heat

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K : over-all coefficient of heat trans-
mission
 k : wave number
 l : peripheral length of chamber
 p : pressure
 Q : heat flux
 R : gas constant
 S_L : absolute speed of heat source
 s : length of heat source region
 T : absolute temperature
 t : time
 u : internal energy
 v : flow velocity
 x : axial distance
 α : characteristic velocity toward down-
stream = $v + c$
 α : characteristic velocity toward upst-
ream = $v - c$
 Δ : minute amount of fluctuation
 γ : specific weight
 θ : temperature rise = $T - T_0$
 κ : specific heat ratio = c_p/c_v
 λ : heat conductivity
 ν : frequency
 ω : angular frequency

Subscripts

e : for entropy mode fluctuation
 p : for pressure mode fluctuation
 0 : for ambient state
 $+,-$: for waves propagating toward down-
stream and upstream respectively
 1 : for kinetic energy component of pres-
sure mode
 2 : for internal energy component of pres-
sure mode

Superscripts

$-$: for time-mean values
 $\sim$: for fluctuating components

Notations of derivative

$D/D_t = \partial/\partial t + v\partial/\partial x$
 $\Delta_+ = \partial/\partial t + \alpha_+\partial/\partial x$
 $\Delta_- = \partial/\partial t + \alpha_-\partial/\partial x$

3. Theoretical explanations of fluctuating mode

3.1 Fundamental equations for fluctuating mode

In what follows, the fluctuating modes in a one-dimensional flow are examined. The definitions of modes and the assumptions for analysis are identical with the explanations that were presented in the previous paper, [1] but, in this paper the following assumptions are added:

(1) As related to the definition of pressure mode, the damping of pressure wave by heat conduction can be ignored owing to the property of the high speed propagation of this mode and to the smallness of the temperature change by this mode.

(2) The strengths of pressure waves which travel in both directions on the negative and positive sides and which are generated by a surface heat source at the position where temperature and pressure have their axial gradient of magnitude are equal to each other.

Equations for conservations of mass, momentum, energy, and for perfect gas law

are expressed respectively as :

Mass conservation :

$$\frac{\partial \gamma}{\partial t} + \frac{\partial}{\partial x}(\gamma v) = 0 \quad (1)$$

Momentum conservation :

$$\gamma \frac{Dv}{Dt} + g \frac{\partial p}{\partial x} = 0 \quad (2)$$

Energy conservation :

$$\gamma \frac{D}{Dt} \left(u + \frac{1}{2} \frac{v^2}{g} \right) + \frac{1}{J} \frac{\partial}{\partial x}(\gamma v) = Q \quad (3)$$

Perfect gas equation of state :

$$p = \gamma RT \quad (4)$$

The characteristic directions which simultaneously satisfy these equations become v , $v+c$, and $v-c$, [8] [9] and thus, these symbols respectively indicates the speed of entropy mode fluctuation, the velocities of pressure modes directed downstream and upstream. In order to obtain the change of each mode, we may as well examine the change along those characteristic directions:

(1) The change along the characteristic direction of entropy mode v is attributed to the original energy equation; namely,

$$\gamma c_p \frac{DT}{Dt} = \frac{1}{J} \frac{Dp}{Dt} + Q \quad (5)$$

This equation stands for the fundamental equation for the entropy mode. Here, Q denotes a heat flux such as heat conduction or heat loss;

$$Q = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) - \frac{Kl}{A} (T - T_0) + \frac{f}{A} \quad (6)$$

Furthermore, from the definition the pure temperature fluctuation should be the one obtained with removal of the temperature fluctuation generated by the pressure waves,

$$\frac{Dp}{Dt} = 0 \quad (7)$$

Under this condition, the relation between the heat release rate and the temperature becomes

$$\gamma c_p \frac{DT}{Dt} = Q \quad (8)$$

(2) For the change along the characteristic direction of pressure mode, the equation for the velocity $v+c$ propagating downstream becomes

$$\Delta_+ p + \frac{\gamma c}{g} \Delta_+ v = J(\kappa - 1)Q \quad (9)$$

and the equation for the velocity $v-c$ propagating upstream becomes

$$\Delta_- p - \frac{\gamma c}{g} \Delta_- v = J(\kappa - 1)Q \quad (10)$$

Moreover, from the assumption (1), it may as well be allowed to treat only the heat release term with respect to Q in the above two equations, hence,

$$Q = \frac{f}{A} \quad (11)$$

3.2 The relation between the entropy mode and the pressure mode

In what follows, the relation between the pressure and temperature waves will be examined. As illustrated in Fig.1, they may be considered as being generated by a heat source which travels at the velocity

