The larger the value of K_p , the larger the frequency and the amplitude of the limit cycle.

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Fatigue Phenomena under Rolling Contact Accompanied with Sliding*

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The problem of fatigue phenomena under rolling contact accompanied with sliding is very important for the gears, rails and tyres, rolling bearings etc., nevertheless little part of the problem has been revealed. So we have carried out some experiments about rolling fatigue accompanied with sliding, and investigated how fatigue cracks occur and progress.

1. Survey of the problem

The theoretical stress-strain analysis of this problem has been carried out by Lundberg⁽¹⁾, Mindlin⁽²⁾, Poritsky⁽³⁾, Smith and Liu⁽⁴⁾, Frocht⁽⁵⁾ etc. They solved the problem under static and elastic conditions, and revealed that the max. shear stress increased and occurred nearer to the surface when a tangential force was applied at the rolling contact point. Okoshi and Sata⁽⁶⁾ proved experimentally that the rolling fatigue limit decreased when sliding was added to the rolling. Hattori and others⁽⁷⁾ showed that the position of the max. work-hardness of the ball bearing race became nearer to the surface when sliding existed at the rolling contact point. So-called shelly cracks of the rails are shown in Fig. 1. Such cracks have been investigated in U.S.A.,⁽⁸⁾ U.S.S.R.⁽⁹⁾ and Japan⁽¹⁰⁾. Nakamura⁽¹¹⁾ showed that the cracks diminished or ceased to increase if wear of rails increased above a certain limit. Enomoto⁽¹²⁾ showed photo-elastic analysis of

contact stresses between cylinder and plate where occurred a sharp surface crack, and he revealed that the shear stress at the tip of a crack increased by the added tangential force. Each of the above mentioned reports is a good clue to solution of the problem, nevertheless we find that the problem has been scarcely solved. It is now quite important to research how fatigue cracks occur and progress under the rolling contact being accompanied with sliding, and how materials and sliding conditions affect the phenomenon.

2. Testing machine, specimens, and testing conditions

2.1 Testing machine and specimens The testing machine is a simple rolling-test machine, upper and lower specimens are driven individually under rolling contact. The specimens are as shown in Fig. 2. Their surfaces are finished by turning. Surface roughness did not affect fatigue phenomenon in our testing. Materials and their combinations are as Tables 1 and 2.

2.2 Testing conditions The max. contact stress and the position of the max. shear stress (distance from the surface) by the Hertz theory are shown

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Fig. 1 Shelly cracks of the rails (Japan National Railways)

Table 1 Materials of specimens

Material	Carbon content	Reference spec.
Tyre steel	0.70%	JIS E 4501
Rail steel A	0.56%	JIS E 1101
Rail steel B	0.70%	"

Table 2 Testing combination of materials

Test No.	Combination of materials
I	Rail steel A and tyre steel
II	Rail steel B and tyre steel

Fig. 2 Shapes and dimensions of specimens

Table 3 Hertz' contact stress and position of max. shear stress

Max. Hertz' contact stress kg/mm ²	49.6	65.6	78.5	89.5	99.3	111.0	121.5	135.9	157.0
Position of max. shear stress (distance from surface) mm	0.05	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16

Table 4 Rotating speed, direction of sliding, sliding ratio and relative sliding velocity of specimens

Specimen	Simple rolling	Rolling accompanied with sliding			
	Rotating speed r.p.m.	Rotating speed r.p.m.	Direction of sliding (Direction of frictional force caused by sliding)	Sliding ratio %	Relative sliding velocity cm/sec
Upper specimen	800±10	890±10	Equal to rolling direction	$(890-800)/890 \times 100 = 10.1$	14.1
Lower specimen	800±10	800±10	Reverse to rolling direction	$(800-890)/800 \times 100 = -11.2$	14.1

Remark : Rolling direction means the travelling direction of rolling contact point

Table 5 Sliding conditions of specimens

Division of test		Material	Setting position of specimen	Sliding direction	Sliding ratio %	Relative sliding velocity cm/sec
Test I	IA(a)	Rail steel A Tyre steel	Lower Upper	Reverse to rolling Equal to rolling	-11.2 10.1	14.1
	IA(b)	Rail steel A Tyre steel	Upper Lower	Equal to rolling Reverse to rolling	10.1 -11.2	14.1
	IB	Rail steel A Tyre steel	Lower Upper	Without sliding	— —	—
Test II	IIA(a)	Rail steel B Tyre steel	Lower Upper	Reverse to rolling Equal to rolling	-11.2 10.1	14.1
	IIA(b)	Rail steel B Tyre steel	Upper Lower	Equal to rolling Reverse to rolling	10.1 -11.2	14.1
	IIB	Rail steel B Tyre steel	Lower Upper	Without sliding	— —	—

Remarks: 1. In test IB the results are approximately equal whether rail steel A is upper or tyre steel is upper.
2. Test IIB was stopped. Because in test IIA no fatigue cracks appeared in consequence of severe wear, comparison the data of IIB with those of IIA was meaningless.

