532. 517 : 533. 691. 011. 3:621-226. 2

On Wakes behind a Single Aerofoil and Cascade*

By Yasuo MORI**

In an axial turbo-machinery, many cascades rotate in the downstream of the preceding cascade. The flow vector variation in the exit of the preceding cascade displays a marked variation of static pressure and flow pattern. It also brings about fatigue of cascade blades, generates noise and becomes a cause of losses. In order to predict and control losses in a cascade, it is necessary to analyse wake characteristics of a cascade and the performance of a cascade in non-steady flow. As a first step in the study of unsteady flow through or in a cascade, our experiment is concerned with wakes behind a two-dimensional single aerofoil and cascade. Experiments were made about wakes in transition and turbulent regions behind a single aerofoil and cascade of chord 30 mm and aspect ratio 3. The mean velocity and velocity fluctuation were measured by a Pitot tube and a hot-wire anemometer. In the transition region, it was shown that a Tollmien-wave interacts with the organ-type acoustic vibration of cascade, and considerable static pressure variation exists in the wake. In the turbulent region, the wake flow is self-preserving even in the immediate neighbourhood of the transition region.

1. Introduction

In an axial turbo-machinery, many cascades rotate in the downstream of the preceding cascades. The flow vector variation in the exit of the preceding cascade displays a marked variation of static pressure and flow pattern. It also brings about fatigue failure of cascade blades, generates noise and becomes a cause of losses. In such a type of cascade as in a turbine where the flow deflection is large, the non-steady phenomena may be explainable by potential field and was theoretically discussed by W. Isay⁽¹⁾. In the other type of cascade as in an axial compressor the deflection of the flow is small and the wake and viscosity phenomena are expected to exert a potent influence. Our attention has been focussed to an unsteady flow through a cascade with small flow deflection.

Consider a simple flow pattern in this kind of cascade. At its exit there is no variation in the flow direction and only the exit velocity changes from maximum to minimum. When the following cascade moves relatively to this preceding cascade, the relative velocity vector varies in its magnitude and direction. In order to predict and control losses in a cascade row with small flow deflection,

it is necessary to analyse the wake characteristics of a cascade and the performance of a cascade in non-steady flow. The first necessary step in such analysis is the determination of the mixing phenomena in the downstream of a cascade. The wake behind a circular cylinder was investigated in many reports, but detailed investigations about the wakes behind a single aerofoil or cascade have been scarcely known.

As a first step in the study of an unsteady flow through or in a cascade, our experiment is concerned with wakes behind a two-dimensional single aerofoil and cascade.

2. Apparatus and procedure

Experiments were made in a 9 cm × 30 cm low turbulent wind tunnel at the Tokyo Institute of Technology which is shown in Fig. 1. Wind speed at the test section was changed from 1 m/sec to 40 m/sec. Longitudinal free stream turbulence level was

Fig. 1 Wind-tunnel

* Received 29th September, 1958.

** Assistant Professor, Tokyo Institute of Technology, Meguro-ku, Tokyo.

measured by a hot wire anemometer and the results are shown in Fig. 2 where U_0 is the main stream velocity and u' the fluctuating velocity.

The symmetrical aerofoil of NACA 65 series was adopted, its chord being 30mm. The aspect ratio was 3 and the pitch chord ratio of the cascade 1. In the test section, the side walls were removed and the parallel top and bottom walls were 200mm long from the aerofoil trailing edge as shown in Fig. 3. At first the top and bottom walls were not installed, but we found that without these walls the two-dimensional character of the flow could not be maintained as S. Corrsin pointed out in his report⁽²⁾ about two-dimensional jets.

The total pressure was measured by a flattened Pitot tube of about 0.2mm opening and the static pressure by a 1mm diameter static tube with 0.3mm diameter pressure hole. The mean velocity was calculated from the total and static pressure readings. A hot wire anemometer was used for velocity fluctuation measurement. Platinum wire, 5 microns in diameter and 3mm length, was used for this purpose. The output of the hot wire was amplified by a conventional compensated amplifying circuit⁽³⁾ as shown in Fig. 4 with a flat frequency response from 20 c/sec to 15kc and con-

tinuously recorded by a cathode ray oscillograph.

