where α , β , and γ are constants and η , ξ are the sum and the number of the terms, respectively. On the other hand when the series converges periodically, the mean value during one period is assumed as their sum.

For the eigenvalue problem as in the cases [A], [D], and [E], this method gives the lower limit to the correct value⁽⁹⁾, because the boundary conditions are taken loosely and on the other hand the conditions of equilibrium in the plate are fulfilled by taking infinite terms in the equations of conditions.

4. Conclusions

In the forgoing discussion the outline of this energy method for the rectangular plates having clamped or supported edges is explained and its accuracy is investigated on some examples.

The numerical calculations in this method are very simple, because the numbers of the unknowns

of the simultaneous equations are small, though the terms treated here are of considerable numbers.

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References

- (1) E. Trefftz: *ZAMM*, Bd. 15 (1935), S. 339.
- (2) B. Budiansky and Pai C. Hu: *NACA TR*, No. 848 (1946), p. 213.
- (3) G. A. Thurston: *Jour. Aero. Sci.*, Vol. 24 (1957), p. 407.
- (4) M. Hamada: *Trans. Japan Soc. Mech. Engrs.*, Vol. 24, No. 139 (1958), p. 148.
- (5) M. Hamada: *Trans. Japan Soc. Mech. Engrs.*, Vol. 23, No. 131 (1957), p. 522.
- (6) M. Naruoka: *Trans. Japan Soc. Mech. Engrs.*, Vol. 17, No. 57 (1951), p. 26.
- (7) *ibid.* (1).
- (8) M. Kuranishi: *HARI HASHIRA ITA* (1943), p. 88.
- (9) R. Courant and D. Hilbert: *Methods of Mathematical Physics*, 1 (1953), p. 407.

678.743:539.374:539.389

On the Yield Behavior of Unplasticized P. V. C.*

(Analogy with the Yielding of Mild Steel)

By Yūzō NAKAGAWA** and Satoshi OKUDA***

We can recognize that there are two steps in the yield process of unplasticized P. V. C.. The first step of yielding is a process, during which the stress-strain curve reaches its maximum point and drops to the lower yield point; we may call it the "initial yielding". The original local contraction can be seen in a part of the specimen. When such local contraction proceeded to a limiting value corresponding to the strain ϵ_1 , a new yielding or contraction of the cross section is caused in the neighbour of yielded part; such yielding or the second step of yield may be called the "progressive yielding".

The above two steps of yielding may be considered as analogous to the process of the upper and lower yieldings of the mild steel. Such phenomena of yielding are remarkably affected by the stretching speed v_{st} .

We established a relation between the stretching speed v_{st} , the yield strain ϵ_1 and the velocity of propagation of plastic strain wave c , as $v_{st} = c \cdot \epsilon_1$.

1. Introduction

Unplasticized polyvinylchloride (P. V. C.) has somewhat singular mechanical properties in many

respects. It seems that these properties give us many suggestions in considering the yielding or the plastic deformation of mild steel.

As described in the previous reports⁽¹⁾⁽²⁾, the sheet specimen of unplasticized P. V. C. shows a stress-strain diagram as shown in Fig. 1 in its tension test. When the load reaches a definite value or its maximum value (i.e. point ①), there

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Testing temperature 20°C, strain velocity 0.15/min
Fig. 1 Load-elongation curve under tension

Fig. 2 Slip lines

appear slip lines distinctly on the surface of specimen (Fig. 2). This is very similar to that of mild steel. According as the load is applied successively, these slip bands increase their widths and finally the cross section of test specimen contracts to a definite magnitude in a part of gage length of the specimen.

When such local contraction proceeds in some range to a definite depth, the work hardening of that portion is remarkable, and further contraction does not take place. Then, the new yielding or the contraction of the cross section is caused in the neighbourhood of the yielded part. This new contraction is propagated gradually in the direction of the tension while the load remains almost constant. Further elongation of the specimen owes mainly to this new yielding and contraction.

