A Dynamic Theory of Piston-Ring Lubrication

(1st Report, Calculation)*

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Near the TDC of the expansion stroke, the piston-ring speed is 0 and its load and temperature reach the maximum. If it were based on the steady-flow-hydrodynamics, the oil-film between the ring and the cylinder would have to be broken down. In practice, however, the fluid-film lubrication is achieved in the high speed region. These phenomena can be explained by a dynamic theory of the hydrodynamics, the piston-ring speed and load vary so extremely that the oil-film thickness changes rapidly, and this rate of change in the oil-film thickness generates a load carrying capacity by the squeeze action. This paper introduces the equations of the above-mentioned theory, and some calculations are carried out for a ring, the surface profile of which consists of the quadratic and the parallel line.

1. Introduction

A striking character of piston-ring lubrication in comparison to general bearing lubrication is that its speed and load change extremely, especially near the TDC of the expansion stroke of the engine, where the piston-ring speed is 0 and its load and temperature reach the maximum, so the critical state in maintenance of the oil-film occurs there. As both the speed and the load vary largely, an instantaneous state of lubrication cannot be studied by the steady-flow hydrodynamics but by a dynamic theory. Now, let us consider that the ring shifts from the state of high speed and light load (the oil-film is thick) to low speed and heavy load, more especially, the critical one that the speed is 0. If it shifted slowly, a load carrying capacity would evidently become 0 namely the oil-film thickness was 0 at the time when the speed was 0. But when the change is very rapid the decreasing rate of the oil-film thickness reaches a considerable value, so the load carrying capacity is generated by the squeeze action even at the critical time, this effect will here be called the "dynamic effect". The author is going to give the explanation how he obtained the theoretical expressions and some examples of numerical calculation in this report, and its experimental proof in the following one.

2. General equations

Now, let us consider Fig. 1 in which the ring loading \( W(t) \) per unit length (in \( x \) direction) is stationary, the cylinder is moving at velocity \( U(t) \) in \( x \) direction, and space between the ring and the cylinder is filled with lubricant of constant viscosity \( \mu \). Neglecting the oil flow in \( z \) direction and following the hydrodynamic theory of the viscous fluid, we obtain

\[
\frac{\partial^2 u}{\partial y^2} = \frac{1}{2\mu} \frac{\partial p}{\partial x} \tag{1}
\]

Integrating Eq. (1), we obtain the velocity distribution

\[
u = \frac{1}{2\mu} \frac{\partial p}{\partial x} (y-h) \tag{2}
\]

From the incompressibility,

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}
\]

From Eqs. (2) and (3), we have

![Fig. 1](image-url)
\[
\begin{align*}
\frac{\partial v}{\partial y} &= -\frac{\partial}{\partial x} \left\{ \frac{\partial p}{\partial x} y(y-h)/2\mu + \frac{\partial}{\partial x} \{ U(t)(h-y)/h \} \right\} \\
\left. v \right|_{y=0} &= V = \frac{\partial h}{\partial t} = -\int_0^h \frac{\partial}{\partial x} \left\{ \frac{\partial p}{\partial x} y(y-h)/2\mu \right\} dy - \int_0^h \frac{\partial}{\partial x} \{ U(t)(h-y)/h \} dy \\
\therefore \quad \frac{\partial p}{\partial x} &= 6\mu U(t)/h^2 + \left\{ \int \frac{\partial h}{\partial t} \, dx + c_1 \right\} 12\mu /h^3
\end{align*}
\]

Assuming that the piston-ring motion is in \( y \) direction only, so \( \partial h/\partial t \) is independent of \( x \), we get

\[
\frac{\partial v}{\partial x} = 6\mu U(t)/h^2 + \left\{ \frac{\partial h}{\partial t} x + c_1 \right\} 12\mu /h^3 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (4)
\]

Pressure distribution in the oil-film is obtained by Eq. (4) as follows:

\[
p = 6\mu U(t) \int \frac{x}{h^3} \, dx + 2\frac{\partial h}{\partial t} \int \frac{1}{h^3} \, dx + c_1 \int \frac{x}{h^3} \, dx + c_2
\]

