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A Study on Non-Steady Combustion*

(3rd Report, On Dynamic Responses of Pipe Systems
Accompanied with Combustion)By Yutaka TANAKA**, Yuzuru SHIMAMOTO***
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This paper deals with calculating methods of dynamic responses about the pipe system in which combustion takes place. The converting equations from heat release rate into pressure wave generation that were proposed in the previous paper, and usual acoustic theories for normal temperature are used in the theoretical analyses. In order to compare the theoretical results with the experimental ones the following combustor and rotary throttle were produced. In this combustor, one dimensional combustion states can be attained. Rotating throttle supplies combustible mixture in a manner close to a sinusoidal oscillating type.

Obtained results show that the amplitude in the combustion state becomes high in comparison to the one in the non-combustion state. It is recognized that the amplitude gain depends on the magnitude of heat release per unit time and that the main source of this gain is a pressure originating from the volumetric expansion by combustion. It is also studied how the length of combustion region, pipe length and volume exert effects on the dynamic responses.

1. Introduction

With respect to the phenomena of acoustic oscillation in a pipe system accompanied by heat action, analyses have been done mainly on the occurrence of high frequency oscillation. By the use of several idealized heat sources, Merk carried out successively several studies on a high frequency oscillatory combustion^[1] which takes place in a longitudinal direction of axis. He also revealed, by the use of the concept of acoustic impedance and of Putnam's criterion on the stability judgement,^[2] the general mechanism of instability and the phenomena of Rijke tube in which the fluctuation of heat transmission is generated under the co-operative actions, which play the role of a driving source, between gauge and pressure wave.^[3] Furthermore, he expressed a heat-driven oscillation in a form transformed into an electrical circuit model;^[4] then, analyzed the occurrence of instability through the consideration of the co-operative actions between flame surface and supplied gas,^[5] and studied the influence of heat transmission between a burner port and a supplied gas on the high frequency oscillation.^[6]

Takeno, et al. explained a high frequency self-excited oscillation,^[7~9] on the basis of the assumption that the strength of the pressure wave at the exhaust port of mixture gas is proportional to the heat release rate of the surface heat source represented by Dirack's δ function; and by considering the axial position of the heat source and the acoustic boundary conditions of both sides of combustion chamber. If various problems with respect to the co-operative action between the heat source and the pressure wave; for instance, the kind of fuel used, the relation between burner and flame shape, the influence of pressure on the combustion velocity, are excluded from consideration, it may be said that the mechanism of high frequency self-excited oscillation has almost been concluded.

The treatment of the low frequency combustion oscillation is comparatively simple, since the feed-back effect that the pressure wave exerts on the flame is not so intensive as the one for high frequency type. However, on the other hand, heat release and temperature distributions which have not so far been taken into account become an important subject. As a result of the occurrence of the fuel flow rate fluctuation, heat release fluctuation is caused; but, detailed explanation has not yet been done on the development of the pressure wave; that is, "Through what process, does this heat release rate come to be detected as a pressure fluctuation after the propagation to each section?"

In this report, by the use of the formula from the heat release to the occurrence of pressure wave, the details of which are described in the previous paper,^[10] and of the established acoustic

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theory for normal temperature, an analytical method of acoustic response at a comparatively low frequency is introduced for a pipe system accompanied with combustion. Furthermore, this report reveals (a) the path way from the flow rate fluctuation of the combustible mixture supplied in a forced oscillation through the heat release fluctuation to the generation of the flow velocity and pressure fluctuations; and (b) the amplifying state of these fluctuations.

2. Main notations (ones specified in the text are excluded)

A : cross-sectional area
 c : sound speed
 d : inner diameter of pipe
 g : gravity acceleration
 h_f : calorific value of fuel
 $j = \sqrt{-1}$
 J : Joule's equivalent of heat
 $l, L, L_0, L_1, L_2, L_f, x$
 : dimensions of length for longitudinal distance (specified in Figures 1 and 2)
 s : Laplace operator
 t : time
 κ : specific heat ratio
 μ : kinematic viscosity
 ν : frequency
 ξ : combustion efficiency
 ρ : density
 ϕ : weight mixing ratio of air to fuel
 ω : angular frequency = $2\pi\nu$

Superscripts

$\cdot$: amplitude
 $-$: averaged value
 $\sim$: fluctuating component

3. Theoretical analysis

3.1 Fundamental equations for pipe system

The pressure and flow velocity fluctuations at the section m in the pipe system are indicated by the symbols $\tilde{p}_m$ and $\tilde{u}_m$ respectively, and those at the section n which lies at the distance l to the right of the section m along the x -axis are indicated by $\tilde{p}_n, \tilde{u}_n$ respectively. Laplace transforming symbols of $\tilde{p}_m, \tilde{u}_m, \tilde{p}_n, \tilde{u}_n$ are written in capital letters: such as $P_m(s), U_m(s), P_n(s), U_n(s)$ respectively.