S_L relative to the fixed point and releases the heat per unit time $\Delta E/\Delta t$ on the x - t plane, in which gas is flowing at the velocity of v . This figure illustrates the state that no waves exist in the x - t plane for $t < 0$ and heat is released within the period of $\Delta_1 t$ from the traveling heat source, which is generated at $t=0, x=0$ and travels at the speed of S_L . The pressure, flow velocity and temperature waves, which are observed after the elapse of time Δt , should be in the hatched state in the figure. Here, it is assumed that the heat release rate is constant throughout the time of elapse of $\Delta_1 t$. Δp_+ and Δp_- illustrated in the hatched parts mean the pressure waves which respectively travel in positive and negative directions. Also, Δv_+ and Δv_- illustrated in the hatched parts indicate similar velocity waves. Temperature changes generated by these pressure waves are ΔT_{p+} and ΔT_{p-} . ΔT_e indicated by hatched part is the temperature wave of entropy mode, which remains in the neighborhood of the heat source.

Pressure mode energies are divided into a kinetic and an internal energy. If we consider that the heat release continues within the time of Δt , the region where fluctuation of energy is generated is limited to the range of $(c+v)\Delta t \geq x \geq (-c+v)\Delta t$. Therefore, the kinetic energy of fluctuation which the heat source brings about in the outside region as a pressure wave becomes;

for the downstream side of heat source:

$$\left(\frac{\Delta E_{p1+}}{\Delta t}\right) \Delta t = \frac{A}{J} (c+v-S_L-\Delta v_+) (\gamma + \Delta \gamma_+) \times \frac{(v+\Delta v_+)^2 \Delta t}{2g} - \frac{A}{J} (c+v-S_L) \gamma \frac{v^2}{2g} \Delta t. \quad (12)$$

and, for the upstream side of heat source:

Fig.1 Occurrence state of pressure, flow velocity, and temperature waves

$$\left(\frac{\Delta E_{p1-}}{\Delta t}\right) \Delta t = \frac{A}{J} (c-v+S_L+\Delta v_-) (\gamma + \Delta \gamma_-) \times \frac{(v+\Delta v_-)^2 \Delta t}{2g} - \frac{A}{J} (c-v+S_L) \gamma \frac{v^2}{2g} \Delta t. \quad (13)$$

Total kinetic energy given as a pressure wave is equal to the summation of ΔE_{p1+} and ΔE_{p1-} .

In addition, the internal energy of pressure mode exerted upon the outer region of the heat source is estimated at

$$\left(\frac{\Delta E_{p2+}}{\Delta t}\right) \Delta t = (c+v-S_L-\Delta v_+) \Delta t (\gamma + \Delta \gamma_+) \times A c_v (T + \Delta T_+) - (c+v-S_L) \Delta t \gamma A c_v T, \quad (14)$$

for the downstream side;

$$\left(\frac{\Delta E_{p2-}}{\Delta t}\right) \Delta t = (c-v+S_L+\Delta v_-) \Delta t (\gamma + \Delta \gamma_-) \times A c_v (T + \Delta T_-) - (c-v+S_L) \Delta t \gamma A c_v T, \quad (15)$$

for the upstream side. Total internal energy ΔE_{p2} given as a pressure mode is obtained by taking the summation of ΔE_{p2+} and ΔE_{p2-} .

Introducing the following equations for the pressure wave into the above mentioned equations;

$$\frac{\Delta p_+}{p} = \kappa \frac{\Delta \gamma_+}{\gamma} = \frac{\kappa}{\kappa-1} \frac{\Delta T_+}{T} = \frac{\kappa}{c} \Delta v_+, \quad (16)$$

$$\Delta p = \Delta p_+ = \Delta p_-, \quad \Delta v = \Delta v_+ = -\Delta v_-, \quad (17)$$

$$\Delta \gamma_+ = \Delta \gamma_+ = \Delta \gamma_-, \quad \Delta T_+ = \Delta T_+ = \Delta T_-, \quad (18)$$

and neglecting the squared-order terms in the case that the fluctuating term is small, one obtains

$$\left(\frac{\Delta E_{p1}}{\Delta t}\right) \Delta t = 4 \frac{A}{J} \gamma v \frac{(v-S_L)}{2g} \Delta v \Delta t = 2 \frac{A}{J} \frac{v}{c} (v-S_L) \Delta p \Delta t, \quad (19)$$

$$\left(\frac{\Delta E_{p2}}{\Delta t}\right) \Delta t = 2(\kappa-1) \gamma A c_v T \Delta v \Delta t = 2 \frac{A}{J} \frac{c}{\kappa} \Delta p \Delta t. \quad (20)$$

Therefore, with respect to the case that $S_L = v$ or $v=0$, the kinetic energy term of which the pressure wave is retained as a whole disappears.