Fig. 3 Fatigue diagram (S-N diagram)

in Table 3. The position of the max. dynamic shear stress by Lundberg and Palmgren⁽¹³⁾ is a little nearer to the surface. Rotating speeds and sliding conditions are shown in Tables 4 and 5. The specimens are lubricated with pure light mineral oil.

3. Testing results

3-1 Wear In the test IA the wear was negli-

Test No.: IA(a) No. 29 (Refer to Table 5), Max. Hertz contact stress: 78.5 kg/mm², Total rotations: 6.50 × 10⁵

Fig. 4 Appearance of contact surface of rail steel A at small contact stress, being accompanied with sliding

gible until fatigue cracks developed, nevertheless in the test IIA the wear progressed rapidly and fatigue cracks did not appear. Thus wear-rates differ by the combinations of the materials and the contact conditions, and they affect fatigue phenomenon variously. For the purpose of simplifying the problem, in this report we will treat only the case that wear is negligible.

3.2 Fatigue diagrams (S-N diagrams) Fatigue diagrams are shown in Fig. 3, where S is the max. contact stress by the Hertz theory, and N is the total rotation until the first visible sign of fatigue cracks appear at the surface. When cracks develop on a specimen, the latter must be changed for a new one (being somewhat rolled for the purpose of omitting the initial effects) to pursue the life of the partner specimen, because such irregularities of the surface affect the partner's fatigue phenomena severely.

3.3 Appearance of surface of specimens after tests Figs. 4 and 5 are examples of the specimens of rail steel, and Fig. 6 is that of tyre steel. Fig. 7 is an example of depressions caused by the worn particles.

3.4 Formations of cracks Fig. 8 is an example of the case of simple rolling without sliding, and Fig. 9 is one of the case being accompanied with sliding. Fig. 10 is a special case where at first sliding accompanies and later ceases to do.

4. Considerations

4.1 Generation of cracks and affecting factors

4.1.1 Origin of cracks and direction of their pro-

gress The origin of each crack is always at the surface, being not concerned with sliding. The direction of progress of each crack is inclined $30\sim 50$ degrees to the tangential line toward rolling direction, as in Figs. 8~10. Figs. 11 and 12⁽¹⁴⁾ show the fatigue cracks of ball bearing inner races, their materials are equal to each other in chemical composition, metallurgical structure, mechanical strength and hardness (RC 62.0~63.0) except forging ratio. In Fig. 11, the origin of cracks is nearer to the surfaces and all cracks connect more easily with one another than in Fig. 12, but in spite of such differences caused by the forging conditions of the materials, cracks always develop at the sub-surfaces. Fig. 13⁽¹⁵⁾ shows an example of ball bearing race where cracks start from the surface and progress inward in the same manner as with Figs. 8~10 (severe sliding occurs at the rolling contact point owing to the poor lubrication). Thus the development and the direction of the cracks differ by the materials and the contact conditions, but the direction of the cracks is not concerned with the direction of the tangential forces caused by sliding friction, and the cracks always progress toward the rolling direction.

Test No.: IA(a) No. 28(Refer to Table 5), Max. Hertz contact stress : 121.5 kg/mm^2 , Total rotations : 4.76×10^5

Fig. 5 Appearance of contact surface of rail steel A at large contact stress, being accompanied with sliding

4.1.2 Life of specimens (Time until first cracks appear) As indicated in Fig. 3 the lives of the specimens decrease with increasing of the Hertz stress, and vary as the kind of materials and the direction of sliding. Thus the stress governing the development of the cracks is a function of the Hertz stress, the property of the materials and the vector (the value and the direction) of the tangential force. When the direction of the tangential forces

caused by the sliding friction is the same with the rolling direction, the lives are longer than in the case where the directions are reverse to each other. This behaviour differs with the materials, namely as shown in Fig. 3, the affection of sliding is severe in the case of the rail steel than the tyre steel.