In this paper, we attempt to compare the yielding phenomenon of P. V. C. with that of mild steel.

2. Plastic deformation behavior of unplasticized P. V. C. under tension

The first local contraction can be seen in a part of the specimen, when the load reaches its maximum value ① and drops to the lower value ② as shown in Fig. 1. The authors call it "initial yielding". The second step of yielding or the successive yielding of the neighbouring part is

Fig. 3 σ_t - q curves at "initial yielding"

a: Stretching speed $v_{st}=12\text{mm/min}$
b: Stretching speed $v_{st}=1.2\text{mm/min}$
c: $v_{st}=6\text{mm/min}$, Stretching is stopped in the midst for a while, and clear knob is formed

Fig. 4 Profiles of plastic deformation under tension

caused after the point ② in Fig. 1. Such yielding may be called "progressive yielding".

Fig. 3 represents the relation between the true stress σ_t and the area contraction q . The curve AA' or BB' ... corresponds to the initial yielding and $A'A'''$ or $B'B'''$ ... to the progressive yielding.

The minimum cross section of specimen was adopted for calculation of σ_t .

The initial yielding is sensitive to the stretching speed and the faster the stretching speed becomes, the higher the yield stress appears. In the initial yielding, σ_t - q curve is straight, and the end point A' or B' ... corresponds to the point ② in Fig. 1.

Up to the point ③ in Fig. 1, the load is nearly constant and the variation of strain is comparatively small. The point ③ corresponds to A'', B'' ... in Fig. 3. Both points A' and A'' or B' and B'' ... are situated closely in the diagram, and it may be considered that the stress in this region is almost constant. But for the end of the stretching, that is, beyond point ③ in Fig. 1, considerable changes of true stress due to area contraction are seen as in $A''A'''$ or $B''B'''$.

Fig. 4 (a) and (b) show the specimens which were pulled up to an equal length, but by different stretching speeds. The larger contraction of cross section is caused by the higher stretching speed, and in such a case large plastic deformation is propagated in the shorter portion of the specimen. Fig. 4 (c) shows the specimen where the stretching was stopped in the way of the deformation, and after a few minutes' rest, the restretching was performed under progressive yielding. In this case, a knob is formed at every halt of the stretching. The stress-strain curve in this case is shown in Fig. 5. During the stopping of stretching, the plastically deformed part of material becomes harder and the heat produced by deformations is dissipated. It seems that the working zone (b) in Fig. 5 is deformed plastically in the primary stretching, but the work hardening prevents it from the further deformation and it remains as a knob.

The front of the plastic strain wave is the so-called "working zone" and change from elastic part to plastic one is caused under an almost constant load. The strain distribution in the working zone is shown in Fig. 6. The "working length" is longer as the stretching speed becomes faster. The next relations exist in the working zone.

Fig. 5 Stress and strain curve and profile of specimen where the stretching was stopped in the way of the deformation

Fig. 6 Strain distribution in the working zone

Fig. 7 σ_t - q and σ_t - ϵ curves in the working zone, that is, in the progressive yielding (25°C)

Fig. 8 Schematic representation of σ_t - q relation

Table 1 Yield point elongation of thin silicon steel plate
Width 16 mm, Thickness 0.35 mm, Gage length 76 mm (J. Miklowitz)

Stretching speed	in/min	8.6×10^{-5}	2.56×10^{-2}	6.16×10^{-1}
Working length	mm	8	10	17
Yield point elongation	%	1.4	1.8	2.8

$$\sigma_t = P_0/A = P_0(1+\epsilon)/A_0 = \sigma_0(1+\epsilon) \quad \text{or} \quad \sigma_t = \sigma_0/(1-q) \quad \dots\dots(1)$$

where, σ_t : true stress, P_0 : tensile load, A : true area of cross section, A_0 : area of initial cross section, σ_0 : conventional stress, q : area contraction ratio.