From the conservation equations of continuity and momentum for the gases in the pipe, the following transfer matrix is obtained.

$$\begin{Bmatrix} P_m(s) \\ U_m(s) \end{Bmatrix} = [\Gamma_n(s)] \begin{Bmatrix} P_n(s) \\ U_n(s) \end{Bmatrix} \quad (1)$$

Fig.1 Pipe system in non-combustion state

Expressing this four-terminal transfer matrix in terms of a characteristic impedance $z(s)$ and a propagation constant $\gamma(s)$.

$$[\Gamma_n(s)] = \begin{bmatrix} \cosh\{\gamma(s)L\} & z(s)A \sinh\{\gamma(s)L\} \\ \frac{1}{z(s)A} \sinh\{\gamma(s)L\} & \cosh\{\gamma(s)L\} \end{bmatrix} \quad (2)$$

$$z(s) = \frac{\rho c^2}{sA} \gamma(s) \quad (3)$$

$$\gamma(s) = \sqrt{s^2 + R_f s} \quad (4)$$

in which, R_f represents the resistance of pipe system.

If the flow rate fluctuation $\tilde{q}_i$:

$$\tilde{q}_i = \tilde{q}_i \exp(j\omega t) \quad (5)$$

is supplied at the section i , the velocity fluctuations are generated, and they are propagated toward the left- and right-hand sides from this section and are respectively denoted by the symbols $\tilde{u}_{i\oplus}, \tilde{u}_{i\ominus}$. Designating the Laplace transforms of $\tilde{q}_i, \tilde{u}_{i\oplus}, \tilde{u}_{i\ominus}$ in the symbols $Q_i(s), U_{i\oplus}(s), U_{i\ominus}(s)$ respectively; we have:

$$Q_i(s) = A[U_{i\oplus}(s) + U_{i\ominus}(s)] \quad (6)$$

The symbol $Y_i^j(s)$ stands for a frequency transfer function, in which the flow rate fluctuation $\tilde{q}_i$ at the section i are referred to the input signal and the pressure fluctuation $\tilde{p}_j$ at the section j are referred to the output one. If we put the real and imaginal parts of this function as X_R and X_I respectively.

$$Y_i^j(s) = P_j(s) / Q_i(s) = X_R + jX_I \quad (7)$$

If the values of X_R and X_I are known, p_j is calculated from the following equations:

$$\tilde{p}_j = \tilde{q}_i [Y_i^j(j\omega)] \exp(j\omega t + j\angle Y_i^j(j\omega)) \quad (8)$$

where, $|Y_i^j(j\omega)| = \sqrt{X_R^2 + X_I^2}$, $\angle Y_i^j(j\omega) = \tan^{-1} \frac{X_I}{X_R}$.

3.2 Transfer function at non-combustion state

As indicated in Fig.1, the relation between the impedance $Z_{i\oplus}$ at the source point i and the impedance $Z_{j\oplus}$ at the point j located in the distance l from the point i , is expressed as:

$$Z_{i\oplus}(s) = \frac{P_i(s)}{U_{i\oplus}(s)} = \frac{Z_{j\oplus}(s) + z(s)A \tanh\{\gamma(s)L\}}{Z_{j\oplus}(s) + z(s)A \tanh\{\gamma(s)L\} + 1} \quad (10)$$

Hence, by the use of the known impedance at the point n of the pipe outlet, that is, $Z_n(s) = 0$, the impedance $Z_{i\oplus}(s)$ at the point i is calculated from the following equation:

$$Z_{i\oplus}(s) = \frac{P_i(s)}{U_{i\oplus}(s)} = z(s)A \tanh\{\gamma(s)L_2\} \quad (11)$$

Similarly, if attention is given to the left-hand side pipe, the relation for the closed pipe becomes;

$$Z_{i\ominus}(s) = \frac{P_i(s)}{U_{i\ominus}(s)} = \frac{z(s)A}{\tanh\{\gamma(s)L_1\}} \quad (12)$$

If the pipe inlet at the left-hand side end is connected to the chamber of volume V ,

$$Z_{i\ominus}(s) = \frac{\frac{A\rho c^2}{V_s} + z(s)A \tanh\{\eta(s)L_1\}}{\frac{\rho c^2}{z(s)V_s} \tanh\{\eta(s)L_1\} + 1} \quad (13)$$

