This classification method is highly estimated by the author as a possible standard for selection of current meters.

In view of these results, the author should like to propose as the next stage the trial production of the combination aerofoil current meter whose characteristics under the influence of a tilted stream conform to the cosine law, and the use of this combination aerofoil current meter in which the defects of old meters under the influence of a tilted stream are eliminated.

In closing, the author wishes to express his sincere thanks to Prof. S. Itaya of the Tokyo

Institute of Technology for his kind guidance, and to Prof. H. Takai of the Hiroshima University for his encouragement, and to Eamele Industrial Co. Ltd. for its financial contribution to the research, and to Messrs. S. Koike, H. Hiramoto, K. Tanaka, and T. Kumazaki for their help in the experiments.

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532.51:621-226.2

On the Deviation of Outlet Flow Angles Caused by Secondary Flows in Turbine Cascades (1st Report)*

By Marusuke GOMI**

This report suggests a simple formula for calculating the mean outlet flow angle deviation caused by the secondary flow in turbine cascades of semi-infinite span, assuming the secondary vortices concentrated on their centroid, then deriving secondary velocities along the trailing edges and computing the actual vane circulation using Smith's equation. As a result, the following conclusions are obtained;

- (1) Deviation of outlet flow angle increases with the increase in the inlet flow angle.
- (2) Deviation of outlet flow angle increases with the increase in the spacing of cascade, especially in impulse cascade of large camber angle.
- (3) Calculated results were compared with some experiments and fairly good coincidence was obtained except the effect of passage vortex of impulse cascade being pushed towards the mid-span.

1. Introduction

It is needless to mention that the knowledges of secondary flows in an axial flow compressor or turbine are very important. If we define that the term "secondary flows" means differences between a potential flow and one with boundary layer, there are miscellaneous sources which bring about them, but in this paper only the one which originates from the boundary layer of inlet flow is investigated. Approximate theoretical formula for

calculating the deviation of mean outlet flow angles caused by secondary flows in turbine blades with semi-infinite span is deduced and the calculated results are compared with some experiments. In the above formula the induced velocities along the trailing edges are calculated, assuming that "passage vortices" of secondary flows are concentrated on their centroid, then the actual circulations around the blades are computed by using Smith's equation, and finally the distributions of mean outlet flow angles along the span are obtained. Furthermore some revisions are made considering that the passage shape bounded by two adjacent blades is deformed from its initial rectangular form after the flow leaving the trailing edge.

* Received 4th November, 1958.

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2. Derivation of approximate formula

Symbols

- V : velocity
 α : angle between the stream line and the perpendicular to the cascade axis
 x : co-ordinate normal both to the span and the stream line
 y : co-ordinate normal to the end wall
 z : complex number
 ζ : vorticity
 Γ : circulation around the blade
 K : circulation around the centroid on which all passage vortices are concentrated
 S : spacing
 W : complex potential
 a : width of the passage bounded by two adjacent blades
 h : span
 Δt : difference between the time required for flow over the blade convex surface and the one over the concave surface
 f : distance concerned with Δt
 e : distance between the tail of the vortex filament and the cascade along the stream line
 ϕ : angle between the vortex filament and the stream line
 s : cross sectional area of the vortex filament cut by the stream surface normal to the end wall
 v : induced velocity along the trailing edge
 δ : boundary layer thickness of the inlet flow
 δ^* : boundary layer displacement thickness of the inlet flow

Suffixes

- 1 : inlet of the cascade
 2 : outlet of the cascade
 ∞ : vector mean
 i : section where $y = \infty$
 s : secondary vortex
 n : normal to the stream line

It is assumed that the flow is incompressible and inviscid, though viscosity is responsible for the production of a non-uniform stream in the form of a boundary layer. There is vorticity in the inlet stream of magnitude $\zeta_1 = dV_1/dy$ and the vorticity vector is at right angles to the streamlines as is shown in Fig. 1. Moving through the cascade, the vortex which has been a straight line in front of the cascade changes into a curved line and is no longer normal to the streamline, but in order to avoid difficulties it is assumed that the vortex remains straight and that it represents the mean

Fig. 1

value over the passage bounded by two adjacent blades.

Now by the Helmholtz Law

$$\zeta_1 \Delta s_1 = \zeta_n \Delta s_2 \dots \dots \dots (1)$$

From the equation of continuity

$$\Delta s_1 / \Delta s_2 = V_1 / V_2 = \cos \alpha_2 / \cos \alpha_1, \dots \dots \dots (2)$$

and from Eqs. (1) and (2)

$$\zeta_s = \zeta_n \cot \phi_2 = \zeta_1 \frac{\cos \alpha_2 (S \sin \alpha_2 + e_2 + f_2)}{\cos \alpha_1 (S \cos \alpha_2)} \dots \dots \dots (3)$$

$$e_2 = e_1 \frac{V_2}{V_1} = S \sin \alpha_1 \frac{\cos \alpha_1}{\cos \alpha_2} \dots \dots \dots (4)$$

From the equation introduced by Smith

$$f_2 = \Delta t V_2 = \frac{V_2 \Gamma}{V_\infty^2} \dots \dots \dots (5)$$

$$V_\infty^2 = \left\{ 1 + \frac{(\tan \alpha_2 - \tan \alpha_1)^2}{4} \right\} V_1^2 \cos^2 \alpha_1 \dots \dots \dots (6)$$

$$\Gamma = S V_1 \cos \alpha_1 (\tan \alpha_1 + \tan \alpha_2) \dots \dots \dots (7)$$