In addition, the energy of pressure wave propagating downstream becomes

$$\left(\frac{\Delta E_{p+}}{\Delta t}\right) \Delta t = \frac{A}{J} c \left[\frac{1}{\kappa-1} \left(\frac{c+v-S_L}{c} - \frac{1}{\kappa} \right) + \frac{v(v-S_L)}{c^2} \left(\frac{v}{2c} + 1 \right) \right] \Delta p \Delta t, \quad (21)$$

and the energy of pressure wave propagating upstream becomes

$$\left(\frac{\Delta E_{p-}}{\Delta t}\right) \Delta t = \frac{A}{J} c \left[\frac{1}{\kappa-1} \left(\frac{c-v+S_L}{c} - \frac{1}{\kappa} \right) + \frac{v(v-S_L)}{c^2} \left(-\frac{v}{2c} + 1 \right) \right] \Delta p \Delta t. \quad (22)$$

Consequently, if $v > S_L$, the pressure propagating downstream becomes greater than that propagating upstream, and this relation gives the result opposite to the case of $v > S_L$.

The energy which all the pressure mode

fluctuations possess amounts to

$$\left(\frac{\Delta E_p}{\Delta t}\right) \Delta t = \left(\frac{\Delta E_{p+}}{\Delta t}\right) \Delta t + \left(\frac{\Delta E_{p-}}{\Delta t}\right) \Delta t \\ = 2 \frac{A}{f} c \left(\frac{1}{\kappa} + \frac{v(v-S_L)}{c^2} \right) \Delta p \Delta t; \dots (23)$$

Meanwhile the total energy ΔE remains constant in spite of the movement velocity of the fluid. If this energy is imparted homogeneously and statically to the pipe closed at both ends whose length is equivalent to the distance from $x=(v-c)\Delta t$ to $(v+c)\Delta t$, in this case the relation of isochore change is applied, and the pressure rise at this time becomes Δp too. At the same time, the homogeneous temperature rise ΔT is expressed as

$$\Delta T = \Delta T_+ + \Delta T_- = T \frac{\Delta p}{p}; \dots (24)$$

and hence the relation between ΔE and Δp becomes

$$\left(\frac{\Delta E}{\Delta t}\right) \Delta t = 2cA\gamma c_v T \frac{\Delta p}{p} \Delta t \\ = 2 \frac{Ac}{f} \frac{1}{\kappa-1} \Delta p \Delta t; \dots (25)$$

From the consideration that the entropy mode energy ΔE_e retains the value of the difference from ΔE to ΔE_p , the following equation can be obtained

$$\frac{\Delta E_p}{\Delta E_e} = \frac{\Delta E_p}{\Delta E - \Delta E_p} \\ = \frac{(\kappa-1) \left\{ \frac{1}{\kappa} + \frac{v(v-S_L)}{c^2} \right\}}{1 - (\kappa-1) \left\{ \frac{1}{\kappa} + \frac{v(v-S_L)}{c^2} \right\}}; \dots (26)$$

If $v=0$ or $v=S_L$, the ratio of ΔE_p to ΔE_e becomes

$$\frac{\Delta E_p}{\Delta E_e} = \left(\frac{\kappa-1}{\kappa} \right) / \left(\frac{1}{\kappa} \right); \dots (27)$$

This means that if energy is added, the $1/\kappa$ parts of the energy are stored as an entropy mode energy in the neighborhood region of the heat source and that the remainder parts of $(\kappa-1)/\kappa$ are propagated beyond this region as a pressure mode energy. Furthermore, it can be understood from Eq.(26) that the pressure mode energy increases with an increase in Mach number and with a decrease in the speed of heat source, and this energy increases in inverse proportion to the decrement of the heat source velocity compared to the flow velocity; and as a result, that the ratio of $\Delta E_p/\Delta E_e$ becomes greater than the value estimated from Eq.(27).

From this result it can be understood that the pressure fluctuation becomes higher for a high flow velocity combustor.

From the above equations, the relations between the heat release energy and the pressure and flow velocity fluctuations become:

$$\Delta p = \frac{(\kappa-1)J}{2cA} \frac{\Delta E/\Delta t}{1 + \kappa \{v(v-S_L)\}/c^2}, \left. \begin{array}{l} \Delta v = \pm \frac{(\kappa-1)J}{2\kappa A p} \frac{\Delta E/\Delta t}{1 + \kappa \{v(v-S_L)\}/c^2} \end{array} \right\} \dots (28)$$

For a pipe system such that the pressure wave never comes back after its liberation from the heat source, the tran-

sient change of the heat release energy per unit time $\Delta E/\Delta t$ can be estimated by the measurement of the transient change of Δp outside the heat source region.