Deciding \( c_1 \) and \( c_2 \) on the boundary conditions of \( p=0 \) at \( x=0 \) and \( x=B \),

\[
p = 6\mu U(t) \left\{ F(x) \frac{G(x)}{G(B)} - H(x) \right\} - 12\mu \frac{\partial h}{\partial t} \left\{ \frac{H(B)}{G(B)} - G(x) \right\}
\]

where

\[
\int_0^B \frac{1}{h^3} \, dx = F(x), \quad \int_0^B \frac{x}{h^3} \, dx = G(x), \quad \int_0^B \frac{x}{h^3} \, dx = H(x)
\]

We can find the load carrying capacity per unit length by Eq. (6)

\[
W(t) = \int_0^B p \, dx = 6\mu U(t) \left\{ [F] - [G] F(B)/G(B) - 12\mu \frac{\partial h}{\partial t} \left\{ \frac{H(B)}{G(B)} [G] - [H] \right\} \right\}
\]

where

\[
\int_0^B F(x) \, dx = [F], \quad \int_0^B G(x) \, dx = [G], \quad \int_0^B H(x) \, dx = [H]
\]

And the frictional force per unit length is obtained by integrating the shearing stress on the surface of \( y=0 \),

\[
R = \int_0^B \tau \, dx = \mu U(t) \left\{ 4D(B) - 3 F(B)^2 G(B) \right\} - 6\mu \frac{\partial h}{\partial t} \left\{ F(B) H(B) - E(B) \right\}
\]

where

\[
\int_0^B \frac{1}{h} \, dx = D(B), \quad \int_0^B \frac{x}{h^3} \, dx = E(B)
\]

Eq. (7) consists of two terms, the first being the wedge action term, and the second the squeeze action term, namely, the load carrying capacity by the dynamic effect.

3. Procedure of calculation

To solve Eqs. (7) and (8) by numerical integration, these are rewritten as follows,

\[
W(t) = 6\mu U(t) X - 12\mu \frac{\partial h}{\partial t} Y \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (7)'
\]

\[
R = \mu U(t) V - 6\mu \frac{\partial h}{\partial t} Z \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (8)'
\]

where each of \( X, Y, V \) and \( Z \) is dimensionless and is a function of the oil-film thickness and its form, so when the profile of the ring surface is given, \( X, Y, V \) and \( Z \) can be obtained with regard to the \( h_0 \).

From Eq. (7)'

\[
\Delta h = (U(t) X - W(t) / 6\mu) \Delta t / 2Y \quad \quad \quad \quad \quad \quad (9)
\]

As the starting point of the calculation of Eq. (9), nearly the middle point of the stroke must be selected because the velocity transition at this point is small and the \( h_0 \) here is obtained by the stationary method, that is, by

\[
X = W(t) / 6\mu U(t) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (10)
\]

as \( X \) is a function of \( h_0 \) only.

Thereby we found the process of change of the oil-film thickness, so \( V \) and \( Z \) at any moment, and so \( R \), moreover the coefficient of friction

\[
f = R/W \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (11)
\]

are obtained.

4. Law of similarity

When the piston-ring has the same size and the same profile, in a stationary state the oil-film thickness and the coefficient of friction are decided only by Eqs. (10) and (11), in a dynamic state, however, we cannot find them so easily. From Eq. (9)

\[
\Delta h = \frac{1}{2Y} \left\{ [6\mu U(t) X / W] - 1 \right\} W \Delta t \quad \quad \quad \quad \quad \quad (12)
\]

From Eqs. (7)' and (8)' we have
\[ f = \frac{\mu_U(t)}{W(t)} Y \left( \frac{3\mu_U(t)}{W(t)} - \frac{X-2}{Y} \right) \] (13)

From these equations, the following remarks can be made: (1) if \( \mu_U(t)/W(t) \) and \( W(t)/\mu \) are the same at every moment, the process of the oil-film thickness is the same; (2) when the stroke \( s \) and the connecting rod ratio \( A \) are given, that process is fixed with \( \mu_U(t)/W(t) \) (\( U_{mean} \) denotes the mean piston speed), in substitution for \( \mu_U(t)/W(t) \), also \( U_{mean} \) is in proportion to \( 1/\dot{A} \). After all, if \( \mu_U(t)/W(t) \) is the same the process of \( h \) is the same; however, if \( s \) or \( A \) is different, this law of similarity does not hold; (3) when the process of \( h \) is the same, that of the coefficient of friction is the same.