The hot wire and Pitot tube were moved in x (flow direction), y (perpendicular to flow direction) and z (span direction). The y and z positions were determined by a precision dial-gauge and x position by callipers.

3. Results and discussions

a) Two-dimensionality and boundary layer around aerofoil

At first, the two-dimensionality of the flow was studied. The displacement thickness δ of the boundary layer on the top and bottom walls was measured and is shown in Fig. 5. It is about 0.8mm and increases by 0.4mm till the end of the test section. Therefore the flow was considered to be absolutely uniform and two-dimensional in the 90 percent region of the span. In order to determine the velocity profile of the boundary layer around an aerofoil, the mean velocity distribution at the trailing edge of an aerofoil was measured. The Reynolds number of our experiment covered the region from 10^4 to 7×10^4 . When c is the distance along the surface from the leading edge to the trailing one, U_0 the main stream velocity, ν the kinematic viscosity and $\eta = y/\sqrt{\nu c/U_0}$ a non-

Fig. 2 Turbulent intensity of main stream

Fig. 3 Schematic view of test section

Fig. 4 Amplifier of hot wire anemometer

dimensional parameter, the measured velocity profile is plotted in Fig. 6. It was found that the Blasius-type velocity profile⁽⁴⁾ was established around a single aerofoil. This seems to be caused by the small thickness chord ratio and negligible static pressure variation along the aerofoil. It was also true for each aerofoil of a cascade when the velocity at the middle point of the neighbouring aerofoil was taken in place of the main flow velocity in the Blasius formula and the data concerning the cascade were also plotted in Fig. 6. The boundary layers around an aerofoil developed into a laminar wake, but the laminar wake was found to exist only in a narrow extent and to immediately transit to a turbulent wake. Therefore a wake is approximately divided into the wake in a transition region and that in a turbulent region.

b) Transition region

The wake remained laminar over a certain distance just downstream from the trailing edge, but in several millimeters downstream from it, a velocity fluctuation was observed and it developed

rapidly. This velocity fluctuation was a regular sinusoidal wave as shown in Fig. 7. The frequency of the Kármán vortex street is proportional to the flow velocity, while the observed frequency increased regularly proportionally to the 3/2 power of the flow velocity. The results are plotted in Fig. 8 where f is the frequency of the sinusoidal wave and U_0 the main flow velocity. The turbulent in the main stream was low, but the sinusoidal fluctuation level reached several percent of the main mean velocity energy. This sinusoidal wave showed the same properties as those in a transition of laminar half jet which was reported lately by H. Sato⁽⁵⁾. Considering that the wake had the free boundary as a jet flow, the same non-dimensional expression of the frequency of the sinusoidal wave in a half jet was adopted. Fig. 9 is the non-dimensional plot of the sinusoidal fluctuation, where θ_0 is the momentum thickness at the trailing edge. $2\pi f\theta_0/U_0$ was about 0.09 and was independent of the boundary layer Reynolds number $U_0\theta_0/\nu$. These results coincided with the experimental ones about a laminar half jet by H. Sato. They also showed that the laminar wake