3.3 Solutions for pressure fluctuations generated by the heat release fluctuation which varies in space and time

Taking the summation of the equation obtained by multiplying both sides of Eq. (9) by Δ_- and the one obtained by multiplying both sides of Eq.(10) by Δ_+ , and neglecting the second order small terms, the following equation of wave motion for pressure wave can be obtained.

$$L(\bar{p}) + \frac{1}{2} \frac{\partial}{\partial x} (\bar{v}^2 - \bar{c}^2) \frac{\partial \bar{p}}{\partial x} \\ + \frac{\bar{\gamma} \bar{c}}{g} \left(-\bar{c} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{c}}{\partial x} \right) \frac{\partial \bar{v}}{\partial x} - \frac{\bar{c}}{g} \frac{\partial (\bar{\gamma} \bar{c})}{\partial x} \frac{\partial \bar{v}}{\partial t} \\ = (\kappa-1) J \frac{D\bar{Q}}{Dt}; \dots (29)$$

With respect to the flow velocity fluctuation, by subtracting side by side the equation of Eq.(9) multiplied by Δ_- from that of Eq.(10) multiplied by Δ_+ ,

$$L(\bar{v}) + \frac{1}{2} \frac{\partial}{\partial x} (\bar{v}^2 - \bar{c}^2) \frac{\partial \bar{v}}{\partial x} + \frac{\bar{v}}{\bar{\gamma} \bar{c}} \frac{\partial (\bar{\gamma} \bar{c})}{\partial x} \frac{\partial \bar{v}}{\partial t} \\ + \frac{(\bar{v}^2 - \bar{c}^2)}{\bar{\gamma} \bar{c}} \frac{\partial (\bar{\gamma} \bar{c})}{\partial x} \frac{\partial \bar{v}}{\partial x} + \frac{g}{\bar{\gamma} \bar{c}} \left(\bar{v} \frac{\partial \bar{c}}{\partial x} \right. \\ \left. - \bar{c} \frac{\partial \bar{v}}{\partial x} \right) \frac{\partial \bar{p}}{\partial x} = -(\kappa-1) J \frac{g}{\gamma} \frac{\partial \bar{Q}}{\partial x}; \dots (30)$$

where, the notation L means:

$$L = \frac{\partial^2}{\partial t^2} + 2\bar{v} \frac{\partial^2}{\partial x \partial t} + (\bar{v}^2 - \bar{c}^2) \frac{\partial^2}{\partial x^2}.$$

The pressure and velocity changes are

$$\left. \begin{array}{l} \frac{d\bar{p}}{dx} = J(\kappa-1) \frac{\bar{v}}{\bar{v}^2 - \bar{c}^2} \bar{Q}, \\ \frac{d\bar{v}}{dx} = -J(\kappa-1) \frac{g\bar{Q}}{\gamma(\bar{v}^2 - \bar{c}^2)}. \end{array} \right\} \dots (31)$$

As a result, pressure head by heat release becomes extremely high when the Mach number in the chamber reaches close to unity. Before solving the above equations for fluctuations, the order of magnitude of each term should be examined in detail. The first term in the left hand side of Eq.(29), that is designated as $L(p)$, has the same order as a squared angular frequency ω^2 , and the remainder has the same order as the product of ω and the longitudinal gradient of sound speed $\partial \bar{c}/\partial x$ or of flow velocity $\partial \bar{v}/\partial x$. Therefore, if the case is excluded of which the steady heat release is quite intensive and the frequency of fluctuation is low, the first term becomes dominant over the other terms. In cases in which the second and the succeeding terms become dominant, attention may as well be given to the changes which occur in the over all system instead of just solving the equation for the propagation of pressure wave, because of its long wave length.

In cases in which the first term is dominant, let the equations of (29), (30) be solved for the following initial condition:

$$\bar{p} = \bar{v} = 0 \quad \text{at} \quad t = 0; \dots (32)$$

On the assumption that $\bar{c}$, $\bar{v}$, and other terms are the values averaged both in time and in space, the following solutions can be drawn for the conditions; no boundary of heat release, no boundary reflecting the pressure wave:

$$\bar{p}(x, t) = \frac{(\kappa-1)J}{2\bar{c}} \left[\int_{x-a, t}^x \int_0^{t-(x-z)/a} \frac{D\bar{Q}(\xi, \tau)}{Dt} d\tau d\xi + \int_x^{x-a, t} \int_0^{t+(x-z)/a} \frac{D\bar{Q}(\xi, \tau)}{Dt} d\tau d\xi \right], \quad (33)$$

$$\bar{v}(x, t) = \frac{(\kappa-1)J}{2\kappa\bar{p}} \left[\int_{x-a, t}^x \int_0^{t-(x-z)/a} \frac{D\bar{Q}(\xi, \tau)}{Dt} d\tau d\xi - \int_x^{x-a, t} \int_0^{t+(x-z)/a} \frac{D\bar{Q}(\xi, \tau)}{Dt} d\tau d\xi \right]. \quad (34)$$

These equations are also obtained from Eq.(28) for finite small wave. Here, the term $D\bar{Q}(\xi, \tau)/Dt$ is calculated by differentiating $\bar{Q}(x, t)$ by D/Dt , and thereafter, by replacing the term of x, t in the obtained derivative by ξ, τ , respectively. Of all terms, the first one represents the wave motion measured at the point (x, t) to which the wave motion generated by the heat release fluctuation source located upstream from the measuring point propagates. The second term is the wave motion generated by the downstream source of heat release fluctuation.