4.2 Progress of cracks and affecting factors

4.2.1 Progressing direction of cracks In the case of simple rolling without sliding, as shown in Fig. 8,

Test No. : IB No. 15 (Refer to Table 5), Max. Hertz contact stress : 135.9 kg/mm^2 , Total rotations : 7.0×10^5

Fig. 6 Appearance of contact surface of tyre steel at large contact stress, in simple rolling (without sliding)

Test No. : IB No. 13 (Refer to Table 5), Max. Hertz contact stress : 111.0 kg/mm^2 , Total rotations : 9.5×10^5
Fig. 8 Progress of fatigue cracks of rail steel A, in simple rolling (without sliding)

Rail steel (Partner specimen of Fig. 6)

Tyre steel (Partner specimen of Fig. 9)

Fig. 7 Depressions caused by worn particles

(a)

(b)

(c)

(d)

×50

Test No. : IA(a) No. 16 (Refer to Table 5), Max. Hertz contact stress : 99.3 kg/mm².
Total rotations : 3.25×10^5

Fig. 9 Progress of fatigue cracks of rail steel A, being accompanied with sliding (Examples of several cracks of the same specimen)

Test No.: IA(a) No. 27 (Refer to Table 5), Max. Hertz contact stress : 121.5 kg/mm^2 ,
Total rotations : 7.19×10^5

Fig. 10 Progress of fatigue cracks of rail steel A. First testing condition was IA(a) in Table 5, then after the development of crack, testing condition was changed to simple rolling, and further test was continued in simple rolling without sliding

(a)

(b)

Test No.: No. 10 of material group 1, Life : 2.0 min

Remarks : Figure (a) and (b) are examples at different positions of the same specimen

Fig. 11 Fatigue cracks of ball bearing races (Cross-section perpendicular to rolling direction)

Test No.: No. 14 of material group 3, Life : 6.5 min

Fig. 12 Fatigue cracks of ball bearing races (Cross-section perpendicular to rolling direction)

the cracks, arriving at the domain where the shear stress is maximum, change their directions and then develop parallel to the tangential line toward the rolling direction. In the case of it being accompanied with sliding, as shown in Fig. 9, cracks reaching to the domain where shear stress is maximum, do not change their directions and still continue to progress inward. Thus the tangential forces caused by the sliding friction aid the cracks to progress inward, preventing their progress at the subsurface (where the shear stress is maximum) parallel to the rolling direction. Also in the case of the bearing steels similar behaviours are seen, Figs. 11 and 12 are the examples of the normal rolling fatigues, and Fig. 13 is an example of rolling fatigue being suffered under severe sliding. In all the tests, the behaviours of the progressing of the cracks are determined only by whether the tangential forces exist or not, and the direction of the tangential forces scarcely affects such phenomenon.

4.2.2 Progressing velocity of cracks The progressing velocities of the cracks can be calculated from the crosssection views of the specimens as shown in Figs. 8~10. Statistical treatments are necessary to calculate them, and our data are a little unsatisfactory for clarifying the problem. So we cannot determine the progressing velocities of the cracks except the fact that they are the functions of the materials and the contact conditions. Figs. 8~10 are shown for reference.

4.2.3 Work-hardness and resulting residual stress Work-hardness was negligible in our experiments, but sometimes it will be quite important.

4.3 Theoretical considerations As in the above mentioned considerations, the values and the directions of the tangential forces caused by the sliding friction can represent the sliding conditions, then a part of the fatigue phenomenon under the rolling contact being accompanied with sliding can be explained by the static and elastic theory of Hertz, Smith and Liu or others. The positions of the appearance of the cracks and the behaviours of the S-N diagrams (Fig. 3) may be analyzed by their theory, but the numerous facts that are involved with the appearance and the progress of the cracks cannot be explained. Why the cracks start from the surface under simple rolling contact without sliding? Why the cracks progress inward in the constant manner? Why the lives until cracks developed differ with the directions of tangential forces? Why the tangential forces aid the cracks to progress inward and prevent their progress in the tangential direction? Why do not the directions of the

→ Direction of rolling
Fig. 13 Fatigue cracks of ball bearing
reces (Cross-section perpendicular to
bearing axis at the center of groove)

tangential forces affect the directions of the cracks? Why does such phenomenon differ with the materials? These behaviours regarding the development and the progress of the fatigue cracks will be explained only by applying the theory of plasticity to the analysis of micro-stress of the materials. In this report we cannot fully explain the problems, but we recognize that such discussions are fundamentally similar in a large part to the case of rolling fatigue under simple rolling contact without sliding, which we have treated in the previous report⁽¹⁴⁾. So we think that the characteristics of the materials must be tough, not only high in the elastic limit and the shear strength, moreover they should be homogeneous and free from impurities for the purpose of preventing the deviations of fatigue lives. Thus the materials will require proper forging, good spheroidizing treatments of carbides, elimination of impurities and gases, right heat treatments, adoption of special chemical compositions⁽¹⁴⁾⁽¹⁶⁾⁽¹⁷⁾ etc.

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