From Eq. (6), the relation between the pressure at the point i and the input flow rate becomes

$$Y_i(s) = \frac{P_i(s)}{Q_i(s)} = \frac{P_i(s)}{A[U_{i\oplus}(s) + U_{i\ominus}(s)]} = \frac{Z_{i\oplus}(s)Z_{i\ominus}(s)}{A[Z_{i\oplus}(s) + Z_{i\ominus}(s)]} \quad (14)$$

If we express the distance from the source to the left-hand measuring point j by l and express the Laplace transform of the pressure $\tilde{p}_j$ at that point by $P_j(s)$, the ratio of $P_j(s)$ to $P_i(s)$ becomes:

$$\frac{P_j(s)}{P_i(s)} = \cosh\{\eta(s)l\} - \frac{z(s)A}{Z_{i\oplus}(s)} \sinh\{\eta(s)l\} = \cosh\{\eta(s)l\} - \frac{\sinh\{\eta(s)l\}}{\tanh\{\eta(s)L_1\}} \quad (15)$$

From Eqs. (14) and (15), $P_j(s)$ is expressed as a ratio to the given fluctuation; $Y_j(s) = \frac{P_j(s)}{Q_i(s)} = \frac{P_i(s)P_j(s)}{Q_i(s)P_i(s)}$

$$= \frac{Z_{i\oplus}(s)[Z_{i\oplus}(s)\cosh\{\eta(s)l\} - z(s)A\sinh\{\eta(s)l\}]}{A[Z_{i\oplus}(s) + Z_{i\ominus}(s)]} \quad (16)$$

3.3 Transfer function at combustion state

As is indicated in Fig. 2, the mixture gas, which is exhausted in a fluctuating manner from the exhaust port placed in a band shape at the peripheral pipe wall, is ignited by the residual high-temperature combustion gas and the grid-type nichrome wires and is burned in a fluctuating manner.

If the flowing toward left-hand side is inhibited, the longitudinal changes of time-mean heat release rate and time-mean temperature become the states such as indicated in the middle figure. As a result of the mixture gas being exhausted in a fluctuating manner, the fluctuating fields of heat release, temperature, and flow velocity, which change both in time and in place, are formed. It can be considered that the pressure fluctuation p_j observed in these fluctuating fields is generated in the following three path ways:

Fig. 2 Model in the combustion system

a) The first one is a pressure wave directly generated from $\tilde{q}_i$ as in the case of the pressure wave in the non-combustion field when the flow rate fluctuation $\tilde{q}_i$ is given, and that is designated by $\tilde{p}_j'$.

b) The second one is a pressure wave at the point j which occurs as a consequence of the generation of heat release fluctuation by the flow rate fluctuation $\tilde{q}_i$ and of the propagation of the pressure wave produced by this heat release. This pressure wave is designated by $\tilde{p}_j''$.

c) The third one is a pressure wave which propagates up to the point j from the driving source at which the volume fluctuation is generated by the heat release fluctuation. This pressure wave is designated by $\tilde{p}_j'''$.

The effect of the latter two types on the heat release field corresponds to the equi-sound-pressure sound source and the equi-flow-velocity sound source respectively. In the case of (c), if there is no ignition lag, the effect of apparent volumetric fluctuation on the surroundings is identical with the case of (a).

The observed pressure wave $\tilde{p}_j$ appears itself through such three path ways and is expressed as a linear combination of $\tilde{p}_j'$, $\tilde{p}_j''$, $\tilde{p}_j'''$.

$$\tilde{p}_j = \tilde{p}_j' + \tilde{p}_j'' + \tilde{p}_j''' \quad (17)$$

3.3.1 In the case that the pressure wave is generated directly from the flow rate fluctuation.

This case is similar to one at non-combustion state except that the temperature and therefore the sound speed are spatially different. If the distance a from the origin of combustion in the longitudinal axis (the point of $x=0$ when the injection point is positioned at $x=L_0$) to the position of the maximum heat release is given, the temperature distribution can be estimated by the use of the empirical formula. [14] In the calculation of the transfer matrix the pipe system is divided into many parts of appropriate length, and it is thought that the distributions of approximately constant-pressure and sound speed are attained in those partitions. That is to say, if the mean density ρ_j^{j+1} and the mean-speed c_j^{j+1} are estimated for the length of partition L_j^{j+1} between the divided positions of j and $j+1$, the transfer matrix between these positions is estimated from the following four-terminal equations.