Fig. 5 Displacement thickness along the top and bottom walls

Fig. 6 Velocity profile at the trailing edge of aerofoil

Time mark: 0.02 sec
Fig. 7 Transition wave

Fig. 8 Relation between stream velocity and frequency of transition wave

did not exist longer and rapidly changed to a turbulent wake. The relation between the frequency of the transition wave and the stream velocity for a cascade wake is the same as that for the wake of a single aerofoil. When increasing the stream velocity in a cascade experiment, strong sound was generated at the stream velocity of 25 m/sec, while in a single aerofoil experiment such strong sound could not be detected at any stream velocity. This strong sound in the cascade experiment was picked up by a microphone placed outside the flow and observed on a cathode ray oscillograph. It had sinusoidal form of almost constant amplitude and the relation between its frequency and stream velocity is plotted in Fig. 8 against that of the transition wave. With the stream velocity increasing, the frequency of the transition wave of the cascade wake increased and at above 22 m/sec a strong sound of the constant frequency 3.9 kc was observed. From 25 m/sec to 30 m/sec the frequency of the transition wave was 3.9 kc and remained constant. At above 30 m/sec it again increased with the $3/2$ power of the stream velocity and no strong sound was detected. The cascade was found to have a resonance frequency of 3.9 kc. The acoustic resonance frequency of the rectangular pipe of the open ends, which consisted of two neighbouring aerofoils and end walls, was calculated by the following formula,

$$f = \lambda a / 2\pi l$$

where l is the chord length corrected for the open-end, λ a non-dimensional parameter expressing the end condition and the mode of vibration and the acoustic velocity. The calculated value of f for our experiment is 3.8 kc and coincides with the observed frequency. As mentioned above, the generated sound in the cascade experiment results from the resonance of the transition wave and the acoustic vibration. At below 25 m/sec, the latter is excited by the former, while at above 25 m/sec vice versa. In a cascade experiments or turbo-machineries,

Fig. 9 Relation between frequency of the transition wave and Reynolds number

when the boundary layers around aerofoils are laminar, it is considered that sound is often generated by the transition wave as discussed above.

Another interesting finding in this transition region of a laminar wake was the existence of remarkable static pressure variation. The static pressure distribution is shown in Fig. 10 where Δp_s is the static pressure departure from that of the main flow and q the dynamic head of the main flow. The static pressure in the close downstream of the trailing edge increased to a high level, that is, to about several percent of the main stream velocity head, then the static pressure in the center part of the wake decreased so that the static pressure defect appeared. The static pressure variation existed only in the transition region and disappeared with a fully turbulent wake developing. The relation between the sinusoidal wave and this static pressure variation must be studied further in future. These phenomena in cascade wakes were the same as those in a wake of a single aerofoil.

The distribution of the mean velocity defect $(U_0 - U)/(U_0 - U_m)$ is non-dimensionally plotted in Fig. 11 using the width b of the wake at the half value of the maximum velocity defect. The solid line in Fig. 11 is the calculated one for the fully turbulent wake, because it is hardly possible to calculate the mean velocity profile in the transition region due to the lack of informations about

Fig. 10 Static pressure distribution

Reynolds stress, etc. The velocity defect against y/b in the transition region has almost the same distribution as that of the fully turbulent wake, but a slight departure from it is seen at larger y/b as expected.

c) Fully turbulent region

In the downstream of the transition region, the fully turbulent wake was observed. It is well known that the mean velocity defect, wake width and Reynolds stresses can be non-dimensionally plotted for the fully turbulent and self-preserving wake behind a circular cylinder. For the fully turbulent wake behind an aerofoil its thickness can not be adopted in place of a cylinder diameter d . In a self-preserving wake, the constant hydrodynamic length is the momentum thickness as long as the static pressure remains invariable in the flow field. The variation of the momentum thickness of wakes of a cascade is shown in Fig. 12. For $x < 75$ mm the flow is in the transition region the extent of which is shown in Fig. 13; while for $x > 150$ mm the thickness increases because of the contamination of the test section boundaries.

The variations of the minimum velocity and the wake width against x are shown in Fig. 14, where b is the width at the half value of the mean velocity defect. It is seen that there exists a

Fig. 11 Velocity distribution in transition region

Fig. 12 Variation of momentum thickness and static pressure

distinct divergence among those variations in the turbulent region, and this seems to be caused by the differences in the extent of the transition region and the rapid attenuation of the mean velocity defect in that region.

The equation of motion about the mean flow of the turbulent wake can be given

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + \frac{\partial \bar{u} \bar{v}}{\partial y} + \frac{\partial}{\partial x} (\bar{u}^2 - \bar{v}^2) = \nu \frac{\partial^2 U}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1)$$

where U and V are the mean velocities along x and y axis, u and v the turbulent velocities, p the pressure, ρ the density, and ν the kinematic viscosity.