5. Form of the oil-film

In general, the surface of the long used ring

\[ X = \frac{rB^2}{2h_2^2} \left[ \left( 1-r \right) \left( 1+3r \right) + 3r^2 - 2 + m \right] \]

\[ Y = \frac{B^2}{48h_2^2} \left[ 2r^4 (13-28\lambda + 8\lambda^2 + 16\lambda^3) + 16r^4 \lambda^2 (5-8\lambda) ight] 
+ 48r\lambda^2 (1-5\lambda^2 + 4\lambda^3) - 16r (2 + 11\lambda^2 + 8\lambda^3) + 32\lambda (\lambda - 1) + 3m (r^2 - 2 + 15\lambda - 33\lambda^2 + 8\lambda^3) 
+ r^2 (-6 - 32 + 25\lambda^2) - 24r^2 (\lambda - 1) + 16r^2 (\lambda - 1) + 9r^2 3m^2 (r^2 - 5\lambda^2 + 3\lambda^2 - 3) \]

\[ V = \frac{2B}{h^2} \left[ 2(1 - r + \mu m) - 3(2\lambda^2 - 2\mu^2 + r + \mu m)^2 \right] 
+ 8\lambda^2 (1-r) + r (2 + 3\lambda^2 - 3\mu^2 m) \]

\[ Z = X \] (17)

For example, the ring \( I \) used for the experiment has approximately \( r = 0.5 \), \( h_1 - h_2 = 0.002 \, \text{mm} \), \( B = 2 \, \text{mm} \); in that case \( X, Y, V \) and \( Z \) to \( h_2 \) are drawn as a logarithmic graph Fig. 2.

6. Examples of calculations

Because the ring of the experimental equipment in the author's next report has the above-mentioned profile, the width \( B = 3 \, \text{mm} \), the elastic tension \( P = 1 \, \text{kg/cm}^2 \) and the crank mechanism has \( S = 90 \, \text{mm} \). \( A = 4.05 \). The author carried out his calculation under these conditions. So the load per unit length is

\[ W(t) = B(P_t + P) = 0.3 (1 + P) \, \text{kg/cm} \] (19)

where \( P \) is the pressure by which the ring is pressed on the cylinder, i.e. back pressure.

A. Case of the constant load

The transition of the oil-film thickness is shown in Fig. 3 on the condition of the constant load \( W = 6.3 \, \text{kg/cm} (P = 20 \, \text{kg/cm}^2) \), the dotted lines showing the stationary value corresponding to velocity at every moment by Eq. (10). It is evident that the dynamic effect due to change of velocity grows largest near the TDC and the BDC, and \( h_{2,\text{min}} \) is thicker with faster revolution. Since acceleration is larger at the TDC than at the BDC, \( h_{2,\text{min}} \) is thicker at the TDC than at the BDC. The variation of the coefficient of friction in the above case is shown in Fig. 4. The dynamic effect on the coefficient of friction is not so remarkable as that on the oil-film thickness. Fig. 5 and 6 show the effect of the load, and Fig. 7 shows the influence of change of the temperature on the viscosity of the lubricant oil. Fig. 8 shows the effect of \( \mu_U(t)/W(t) \) on \( h_2 \), which is explained in 4th section. Influence of change of the stroke is indicated has the profile that is almost symmetrical from left to right being flat in the middle part and rounded off at both ends.

Let us assume that the profile consists of the quadratic and the parallel straight line as shown in Fig. 1, and the effective profile is 1, 2 to 3, namely the part 3 to 4 is left out of consideration, but the back pressure is loaded all over \( B \). Then we can express the oil-film thickness, part 1 to 2 as

\[ h = h_1 \frac{2(h_1 - h_2)}{a} x + h_1 - h_2 \]

\[ a = \frac{1}{\sqrt{\lambda - 1}} \]

Putting

\[ h_1 = \lambda h_2, \quad a = rB, \quad m = \frac{1}{\sqrt{\lambda - 1}} \tan^{-1}(-\sqrt{\lambda - 1}) \]

\( X, Y, V \) and \( Z \) corresponding to the oil-film having such a profile will be given by

\[ B = 2, \quad h_1 - h_2 = 0.002 \, \text{mm} \]

\[ r = 0.5 \]

Fig. 2
in Fig. 9, where $U_{\text{mean}}=9\text{ m/sec}$, $\mu U(t)/W(t)$ of each one are constant, and the stroke and number of rotation are varied. It tells that the higher the revolutions is, the more noticeably the dynamic effect appears.