$$\begin{bmatrix} P_j(s) \\ U_j(s) \end{bmatrix} = [\Gamma_j^{j+1}] \begin{bmatrix} P_{j+1}(s) \\ U_{j+1}(s) \end{bmatrix} \quad (18)$$

$$[\Gamma_j^{j+1}] = \begin{bmatrix} \cosh\{\eta_j^{j+1}(s)L_j^{j+1}\}, & \frac{1}{z_j^{j+1}(s)A} \sinh\{\eta_j^{j+1}(s)L_j^{j+1}\}, \\ z_j^{j+1}(s)A \sinh\{\eta_j^{j+1}(s)L_j^{j+1}\}, & \cosh\{\eta_j^{j+1}(s)L_j^{j+1}\} \end{bmatrix} \quad (19)$$

$$\text{here, } z_j^{j+1}(s) = \frac{\rho_j^{j+1}(c_j^{j+1})^2}{sA} \eta_j^{j+1}(s) \quad (20)$$

$$\eta_j^{j+1}(s) = \sqrt{s^2 + R_f s / c_j^{j+1}} \quad (21)$$

For a laminar flow, the resistance R_f is estimated as

$$R_f = 32 \mu / (\rho_j^{j+1} d^2). \quad (22)$$

Expressing the impedance directed toward right-hand side at the point j by that at the point $j+1$, one obtains

$$Z_{j+1}(s) = \frac{Z_{j+1}(s) + z_j^{j+1}(s) A \tanh(\eta_j^{j+1}(s) l_j^{j+1})}{\frac{Z_{j+1}(s)}{z_j^{j+1}(s) A} \tanh(\eta_j^{j+1}(s) l_j^{j+1}) + 1} \quad (23)$$

The relation between the pressure at the point j and the one at the point $j+1$ becomes

$$\frac{p_{j+1}(s)}{p_j(s)} = \frac{Z_{j+1}(s)}{Z_{j+1}(s) \cosh(\eta_j^{j+1}(s) l_j^{j+1}) + z_j^{j+1}(s) A \sinh(\eta_j^{j+1}(s) l_j^{j+1})}. \quad (24)$$

For a pipe directed to the left-hand side, a similar relation to that in the equations from (18) to (24) is applied by assigning the positive value to the direction of flow velocity and to the length directed to the left-hand side.

If the impedance at the pipe outlet is provided, the impedance inside the pipe can be estimated successively by the use of Eq.(23). Therefore, both impedances directed to the left- and right-sides at the point j which lies in the mixture injection port are decided; and then, the pressure fluctuations at arbitrary points are calculated from Eq.(16). By inverse-transformation of this term the pressure fluctuation $\tilde{p}_j$ at the point j can be calculated for a given $\tilde{q}_i$.

3.3.2 In the case that the pressure and volume fluctuations generated by the heat release fluctuation act as a driving source

It was previously mentioned that a heat release region develops on each side of the mixture exhaust place and that the

$$(\Delta \tilde{p}_\oplus)_{x=0} = \frac{(\kappa-1)Jf}{4c} \left\{ \frac{1}{\omega/(c-\bar{u}) - (\pi/L_f)} \left(1 - \frac{\pi \bar{u}}{\omega L_f} \right) - \frac{1}{\omega/(c+\bar{u}) + (\pi/L_f)} \left(1 + \frac{\pi \bar{u}}{\omega L_f} \right) \right\} B_\oplus \exp[j\omega(t-t_i) + \alpha_\oplus], \quad (28)$$

$$(\Delta \tilde{p}_\ominus)_{x=L_f} = \frac{(\kappa-1)Jf}{4c} \left\{ \frac{1}{\omega/(c+\bar{u}) - (\pi/L_f)} \left(1 - \frac{\pi \bar{u}}{\omega L_f} \right) - \frac{1}{\omega/(c-\bar{u}) + (\pi/L_f)} \left(1 + \frac{\pi \bar{u}}{\omega L_f} \right) \right\} B_\ominus \exp[j\omega(t-t_i) + \alpha_\ominus], \quad (29)$$

$$B_\oplus = \sqrt{\left(1 + \cos \frac{\omega L_f}{c+\bar{u}} \right)^2 + \left(\sin \frac{\omega L_f}{c+\bar{u}} \right)^2}, \quad (30)$$

$$\alpha_\oplus = -\tan^{-1} \left(\frac{\sin \frac{\omega L_f}{c+\bar{u}}}{1 + \cos \frac{\omega L_f}{c+\bar{u}}} \right). \quad (31)$$