Among these terms in Eq. (1), $V \frac{\partial U}{\partial y}$, $\frac{\partial}{\partial x} (\bar{u}^2 - \bar{v}^2)$,

$\nu \frac{\partial^2 U}{\partial y^2}$ and $\partial p / \partial x$ are negligible, so by introducing $u_1 = U_0 - U$ and the turbulent viscosity κ defined by $\bar{u} \bar{v} = \kappa \frac{\partial U}{\partial y}$, Eq. (1) can be written as:

$$U_0 \frac{\partial u_1}{\partial x} = \kappa \frac{\partial^2 u_1}{\partial y^2} \quad (2)$$

where U_0 is the main stream velocity.

The turbulent region does not have its origin at $x=0$, but its virtual origin can be represented by x_t . When θ is the momentum thickness of the turbulent region, the solution of Eq. (2) becomes

$$u_1 = c_1 \{ (x - x_t) / \theta \}^{1/2} \exp \{ -U_0^2 y^2 / 4 (x - x_t) \kappa \} \quad (3)$$

Fig. 13 Extent of transition region

Fig. 14 Variation of minimum velocity and characteristic width

Fig. 15 b/θ and $(1-U_m/U_0)^{-2}$

where c_1 is a constant. By using the minimum velocity U_m , Eq. (3) is written as

$$\frac{(U_0 - U)}{(U_0 - U_m)} = \exp \{ -c_2 (y^2/b^2) (b^2/\theta^2) [\theta/(x-x_t)] \} \quad \dots (4)$$

where c_2 is a constant. From Eq. (3) we obtain

$$\{1 - (U_m/U_0)\}^{-2} = \{(x-x_t)/\theta\} / c_1^2 \quad \dots (5)$$

The relation between b^2/θ^2 and $\theta/(x-x_t)$ obtained from the experiment at $U_0 = 31.5$ m/sec is shown in Fig. 15 where x_t' is the virtual origin for b^2/θ^2 .

Using this relation it is shown that the distribution of the velocity defect is given by

$$\frac{(U_0 - U)}{(U_0 - U_m)} = \exp(-c_3 y^2/b^2) \quad \dots (6)$$

The mean velocity distribution measured in the experiments of a single aerofoil and a cascade is shown in Figs. 16 and 17 with the solid lines expressing Eq. (6). In Fig. 15 the relation between $\{1 - (U_m/U_0)\}^{-2}$ and $(x-x_t)/\theta$ obtained from the experimental measurement is also shown to prove Eq. (5).

When the ratio of the momentum thickness θ to the pitch t of a cascade is not large, the phenomena in the wakes of a single aerofoil and a cascade are expected to be almost the same. This ratio in our experiment was small as in a usual cascade and less than several percent. For the fully turbulent wakes of a cascade of small θ/t , the same relations as those applied to a wake of a single aerofoil were found to be adopted as seen in Figs. 15, 16, and 17.

4. Conclusion

Knowledge of blade wake characteristics may be helpful in compressor or turbine research. In order to obtain knowledges about wakes, fundamental, experimental investigation of two-dimensional wakes of a single aerofoil and cascade was performed and the the following conclusions reached.

1. The boundary layer at the trailing edge can

Fig. 16 Velocity distribution in turbulent region

Fig. 17 Velocity distribution in turbulent region

be considered to have the Blasius-type velocity profile on a flat plate so long as the thickness chord ratio is small. The wake can be divided into ones in a laminar, transition and turbulent region. This is also true for each aerofoil of a cascade.

2. The laminar wake is unstable and soon breaks down to be turbulent. Let the frequency of the transition wave (the Tollmien-Schlichting wave) be f c/sec, the main stream velocity U_0 m/sec, the momentum thickness θ_0 m, then $2\pi f \theta_0 / U_0$ is about 0.09 and independent of the boundary layer Reynolds number $U_0 \theta_0 / \nu$; f increases with the $3/2$ power of U_0 .