Moreover, the solutions for the conditions such as: (1) the initial condition is accompanied by arbitrary fluctuations, (2) the boundary conditions are added, or (3) the heat release region is defined, can be determined respectively by adding those waves generated by (1) initial condition, (2) boundary condition, or (3) by altering integral interval.

3.4 Examples for pressure wave generation

As an example of the heat release profile several representatives will be illustrated and the numerical equation for the converting rate from the heat release into the pressure fluctuation will be introduced.

(1) With respect to the case in which the heat release profile is a rectangular

The pressure wave at $x=s$ is expressed as: when $k_i \neq \omega/\alpha_+$,

$$\bar{p}_{x=s} = \frac{(\kappa-1)Jf_m}{4\bar{c}A\omega} \left\{ \omega \left[\frac{1}{k_i - \omega/\alpha_+} + \frac{1}{k_i + \omega/\alpha_+} \right] - k_i \bar{v} \left[\frac{1}{k_i - \omega/\alpha_+} - \frac{1}{k_i + \omega/\alpha_+} \right] \right\} \times \sqrt{\left[(-1)^i - \cos \omega \frac{s}{\alpha_+} \right]^2 + \left[\sin \omega \frac{s}{\alpha_+} \right]^2} \sin \left(\omega t - \tan^{-1} \frac{\sin \omega(s/\alpha_+)}{\cos \omega(s/\alpha_+) - (-1)^i} \right), \quad (44)$$

and when $k_i = \omega/\alpha_+$, $\bar{p}_{x=s} = \frac{(\kappa-1)Jf_m}{4\bar{c}A} s \left(1 - \frac{\bar{v}}{\alpha_+} \right) (-1)^i \cos \omega t$.

If we express the maximum amplitude at low flow rate in a type of Eq.(36), the converting rate C_h becomes:

$$\text{when } k_i \neq \omega/\alpha_+, \quad (C_h)_{\bar{v}=0} = \frac{(\kappa-1)J\pi}{2\sqrt{2}\bar{c}} \frac{k_i}{|(\omega/\bar{c})^2 - k_i^2|} \sqrt{1 + (-1)^{i+1} \cos(\omega s/\bar{c})},$$

$$\text{and when } k_i = \omega/\alpha_+, \quad (C_h)_{\bar{v}=0} = \frac{(\kappa-1)J\pi s}{8\bar{c}} \quad (45)$$

pulse type and its amplitude changes in a sinusoidal form.

The heat release profile is expressed in the following type of equation:

$$f = f_m \sin \omega t \quad \text{for } 0 \leq x \leq s, \\ f = 0 \quad \text{for } x < 0, x > s, \quad (35)$$

Let $|\bar{F}|$ be the heat release amplitude averaged spatially over the length of the heat release, then,

$$|\bar{F}| = \int_0^s f_m dx/s = f_m \quad (36)$$

The pressure wave form $\bar{p}$, if it is observed at the end of this region $x=s$, becomes:

$$\bar{p}_{x=s} = \frac{(\kappa-1)Jf_m}{2\bar{c}A} \int_0^s \int_0^{t-(x-z)/a} \omega \cos \omega \tau d\tau d\xi \\ = \frac{(\kappa-1)Jf_m}{\bar{c}A} \frac{\sin(\omega s/2\alpha_+)}{\omega(1/\alpha_+)} \sin \omega \left(t - \frac{s}{2\alpha_+} \right). \quad (37)$$

Let $|\bar{p}_s|$ be the notation of the amplitude of pressure wave

$$|\bar{p}_s| = C_h \frac{|\bar{F}|}{A} \quad (38)$$

Here the notation C_h was designated as the converting rate from heat release fluctuation into pressure wave, therefore, in this case;

$$C_h = \frac{(\kappa-1)J}{\bar{c}} \frac{|\sin(\omega s/2\alpha_+)|}{(\omega/\alpha_+)}. \quad (39)$$

The expression $|\bar{F}|/A$ represents the amplitude of combustion strength, and has the dimension of [calorific value/(length)³.time]. For the low frequency range of fluctuation or for the short length of the heat release region, the expression of C_h is simplified as:

$$C_h = \frac{(\kappa-1)Js}{2\bar{c}}, \quad (40)$$

therefore, the equations (36) and (38) coincide with Eq.(27).

(2) With respect to the case in which the heat release profile has sinusoidal form and its amplitude changes in a sinusoidal form.