For the combustor such as being used in this experiment, the injection port coincides with the position of maximum heat release, and hence, the condition of $L_0 = L_f/2$ can be expected to hold. If the above-mentioned pressure and flow velocity waves arriving at both ends of combustion region are replaced by those at the position of mixture gas injection, that is, $x=L_f/2$, the phase difference terms $\alpha_\oplus$, $\alpha_\ominus$ caused from the time necessary for the wave to propagate the distance $L_f/2$ can be eliminated. At the low Mach number, the pressure wave $\Delta \tilde{p}_i$ and the flow velocity wave $\Delta \tilde{u}_{i\oplus}$ and $\Delta \tilde{u}_{i\ominus}$ generated at the point of $x=L_f/2$ by the combustion become.

$$\Delta \tilde{p}_i = \frac{B}{c} f \exp[j\omega(t-t_i)] \quad (34)$$

heat release distribution results in the form of $\tilde{f}$ as illustrated in Fig.2. It is assumed that the heat release fluctuation $\tilde{f}$ takes place in the region between the origin of combustion, that is, $x=0$ and the place $x=L_f$, and that the following equation is applied for the expression of $\tilde{f}$ with respect to the flow rate fluctuation given by Eq.(5).

$$\left. \begin{aligned} x < 0, \quad x > L_f \quad \tilde{f} &= 0, \\ 0 \leq x \leq L_f \quad \tilde{f} &= f \sin \left(\frac{\pi}{L_f} x \right) \exp[j\omega(t-t_i)]. \end{aligned} \right\} \quad (25)$$

Where t_i represents the time lag from the generation of flow rate fluctuation to that of heat release fluctuation, namely, it expresses the ignition delay.

Between the amplitudes of flow rate $\tilde{q}_i$ and the heat release $\tilde{f}$, a simple relationship holds:

$$\tilde{f} = \frac{\rho g h_f \tilde{q}_i}{A L_f (\phi+1)}. \quad (26)$$

When the wave reflected in the pipe is neglected for the heat release fluctuation given by Eq.(25), if we respectively designate $(\Delta \tilde{u}_\oplus) = 0$, $(\Delta \tilde{p}_\ominus) = 0$ for the flow velocity and pressure, both of which are observed at the point $x=0$, and in which the left-hand direction is taken as positive, and if we designate $(\Delta \tilde{u}_\ominus) = 0$, $(\Delta \tilde{p}_\oplus) = 0$ for those observed at the point $x=L_f$ in the same use of positive direction, the equations for pressure waves are represented as

$$\left. \begin{aligned} (\Delta \tilde{u}_\oplus)_{x=0} &= \frac{c}{\kappa p} (\Delta \tilde{p}_\oplus)_{x=0}, \\ (\Delta \tilde{u}_\ominus)_{x=L_f} &= \frac{c}{\kappa p} (\Delta \tilde{p}_\ominus)_{x=L_f}, \end{aligned} \right\} \quad (27)$$

and the right- and left-hand pressure waves become:

$$B_\oplus = \sqrt{\left(1 + \cos \frac{\omega L_f}{c-\bar{u}} \right)^2 + \left(\sin \frac{\omega L_f}{c-\bar{u}} \right)^2}, \quad (28)$$

$$\alpha_\oplus = -\tan^{-1} \left(\frac{\sin \frac{\omega L_f}{c-\bar{u}}}{1 + \cos \frac{\omega L_f}{c-\bar{u}}} \right), \quad (29)$$

$$B_\ominus = \sqrt{\left(1 + \cos \frac{\omega L_f}{c+\bar{u}} \right)^2 + \left(\sin \frac{\omega L_f}{c+\bar{u}} \right)^2}, \quad (30)$$

$$\alpha_\ominus = -\tan^{-1} \left(\frac{\sin \frac{\omega L_f}{c+\bar{u}}}{1 + \cos \frac{\omega L_f}{c+\bar{u}}} \right). \quad (31)$$

$$\Delta \tilde{u}_{i\oplus} = \Delta \tilde{u}_{i\ominus} = \frac{B}{\kappa p} f \exp[j\omega(t-t_i)], \quad (32)$$

$$B = \frac{(\kappa-1)J}{2\sqrt{2}} \left(\frac{1}{\omega/(c-\pi/L_f)} - \frac{1}{\omega/(c+\pi/L_f)} \right)$$

$$\times \sqrt{1 + \cos \frac{\omega L_f}{c}}. \quad (33)$$

The pressure wave which arises at the point j for the driving force of $\Delta \tilde{p}_i$ is put as $\tilde{p}_j$ and the one which arises at the point j for the driving forces of $\Delta \tilde{u}_{i\oplus}$ and $\Delta \tilde{u}_{i\ominus}$ put as $\tilde{p}_{j\oplus}$.