**B. Case in which velocity and load both change**

The method and conditions of calculation in this case are the same as the previous section and the only difference is that $W(t)$ varies.

When the transition of $P$ in Eq. (19) is taken as equal to shown in Fig. 10, which is the one in the indicator diagram of a gasoline engine. Fig. 11 gives one of the results of the calculations, Figs. 12 and 13 show the transition of the oil-film thickness and the frictional force respectively when the phase of the pressure diagram is put forward by 25° from that in Fig. 10 and stands at +25°, 0° and -25°. It becomes evident from these results that the dynamic effect by variation of load is

**Table 1**

<table>
<thead>
<tr>
<th>Period of $P_{\text{max}}$ (after TDC)</th>
<th>r.p.m.</th>
<th>Press. at TDC $P_r\text{ kg/cm}^2$</th>
<th>$\beta_2\text{ min}$</th>
<th>$1/1000\text{ mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+25^\circ$</td>
<td>1 000</td>
<td>12.4</td>
<td>1.83</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>3 000</td>
<td></td>
<td>3.30</td>
<td>3.38</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>1 000</td>
<td>20.0</td>
<td>1.46</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>3 000</td>
<td></td>
<td>2.76</td>
<td>2.60</td>
</tr>
<tr>
<td>$-10^\circ$</td>
<td>1 000</td>
<td>18.5</td>
<td>1.38</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>3 000</td>
<td></td>
<td>2.65</td>
<td>2.74</td>
</tr>
<tr>
<td>$-25^\circ$</td>
<td>1 000</td>
<td>15.7</td>
<td>1.41</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>3 000</td>
<td></td>
<td>2.76</td>
<td>3.60</td>
</tr>
</tbody>
</table>

Fig. 4 Transition of the coefficient of friction, constant load $W=6.3\text{ kg/cm}$ ($P=20\text{ kg/cm}^2$), $\mu=20.5\times10^{-6}\text{ kg/s/cm}^2$

Fig. 5 Effect of the load on the oil-film thickness $h_2$, $\mu=20.5\times10^{-6}\text{ kg/s/cm}^2$, $n=3000\text{ rpm}$

Fig. 3 Effect of the number of revolution on the oil film thickness, the constant load acting $W=6.3\text{ kg/cm}$, $\mu=20.5\times10^{-6}\text{ kg/s/cm}^2(80^\circ\text{C oil})$
Fig. 6 Effect of the load on the frictional force,
μ=20.5×10^-8 kg·s/cm², n=3 000 rpm

Fig. 7 Effect of the oil temperature on hₙ,
P=20 kg/cm², n=3 000 rpm.
Oil viscosity μ=150.7 ×10^-8 kg·s/cm² at 40°C
" 20.5 " 4.1 " 140°C

Fig. 8 Relation of μU/W and hₙ, when steady
load acts, B=2 mm, h₁-hₙ=0.0026 mm, r=0.5.
S=90 mm, and A=4.05

Fig. 9 When μU/W is constant and stroke S
varies, P=20 kg/cm², μ=20.5×10^-8 kg·s/cm²

Fig. 10 Diagram of pressure P and
velocity U
much smaller than that by variation of velocity in the case of engine. In order to make it more clear the author compared in Table 1, the values of $h_{2\text{min}}$ under the changeable load and those under constant load acting through the whole stroke whose value equals the load at the TDC under changeable conditions.

7. Conclusions

From these results the author obtained the following conclusions: (1) The piston-ring lubrication is explainable on the basis of not the stationary theory but the dynamic one of hydrodynamics. (2) The dynamic effect based on variation of velocity is specially large. (3) But that on the variation of load is comparatively small, so $h_{2\text{min}}$ in changeable load nearly equals $h_{2\text{min}}$ in stationary load which is equivalent to that at the TDC in changeable load. (4) When the stroke and $A$ are given and $\mu U/W$ is the same, the characteristics of lubrication is the same even by the dynamic theory. (5) In the case that $\mu U/W$ is the same and the stroke is different, the larger the number of rotation, the more remarkable the dynamic effect becomes.