The heat release profile is expressed

$$\text{as: } f = f_m \sin \omega t \sin k_i x \quad \text{for } 0 \leq x \leq s, \\ f = 0 \quad \text{for } x < 0, x > s, \quad (41)$$

where, $k_i = i\pi/s$ ($i=1, 2, 3, \dots$) (42)

For this heat release profile, the heat release amplitude averaged spatially $|\bar{F}|$ becomes:

$$|\bar{F}| = f_m \frac{i}{s} \int_0^s |\sin k_i x| dx = \frac{2f_m}{\pi} \quad (43)$$

Also, in the case of $k_i = i\pi/2s$ ($i=1,3,5,\dots$),(46)
similar analysis gives;

when $k_i \neq \omega/\alpha_+$,

$$(C_h)_{\bar{v}=0} = \frac{\kappa-1}{4\sqrt{2}} \frac{J\pi}{\bar{c}} \times \sqrt{\frac{1}{(\omega/\bar{c}-k_i)^2} + \frac{1}{(\omega/\bar{c}+k_i)^2} + (-1)^{(i+1)/2} \left\{ \frac{1}{(\omega/\bar{c}-k_i)^2} - \frac{1}{(\omega/\bar{c}+k_i)^2} \right\} \sin(\omega s/\bar{c})}, \dots(47)$$

and when $k_i = \omega/\alpha_+$,

$$(C_h)_{\bar{v}=0} = \frac{(\kappa-1)Js}{4i\bar{c}} \sqrt{1 + \left(\frac{i\pi}{2}\right)^2}$$

In the equations (44), (45) and (46), the particular wave number for $k_i = \omega/\alpha_+$ was treated as a singular point. However, the value at this point lies on the curve which connects smoothly the two curves for $k_i = \omega/\alpha_+$, which lie on both sides of that particular wave number. Therefore, in the following analysis, this wave number k_i will not be treated as a particular point.

(3) With respect to the case in which the heat release profile has a cosine wave form and its amplitude changes sinusoidally.

The heat release profile is expressed as :

$$f = f_m \sin \omega t \cos k_i x, \quad \text{for } 0 \leq x \leq s, \quad \text{and} \quad f = 0 \quad \text{for } x < 0, x > s. \dots(48)$$

For the wave number of $k_i = i\pi/s$, ($i=1,2,3,\dots$)(49)
the converting rate C_h becomes:

$$(C_h)_{\bar{v}=0} = \frac{(\kappa-1)J\pi}{2\sqrt{2}} \frac{\omega/\bar{c}}{[(\omega/\bar{c})^2 - k_i^2]} \sqrt{1 + (-1)^{i+1} \cos \frac{\omega s}{\bar{c}}}, \dots(50)$$

Next, for the case of $k_i = \frac{i\pi}{2s}$, ($i=1,3,5,\dots$),(51)

the converting rate becomes;

$$(C_h)_{\bar{v}=0} = \frac{(\kappa-1)J\pi}{4\sqrt{2}} \frac{J\pi}{\bar{c}} \times \sqrt{\frac{1}{(\omega/\bar{c}-k_i)^2} + \frac{1}{(\omega/\bar{c}+k_i)^2} + (-1)^{(i+3)/2} \left\{ \frac{1}{(\omega/\bar{c}-k_i)^2} - \frac{1}{(\omega/\bar{c}+k_i)^2} \right\} \sin(\omega s/\bar{c})} \dots(52)$$

4. Results and considerations

4.1 Frequency characteristics for the converting rate from heat release to pressure wave

Figures 2, 3, 4, and 5 indicate the curves of the converting rate C_h and the rate which represents the indication of the

strength of the pressure propagating from the heat source region toward its outside. In these figures, the mean amplitude of heat release is kept constant for the same length of s even though the heat release forms are different from each other; as a consequence, the high value of C_h means that a strong pressure wave would be generated.

Fig.2 Frequency characteristic of the conversion rate

Fig.4 Frequency characteristic of the conversion rate

Fig.3 Frequency characteristic of the conversion rate

Fig.5 Frequency characteristic of the conversion rate

For the rectangular pulse type of heat release, as shown in Fig.2; the curve C_h remains constant up to the frequency of 100 Hz. Beyond this frequency, the value of C_h decreases rapidly. In the higher frequency range, C_h shows the fluctuating behavior in the narrow frequency region, but its amplitude observed in the figure decreases gradually as the frequency increases.

Figure 3 indicates the C_h curve for the cosine type of heat release profile. This figure is applied for the case of $i=1$ and therefore the half wave length, and three curves in it have different values of s and $\bar{c}$. These curves resemble those for the rectangular pulse type of heat release profile.