Representing the Laplace transforms of $\Delta \tilde{p}_i$, $\Delta \tilde{u}_{i\oplus}$, $\Delta \tilde{u}_{i\ominus}$ as $\Delta P_i(s)$, $\Delta U_{i\oplus}(s)$, $\Delta U_{i\ominus}(s)$ respectively, then we have

$$\frac{\Delta P_i(s)}{Q_i(s)} = \frac{B \rho g h_f \tilde{f}}{c A L_f (\phi+1)} \exp(-j\omega t_i), \quad (34)$$

$$\frac{\Delta U_{i\oplus}(s)}{Q_i(s)} = \frac{\Delta U_{i\ominus}(s)}{Q_i(s)} = \frac{B \rho g h_f \tilde{f}}{\kappa P A L_f (\phi+1)} \exp(-j\omega t_i). \quad (35)$$

Since the pressure waves observed at the point j are defined by Eq.(17), the transfer function $Y_{ij}(s)$, for which the injection rate $Q_i(s)$ at the point i is the

input and the pressure wave $P_j(s)$ observed at the point j is the output, is represented by the following equation by the use of the results obtained for the above mentioned pathways of 3.3.1 and 3.3.2.

$$Y_i(s) = \frac{P_j(s)}{Q_i(s)} = \frac{P_j(s)}{P_i(s)} Y_i'(s) + \frac{\Delta P_i(s)}{Q_i(s)} \times \left[\frac{\Delta U_{i1}(s) + \Delta U_{i2}(s)}{Q_i(s)} Y_i'(s) \right], \dots (39)$$

4. Apparatus and experimental method

Systematic view of the apparatus used is shown in Fig.3. The air supplied from an air compressor 1 and the propane gas

1. Compressor
2. Propane gas bomb
3. Orifice flow-meter
4. Mixing chamber
5. Rotary throttle
6. Photo-transistor
7. Variable speed motor
8. Measuring point at throttle outlet
9. Combustion chamber
10. Electric transformer for igniter
11. Electric linearizer for anemometer
12. Hot wire anemometer
13. Balancer of power supplier
14. Hot wire probe
15. Solid state pressure transducer
16. Synchroscope
17. Amplifier
18. Oscillograph

Fig.3 Systematic view of the apparatus

Fig.4 Rotary throttle

from a bomb 2 are supplied respectively to a mixing chamber 4 via flow rate adjusting valves and flow-rate measuring orifices. For the sake of safety, reverse-flow inhibiting fluid diodes are connected both at the back and the front of the mixing chamber. Mixture is supplied in a cyclic way of 1, 2, 3, or 6 pulses per revolution by a variable speed electric motor 7 and by a rotating throttle valve 5, the detailed view of which is shown in Fig.4. This time, the flow rate fluctuation supplied for the combustor is approximated by a sine wave of zero minimum flow rate, hence, of no reverse flow. There is a measuring section of fluctuating flow rate 8 between the rotating throttle valve and the combustion chamber 9, in which a probe of hot wire anemometer is positioned. The leading pipe from this section to the chamber has a length of 30 cm and an inner diameter of 8 mm. Mixture gas is injected from 140 holes of 1.8 mm diameter arranged within a 20 mm wide surface around the combustion chamber toward the center of the chamber; thereafter, it is ignited both by the residual reacted gas and by the ignition nichrom wire-grids concentrically arranged inside the wall of the chamber.

5. Numerical values used and calculating procedure

The dimensions of the apparatus are shown in Fig.1. Forced frequency is altered in the range from 10 Hz to 50 Hz. Commercial propane gas was used as fuel and then the mixture ratio was retained around the value of stoichiometric ratio, that is, $\phi=15.5$, throughout the experiment.

For the flow rate fluctuating amplitude which is identical with the mean flow rate, the frequency characteristics of pressure wave $\tilde{p}_j$ for non-combustion state are examined by the use of the method which is similar to that in the equations (8) and (9), and than, by substituting the relation of $s=j\omega$ in the equations from (14) to (16). Similar calculations are also done for the combustion state, but, in this case the temperature in the pipe, consequently, the propagation constant $\eta(s)$ and the characteristic impedance respectively take considerably different values for different longitudinal positions. Consequently, the calculation procedure presented in a previous paper^[14] is applied for the estimation of the combustion temperature.