Fig. 7 shows σ_t - q curves in the working zone. Since the progressive yielding is caused in the working zone, it also corresponds to σ_t - q curve in the progressive yielding, and σ_t - ϵ curves are added there. Those curves for this case are affected slightly by the stretching speed. It requires the greater stress to get an equal strain under the yielding by the smaller stretching speed. Fig. 8 shows the schematic representation obtained by combining Fig. 3 with Fig. 7. The breaking true stress is $\sigma_t = 1100 \sim 1200 \text{ kg/cm}^2$. The contraction of area at the breaking point or the breaking strain at the minimum cross section increases with the increase of stretching speed. However, on the contrary, the breaking elongation decreases according as the plastic deformed portion becomes shorter. The true stress σ_t for the same contraction q is influenced by the temperature. Both stresses in the initial and the progressive yielding become larger and the area contraction ratio q at the breaking point becomes smaller, as the temperature is decreased.

3. Comparison of both yielding phenomena of P. V. C. and mild steel

For mild steel, the upper yield point is not stable and has not always a constant value, but the lower yield stress has almost a fixed value. At the lower yield point, the yielded portion is mixed with elastic and plastic parts, and the neighbouring part is successively yielded under the constant load.

J. Miklowitz⁽³⁾ observed the working zone in a thin silicon steel specimen and measured its length and the elongation at the yield point. The results are shown in Table 1. The working

Fig. 9 Relation between strain velocity and yield strain of 0.04% carbon steel (J. Winlock and R. W. Leiter)

Fig. 10 Relation between stretching speed and yield strain of unplasticized P. V. C. (25°C)

length and elongation at the yield point vary their values with various stretching speeds. Such a behavior is also seen in the case of unplasticized P. V. C.. J. Winlock and R. W. Leiter⁽⁴⁾ also re-

ported the relation between the rate of strain and the elongation at the yield point as rearranged in Fig. 9. In the case of unplasticized P. V. C., the relation between the yield point elongation and the stretching speed is similar to that as seen in Fig. 10, where the elongation at the end of the initial yielding is adopted as the yield point elongation.

Although the molecular mechanism causing the discontinuity of the stress-strain curve at the yield point in one material is very different from that of the other, the yielding mechanism may be considered as similar.

4. The propagation of plastic wave front

As described in the above articles, the plastic elongation in the tension test results from the movement of the working zone.

When a specimen is pulled by the constant speed v_{st} , the spreading speed of the working zone, i.e. the speed of propagation of plastic wave front c , may be obtained by measuring the length of elastically deformed part of the specimen.

According to the movement of the working zone, the elastic portion with strain ϵ_0 is deformed plastically up to the strain ϵ_1 as shown in Fig. 11.

Now, we are considering the transition from the elastic part to the plastic one. Therefore, we adopt an initial value in the necked part or that in the bottom of the working zone, as ϵ_1 notwithstanding the fact that the strain of already necked part would be slightly increased during further extension.

The value of this strain ϵ_1 was almost constant throughout the progressive yielding. Now, let the elongation caused in time dt be Δl .

If we express the distance of the plastic wave front at any time from the initial position by x_1 , the following relations can be obtained in the tension test with constant stretching speed.

$$\int_0^{x_1} \epsilon_1 dx = \Delta l = \int_0^T v_{st} dt \quad \dots\dots\dots (2)$$

then

$$\epsilon_1 x_1 = \Delta l = v_{st} \cdot T \quad \dots\dots\dots (3)$$

$$v_{st} = \epsilon_1 \cdot x_1 / T = \epsilon_1 \cdot c \quad \dots\dots\dots (4)$$

Fig. 11

Fig. 12 Δl - x - t diagram (25°C)

Eq. (4) represents the relation between the stretching speed v_{st} , the yield strain ϵ_1 and the velocity of propagation of plastic wave front c .

Fig. 12 shows the relations between two of the following variables, that is, the moved distance of plastic wave front x , the total elongation Δl and time t .