Length of heat release region indicated by a solid line in the figure is four times as long as the condition indicated by a broken line. Therefore the former condition has four times as strong heat release fluctuation as the latter condition. In order to evaluate both conditions by the same measure of heat release, comparison should be made between both values in the ratio of the converting rate C_h to the length s , and hence by doing so, the value of C_h/s at low frequency coincides with each other. However the curve of C_h/s takes a profile shifted toward the higher frequency side with a decrease in s . From this result, it is understandable that the pressure fluctuation is liable to be generated up to the higher frequency range and that the degree becomes stronger in inverse proportion to the length of the heat release region.

An instance for a higher sound velocity is illustrated by the dashed line. This condition is equivalent to the state that the temperature in the combustion chamber becomes higher. In this case, the value of C_h is low, compared with the condition for the broken line, and the profile of C_h curve is shifted toward the higher frequency side. Therefore, this fact means that in two cases, each of which has a high and a low temperature respectively but has the same heat release fluctuations, the former gives a lower pressure wave than the latter, but gives a higher pressure frequency.

In connection with the explanation stated in this section, probable pressure wave strength will be calculated. For instance, the value of C_h for the case of $s=2m$, $\bar{c}=340m/s$, and $f=$ about 50 Hz, gives $C_h=0.5$ kgms/kcal. The combustion load in the usual chamber is estimated at about 10^6 kcal/m h, but, it seems that the fluctuation approximately as much as 5 % of this value may constantly be generated even if the case of oscillatory combustion is excluded. The amplitude of pressure wave is estimated as $|\tilde{p}| = 7$ mmAq by the use of Eqs.(38) and (45). This value coincides approximately with the actual value.

The condition of the figure just indicated was applied for the heat release profile at $i=1$ of half wave length, but

for comparison with this figure, the C_h curves for different wave numbers are illustrated in Fig.4. In this case, both of the high and low parts of heat release exist in the same region. In the low frequency region the value of C_h is low, then, this value increases gradually as v increases and it reaches 0.1 at 550 Hz, this C_h value being close to the maximum value in the case of heat release profile of half wave length. Further increase in v causes C_h value to gradually decrease in an oscillatory manner.

The case of $i=3$ gives the heat release profile of 1.5 wave length. In the low frequency region, the C_h value maintains a nearly constant value, but, in the higher frequency range a certain range of frequencies appears at which the C_h value becomes higher than that in the low frequency region, reaching close to 0.1 at about $v = 1000$ Hz. The C_h curve at $i=4$ resembles that of $i=2$.

Figure 5 illustrates the instance of the heat release profile of cosine type of half wave length $i=1$. Just as the instance of sine type profile, also this case shows the C_h curves in which the value of C_h is small at low frequency region and takes an oscillatory behavior in the higher frequency range. If we consider this fact in correspondence with the combustion in the chamber, such a heat release profile seldom takes place in the longitudinal direction, but may occur in the lateral direction. In fact, few report state that the the middle- or high-frequency oscillation takes place as a result of the lateral heat release fluctuation. [6] [7]

4.2 Qualitative consideration

The general solutions of Eqs.(28),(33),(34) mean that the heat release fluctuation causes the pressure wave. At the same time, this relation expresses that if the time necessary for the pressure wave to propagate from the heat release region to the measuring point of pressure and flow velocity is short enough in comparison to a period of oscillation, and if there is no interaction between the pressure wave and the combustion, the pressure at both of the up- and down-stream measuring points and the flow velocity at the down-stream point are in the same phase with the heat release fluctuation, and the down-stream velocity is in the opposite phase with the heat release fluctuation. This relation between the heat release and the pressure wave should be held with no regard to the period from the fuel exhaust to the heat release. This fact has qualitatively been long established, but quantitatively it was seldom confirmed. For the confirmation, the changes of the pressure wave, in this case, the changes of flow velocity that occur at a frequency which deviates from the acoustic natural frequency of the combustion system, especially for the case of changing forcibly the heat release state in a low frequency range will be examined in the following discussion.

The high speed schlieren pictures for the case that the rate of fuel exhaust is altered by forcibly oscillating the pressure in the fuel plenum chamber are shown in the first figure of Fig.6. The second figure of Fig.6 illustrates the pressure wave form in the plenum chamber, the wave form of the surrounding flow velocity (measured at 100mm upstream from the burner port position), and the traces of the fluid (in which solid lines indicate the curves calculated by the equation presented in the previous paper^[1]). The pressure and flow velocity wave forms indicated respectively by different solid lines are continuously recorded on a 16 mm films together with the flame shape, the broken lines indicating the wave forms drawn as smooth curves by eliminating the small fluctuating parts of the solid curves. The component of the fluctuation which corresponds to the natural acoustic resonance frequency (28 Hz), together with a frequency higher than this resonance frequency, is superimposed on each fluctuating wave form. Actually, however, we examine only the wave form of a broken line which coincides with the forced frequency.