Tab.1 Main parts dimensions of the apparatus used

Pipe diameter	d cm	5.2
Pipe length	L cm	323, 404, 687
Volume	V cm ³	0,5000, 10 ⁴
Position of exhaust port of mixture gas	L_1 cm	13
Measuring position	l cm	95

By the use of this calculation result, the pipe length is divided into many sections each with 100°C difference in temperature; and for these sections of L_j^{j+1} length, the temperature is regarded as constant, and hence, the sound speed c_j^{j+1} and the density ρ_j^{j+1} are estimated. Then by the use of the known boundary impedance at the pipe ends, the impedance $Z_{j0}(s)$ or $Z_{je}(s)$ is successively determined from these ends up to the point j at the mixture injection point. By substituting the equations (24), (37), (38) into the equation (39), the transfer function in the combustion state is obtained. The calculation in obtaining $\hat{p}_j$ is similar to that at the non-combustion state.

6. Results and considerations

Figure 5 shows the pressure wave at the open side position of 95 cm measured from the injection point situated at 13 cm from the closed pipe end of length $L=323$ cm or 687 cm. Mixing ratio at the combustion state is $\phi=14.4$. One unit of scale on

Abcissa : time, 1 div. = 0.1 sec.
Fig.5 Pressure wave forms

Fig.6 Frequency spectrum

the abscissa corresponds to the time of 0.1 sec, and the left- and right-hand figures represent the pressure waves in the non-combustion and combustion states respectively. The numerical values indicated in the middle part of the figure are the forced frequency of the mixture flow rate oscillation. Figure 6 shows the result of frequency analysis at the same condition. Amplitude and its change of pressure wave at the combustion state become considerably large in comparison to that at non-combustion state.

For the measurement of frequency transfer function, pure tone analysis is desirable. Consequently, this case where several peaks are observed in the region which departs from the forced frequency, as is shown in the figure, means a defect of the experimental apparatus. However, it is very difficult to inject the mixture, and at the same time, to provide the monotone fluctuation to the flow rate. In the following arrangement of the obtained data, the experimental error of the amplitude variation owing to the presence of plural waves is controlled to within the minimum allowable limits by the employment of the values of pressure and flow rate amplitudes at an identical frequency with the forced frequency.

Table 2 shows the resonance frequencies in the non-combustion and combustion states for the forced oscillation of the supplied mixture flow rate. Experimental results coincide well with the theoretical ones irrespective of the presence of the volume. Owing to the rapidness of the sound speed in the combustion state compared with that in the non-combustion state, the former shows a similar effect to the case that the pipe length becomes shorter. Therefore, the resonance frequency becomes higher, and, in this point the theoretical results coincide well with the experimental ones. As a whole, the maximum error between them is approximately 1 Hz.

Figure 7 shows the results of the amplitude ratio of pressure to flow rate which is generated through three exciting pathways, and the result of the resultant amplitude ratio of pressure to flow rate and the phase difference, then the relation among them is obtained for the conditions $L=687$ cm, $\bar{q}=176$ cm³/s, $\phi=17.4$, $t_1=0$ sec in

Tab.2 Resonance frequency

Pipe length cm	Volume cm ³	Non-comb.		Combustion	
		Cal. Hz	Exp. Hz	Cal. Hz	Exp. Hz
323	0	25.8	25.5	26.4	26.7
404	0	20.7	20.3	21.3	21.6
687	0	12.2	12.2	12.2	12.2
		36.6*	36.3*	37.5*	36.5*
687	5×10^3	9.4	9.4	9.5	9.5
687	10^4	7.6	7.7	7.9	7.8

* Note. Third order resonance

the combustion state and at the location $z = 95$ cm.

The peaks of $|Y_1^j|'$ and $|Y_1^j|''$ lie in approximately the same frequency range, and that of $|Y_1^j|'''$ lies at the frequency adjacent to the location of the minimum value of both of $|Y_1^j|'$ and $|Y_1^j|''$. When the frequency approaches zero, both $|Y_1^j|'$ and $|Y_1^j|''$ tend to zero; on the other hand, $|Y_1^j|'''$ doesn't reach zero. The phase differences $\angle Y_1^j'$ and $\angle Y_1^j''$ take the values of 90° or -90° with correspondence to the frequency range for the case of zero ignition lag or zero viscosity. On the other hand, $\angle Y_1^j'''$ is then zero, therefore, the pressure wave generated is in the same phase with the input flow rate. As a result, from the magnitude relation among $|Y_1^j|'$, $|Y_1^j|''$ and $|Y_1^j|'''$, the phase $\angle Y_1^j$ of the resultant pressure wave takes the forms as indicated in the figure.