The load P vs. elongation Δl curve is added also in Fig. 12. The measurement of x is performed by photographing the specimen.

In this diagram, the inclination of Δl vs. x curve corresponds to the plastic strain ϵ_1 , and that of Δl vs. t curve to the stretching speed v_{st} , and that of x vs. t curve to the velocity of propagation of plastic wave front c .

The value of ϵ_1 varies rapidly its value until the end of the initial yield, and thereafter, the variation of ϵ_1 in the progressive yielding does no more change than that of initial yielding.

Table 2 Relation $v_{st} = c \cdot \epsilon_1$ at the progressive yielding

Stretching speed v_{st} mm/min	Propagation speed of plastic strain wave c mm/min	Plastic strain ϵ_1	$c \times \epsilon_1$ mm/min
1.2	0.65	1.9	1.23
6.0	3.1	1.9	5.9
9.0	4.0	2.5	10.0

Therefore, we adopted the mean value of ϵ_1 in the progressive yielding as the measure of plastic strain, and confirmed the establishment of Eq. (4) as shown in Table 2.

We may write $dx=c \, dt$. Then

$$\dot{\epsilon}=d\epsilon/dt=c \cdot d\epsilon/dx \dots\dots\dots (5)$$

where, $d\epsilon/dx$ is constant and $\dot{\epsilon}$ is proportional to the velocity of plastic wave front c .

5. Conclusions

We recognized that there are two steps in the yielding of unplasticized P.V.C., that is, the "initial yielding" and the "progressive yielding", in this paper.

And these two steps may be considered as analogous to the upper and lower yield point of mild steel.

They are remarkably affected by the stretch-

ing speed. We also established a relation between the stretching speed v_{st} , the yield strain ϵ_1 and the velocity of propagation of plastic wave front c , as $v_{st}=c \cdot \epsilon_1$.

These analogies may give us some suggestions in considering the yielding phenomena of mild steel.

This paper is concerned with unplasticized P. V. C., but even to thermoplastics such as polyethylene and nylon, the same treatment may be applied.

References

- (1) Y. Nakagawa and S. Okuda: *Jour. Japan Soc. Testing Materials*, Vol. 5, No. 34 (1956), p. 404.
- (2) Y. Nakagawa and S. Okuda: *Jour. Japan Soc. Testing Materials*, Vol. 5, No. 36 (1956), p. 524.
- (3) J. Miklowitz: *Jour. Appl. Mech.*, Vol. 14 (1947), p.A-31.
- (4) J. Winlock and R. W. Leiter: *Trans. ASM*, Vol. 25 (1937), p. 163.

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Effect of Stress Reduction on Secondary Creep of Mild Steel*

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The transient behavior of 0.14 per cent carbon steel, which appeared by a partial removal of applied stress at the stage of secondary creep, was studied at 450°C. The strain rate after the partial reduction of stress was found smaller than the true steady creep rate for the reduced stress, however it gradually increased with time and appeared to tend to the steady creep rate. It was also found that the higher was the applied stress before the stress reduction, the lower rate of strain appeared after the reduction at a given strain and the longer period was necessary before the strain rate reached the steady creep rate.

These behaviors were ascertained to be well interpreted by the authors' theory of the plastic deformation of metals at high temperatures.

1. Introduction

The authors have reported the effect of stress change on the creep in primary stage of mild steel⁽¹⁾⁽²⁾. It was found that the strain rate after the change in stress at a given strain was higher when the stress was increased, and lower

when the stress was decreased, as compared with the strain rate in the reference test, i.e. the creep test under the constant stress, the magnitude of which is the same as that of stress after the change in stress. In other words, the behaviors just after the stress change did not obey the law of the mechanical equation of state in solid⁽³⁾. However it was also observed that the strain rate after the stress change drew closer gradually with increase in strain (or with lapse of time) to that in the reference test, and came to obey the law of the mechanical equation of state.

These phenomena were well interpreted by the authors employing the idea of change in inter-

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