The pressure fluctuation in the plenum chamber causes the fuel exhaust rate to vary in the same phase. Consequently, at the position of the schlieren pictures 2, 6, 10, 14 the fuel exhaust velocity is equal to the time-mean value, that is, $v_1 = 0.24$ m/s. The maximum heat release position which corresponds to this exhaust velocity lies in the position of $x = 0.11$ m located downstream from the burner port. The fuel nodule exhausted in an oscillatory motion flows from a position of picture number 2 toward downstream direction along the trace of the solid line. In this case, the dense part of the shadow in the schlieren pictures coincides with the sharp density gradient, that is, approximately with the sharp fuel concentration gradient or of the high heat release rate. Within an initial stage of period from the numbers 4 to 7, fuel nodule exhausted at the number 4 continues to react gradually at the surface in contact with the surrounding air stream; within a period from 9 to 10, it spreads over the whole section of the combustion chamber during its travel toward downstream direction, then it reaches the stage of maximum heat release at the picture of 12. The last position of 12 coincides with the maximum heat release position calculated from the time-mean velocity. The upstream flow velocity becomes minimum at this maximum heat release. Consequently, it is understood that there is a time lag of one cycle period within the time from the maximum fuel exhaust to the maximum heat release, but that there is just a phase difference of 180° between the phases of the maximum heat release and the upstream flow velocity. This experimental fact satisfies the

previously described relations.

5. Conclusions

Conversion process from the heat release mode into the pressure mode was formulated. On the basis of the analytical equations obtained, the frequency characteristics of the conversion rate of this process is also presented for several possible representative patterns of heat release fluctuation.

By the summarization of the obtained results, the following conclusions were drawn:

(1) When a flame propagates toward downstream direction, the energy possessed by the pressure wave which propagates toward upstream becomes larger than that by the pressure wave which propagates toward downstream.

(b) Wave forms and traces of flame nodule

Fig.6 (a) High speed schlieren pictures of fluctuating flame

(2) In such a case that there is no flow or that the flame surface is in an apparent state of rest, if energy is added, the $1/\kappa$ parts of which are stored, as a temperature wave energy of the entropy mode, in the neighborhood of the heat source: and the remainder $(\kappa-1)/\kappa$ parts are propagated at a sound speed as a pressure mode energy, from the heat source toward the outside region.

In addition, the energy ratio provided to the pressure mode becomes greater than the ratio of $(\kappa-1)/\kappa$, and the energy ratio increases with a decreasing velocity of heat source and/or with an increasing Mach number of the flow. Therefore, it can be said that a higher load combustor is liable to directly convert the heat release fluctuation into the pressure wave.

(3) From the observation of the simultaneously-photographed pictures of the fluctuating flame, together with the pressure wave and flow velocity wave forms, it was revealed that the observed increasing state of fuel nodule accompanying heat release fluctuation agreed well with the results calculated by using the equation presented in the previous paper. With respect to the relation between the flow velocity and the heat release fluctuation, as is expected in this paper, it was confirmed that the flow velocity at the upstream of the heat source decreases with an increase in strength of the heat release.

(4) In cases that the heat release profile is the single rectangular pulse type; the frequency characteristics shows such pattern that: 1) the converting rate C_h maintains a high and constant value in the low frequency region; and 2) makes an oscillatory behavior in its amplitude during the change from middle- to high-frequency, but, its extreme value gradually decreases with the frequency rise.

The heat release profiles both of the sine and cosine wave forms, when the wave lengths of which are: $1/2$, $1-1/2$, $2-1/2$, ... in the former cases; and 1 , 2 , 3 , ... in the latter case; show similar curves to those described for the rectangular pulse type. However, the value of C_h is low in the low frequency region both for the cases of sine- and cosine-wave forms, when the wave lengths of which are: 1 , 2 , 3 , ... in the former case; and $1/2$, $1-1/2$,

$2-1/2$, ... in the latter case; becomes greater with an increasing frequency, and after taking the extreme value it decreases in an oscillatory manner.

Furthermore, the value of C_h decreases with an increase in temperature, together with a decrease in the length of the heat release region s ; but its frequency characteristic profile is shifted toward the higher frequency side.

Summing up the conclusions mentioned above, and those of the previous paper which treated the entropy mode, with respect to the occurrence of the combustion instability and its prediction the following conclusions are obtained:

(5) The occurrence of low frequency oscillatory combustion is closely related to the heat release and the temperature fluctuation. The occurrence of middle- or high-frequency oscillations depends on the heat release fluctuation and its distributing profile. Furthermore, it must be noted that the occurrence of the combustion instability can be forecast by the consideration of the boundary condition and of the fuel- and air-exhaust characteristics; and through the examination both of the conversion and combination effects among the modes.

6. References

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