3. A cascade usually has such a construction as to have an acoustic resonance frequency. When its frequency coincides with the frequency of the transition wave, a very strong resonating sound is observed.

4. In the transition region of a wake, there exists a remarkable static pressure distribution perpendicular to the flow.

5. In the fully turbulent wake, the mean velocity and the characteristic width of the wake are of self preserving type. The mean velocity has the

distribution of an exponential type which can be calculated by assuming the turbulent viscosity constant.

References

- (1) W. Isay: *Z.A.M.M.*, Bd 33, Nr. 12 (1953).
 (2) S. Corrsin: *N. A. C. A. ACR* No. 4H24.

- (3) Y. Mori: *Preprint of extraordinary meeting of J.S.M.E. at Okayama*, (1956).
 (4) H. Blasius: *Z. Math. Physik*, 56 (1908).
 (5) H. Sato: *Jour. Phy. Soc. Japan.*, Vol. 11, No. 6, (1956).
 (6) A.A. Townsend: *The structure of turbulent shear flow*, Cambridge Univ. Press, (1956).

532.51

On the Fundamental Formula of Non-Newtonian Flow*

By Yukio TOMITA**

In this paper, we first deduce the fundamental equations of motion for pseudo-plastic and dilatant fluids. Secondly, it is proved that the equations of minimum dissipation of energy for these fluids are equivalent to the equations of motion, if inertia terms are neglected and external forces derived from a potential or zero. Then, the flow past sphere is approximately solved using this energy method.

1. Introduction

As a result of the development of modern chemical engineering, a large number of fluids whose stress components are expressible by arbitrary scalar functions of the rate-of-strain components, so-called non-Newtonian fluids, have been produced and engineers have actively studied the flow of these fluids. However, very few have treated non-Newtonian flows except the pipe flow. The reason is mainly the complication of the fundamental equations of motion for non-Newtonian flow, though, of course, it is partly the engineering importance of the pipe flow.

In this paper, the fundamental equations of motion for pseudo-plastic and dilatant fluids are firstly deduced and then it is proved that the equations of minimum dissipation of energy for these fluids are equivalent to the equations of motion, if inertia terms are neglected and external forces derived from a potential or zero. Then, the flow past sphere is approximately solved using this energy method. The non-Newtonian fluids in this paper are mainly pseudo-plastic and dilatant fluids, and these fluids are treated as incompressible ones.

2. Fundamental equations of motion for pseudo-plastic and dilatant fluids

The simple shear flow of pseudo-plastic and

dilatant fluids is expressed with considerable exactness by Waele-Ostwald's formula⁽¹⁾ as follows:

$$\frac{du}{dy} = \frac{\tau^n}{\mu_{psu}} \dots \dots \dots (1)$$

where μ_{psu} is the similar quantity to the viscosity of Newtonian fluid and its dimension is given by $[\mu_{psu}] = [\text{kg}^n/\text{m}^n \cdot \text{sec}^{2n-1}]$, τ is the shearing stress, du/dy is the velocity gradient, n is the rheological constant. The flow of pseudo-plastic fluids is expressed by $n > 1$, and that of dilatant fluids is given by $n < 1$ in the above equation, respectively.

As far as the author knows, the fundamental equations of motion using the same expression as Eq. (1) for these fluids do not yet be deduced except for the simple shear flow. Therefore, the fundamental equations of motion for these fluids are here deduced. In the first place, the relation between the stress components and rate-of-strain components for non-Newtonian fluids is considered. This relation is appropriately expressed as follows⁽²⁾:

$$p_{ik} = \eta e_{ik} - p \delta_{ik} \dots \dots \dots (2)$$

where p_{ik} is a component of the stress tensor (p_{ik}), as indicated in the following array:

$$(p_{ik}) = \begin{pmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{pmatrix} \dots \dots \dots (3)$$

and e_{ik} is a component of the rate-of-strain tensor (e_{ik}), which is expressed as follows:

* Received 12th May, 1958.

** Research Assistant, Tokyo Institute of Technology, Meguro-ku, Tokyo.