Fig.7 Exciting pathway, pressure wave, and its phase difference

Fig.8 Frequency responses in the states of non-combustion and combustion

In addition, the difference between the results with and without consideration of laminar flow viscosity effect is very little both in the amplitude and in the phase. However, for the effect of the ignition lag t_i , the Y_1^j and Y_1^j include directly t_i in the form of the equations (37), (38); and hence, $\angle Y_1^j$ changes on the basis of the magnitude relation among $|Y_1^j|'$, $|Y_1^j|''$, and $|Y_1^j|'''$. From the measured results, the value of $\angle Y_1^j$ at a remote frequency from the resonance one coincides with the values of $\pm 90^\circ$; whence, it is understood that the ignition lag in this case is quite little.

Furthermore, as shown in the figure, in the usual combustion state the component of pressure wave Y_1^j based on the generation of flow velocity wave by combustion is considerably large in comparison with the other two components which take the other pathways. This is owing to the fact that the terms of $\Delta U_{\phi i}(s)/Q_i(s)$ or $\Delta U_{\phi i}(s)/Q_i(s)$ in the equation (39) become by far greater than $1/2$. In view of the fact that the term B/L_f used in the equations (37), (38) remains constant at low frequency region, it can be understood that the resultant magnitude $|Y_1^j|$ is correlated mainly with the magnitude of the term $\xi h_j/(\phi+1)$, that is, the rise in volume by combustion. In addition, in the present analysis the mole number changes owing to the combustion being disregarded, but, if these are taken into account the above mentioned increase rate in volume will change by a fraction of the change in mole number. However, the influence of this mole number change is small for the usual combustion reaction.

Figures 8 and 9 show the frequency response of the pressure wave, which is measured at the location of $z=95$ cm and for the condition of closed pipe end. Broken and solid curves indicate the theoretical results and correspond respectively

Fig.9 Frequency responses in the states of non-combustion and combustion

Fig.10 Frequency response under the condition of connecting the reservoir

with the state of non-combustion and of combustion. Solid and open circles indicate the experimental values respectively corresponding to each state. It is clearly known from the figure that the pressure amplitude in the combustion state is far larger than that in non-combustion state.

From the whole view of these figure, it is understandable that the theoretical results agree well with the experimental ones. Therefore, it may be assumed that the distribution form of the heat release fluctuation estimated in the form of equation (25) can exactly express the phenomena.

Figure 10 indicates the three cases of frequency responses for one kind of pipe length of $L=687\text{cm}$, two kinds of volume, or no volume but closed pipe end. By connecting the reservoir to the pipe end, the resonance frequency is decreased, still further, the frequency band width which gives the same amplitude ratio at the standard frequency region becomes narrower. Furthermore, the resonance frequency, which corresponds to the third order resonance frequency for closed end condition, is decreased by connecting the reservoir; as a result, the third resonance becomes scarcely discernable for the condition of $V=10^4\text{ cm}^3$.

It can be therefore supposed that the method of connecting the volume to the pipe system is one of the effective methods for the purpose of suppressing the oscillation. Putnam [15] and Suzuki [16] has already confirmed this experimentally.

7. Conclusions

This paper deals with an analytical method of the frequency response of the pressure wave in the case that the mixture is injected fluctuatedly into the pipe system and is burned. The comparison between the theoretical calculation result and the experimental one obtained by the use of the rotary valve and the combustion chamber in which the one-dimensional combustion is attained, is also presented.

By summarization of the obtained results, the following conclusions are drawn:

(1) The pressure fluctuation in the combustion state can be expressed in a considerably exact form as the one order combination of the following three pressure waves:

(a) The one is a pressure wave generated directly by a given flow rate oscillation.

(b,c) The other two come through the heat release fluctuation generated by a given flow rate oscillation; thus, (b) one of which is a pressure wave generated with a heat release and (c) the last one is a pressure wave generated by a volumetric fluctuation caused by this heat release.

Among these three pathways, the component due to volumetric fluctuation is greater than the other two; and hence, the dynamic response can be estimated only by this component.

(2) If the flow rate oscillation is equal for the combustion and non-combustion states, the pressure fluctuation in the combustion state becomes greater than that in the non-combustion state.

The degree of this rise depends mainly on the magnitude of the term $\xi h_f/(\phi+1)$, that is, the strength of the heat release fluctuation in the combustion region.

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