Substitution of Eqs. (6) and (7) into Eq. (5) gives

$$f_2 = \frac{4S(\tan \alpha_1 + \tan \alpha_2)}{\{4 + (\tan \alpha_2 - \tan \alpha_1)^2\} \cos \alpha_2} \dots \dots \dots (8)$$

Substitution of Eqs. (4) and (8) into Eq. (3) yields

$$\zeta_s = \frac{\zeta_1}{\cos \alpha_1 \cos \alpha_2} \left\{ \sin \alpha_1 \cos \alpha_1 + \sin \alpha_2 \cos \alpha_2 + \frac{4(\tan \alpha_1 + \tan \alpha_2)}{4 + (\tan \alpha_2 - \tan \alpha_1)^2} \right\} \dots \dots \dots (9)$$

It is assumed that the span is semi-infinite and if vortices of which the circulation is $\zeta_s dx' dy'$ distribute in the passage bounded by two adjacent blades, the complex potential W at point z is

$$W = \int_0^\delta \int_0^a \frac{i \zeta_s}{2\pi} \left\{ \ln \frac{\sin \frac{\pi}{2a}(z - x' - iy')}{\sin \frac{\pi}{2a}(z - x' + iy')} - \ln \frac{\sin \frac{\pi}{2a}(z + x' - iy')}{\sin \frac{\pi}{2a}(z + x' + iy')} \right\} dx' dy' = \phi + i\psi \dots (10)$$

hence

$$\Psi = \int_0^\delta \int_0^a \frac{\zeta_s}{4\pi} \left\{ \ln \left| \frac{\cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(x-x')}{\cosh \frac{\pi}{a}(y+y') - \cos \frac{\pi}{a}(x-x')} \right| - \ln \left| \frac{\cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(x+x')}{\cosh \frac{\pi}{a}(y+y') - \cos \frac{\pi}{a}(x+x')} \right| \right\} dx' dy' \quad (11)$$

$$v = \left(\frac{\partial \Psi}{\partial x} \right)_{x=a} = \int_0^\delta \int_0^a \frac{\zeta_s}{4a} \left\{ \frac{\sin \frac{\pi}{a}(a-x')}{\cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(a-x')} - \frac{\sin \frac{\pi}{a}(a-x')}{\cosh \frac{\pi}{a}(y+y') - \sin \frac{\pi}{a}(a-x')} \right. \\ \left. - \frac{\sin \frac{\pi}{a}(a+x')}{\cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(a+x')} + \frac{\sin \frac{\pi}{a}(a+x')}{\cosh \frac{\pi}{a}(y+y') - \cos \frac{\pi}{a}(a+x')} \right\} dx' dy' \quad (12)$$

If ζ_s does not vary with x' , this gives by integration with respect to x'

$$v = \int_0^\delta \frac{\zeta_s}{4\pi} \left\{ -\ln \left\{ \cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(a-x') \right\} + \ln \left\{ \cosh \frac{\pi}{a}(y+y') - \cos \frac{\pi}{a}(a-x') \right\} \right. \\ \left. - \ln \left\{ \cosh \frac{\pi}{a}(y-y') - \cos \frac{\pi}{a}(a+x') \right\} + \ln \left\{ \cosh \frac{\pi}{a}(y+y') - \cos \frac{\pi}{a}(a+x') \right\} \right\} dy' \\ = \int_0^\delta \frac{\zeta_s}{\pi} \ln \left| \frac{\sinh \frac{\pi}{a} y + \sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y - \sinh \frac{\pi}{a} y'} \right| dy' \quad (13)$$

Let us consider the case where vortices concentrate near the end wall. As the vortices at the points where y is far greater than y' are predominant, Eq. (13) is given by the following series

$$v = 2 \int_0^\delta \frac{\zeta_s}{\pi} \left\{ \frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} + \frac{1}{3} \left(\frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} \right)^3 + \dots \right\} dy' \\ = 2 \int_0^\delta \frac{\zeta_s}{\pi} \left(\frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} \right) \left\{ 1 + \frac{1}{3} \left(\frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} \right)^2 + \dots \right\} dy' \quad (14)$$

In the case where $y' \ll y$, we can put

$$1 + \frac{1}{3} \left(\frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} \right)^2 + \dots \approx 1$$

or

$$1 + \frac{1}{3} \left(\frac{\sinh \frac{\pi}{a} y'}{\sinh \frac{\pi}{a} y} \right)^2 + \dots = 1 + R$$

where R is small. Under the assumption that $\pi y' \leq a$, we can put

$\sinh \frac{\pi}{a} y' = \frac{\pi}{a} y'$, hence Eq. (14) gives

$$v = \frac{2(1+R)}{a \sinh \frac{\pi}{a} y} \int_0^\delta \zeta_s y' dy' \quad (15)$$

Also

$$\delta^* = \frac{\int_0^\delta \zeta_s y' dy'}{\int_0^\delta \zeta_s dy'} \quad (16)$$

$$K = a \int_0^\delta \zeta_s dy' \quad (17)$$

Fig. 2

From Eqs. (15), (16), and (17)

$$v = \frac{2(1+R)K\delta^*}{a^2 \sinh \frac{\pi}{a} y} \quad (18)$$

If vortices exist in an infinitely wide space, they rotate about their centroid and the induced velocity is obtained in the following manner. As we see from Eq. (16), the centroid of vortices coincides with the displacement thickness of the boundary layer, hence the complex potential is

$$W = \frac{K}{2\pi} \left\{ \ln \frac{\sin \frac{\pi}{2a} \left(z - \frac{a}{2} - i\delta^* \right)}{\sin \frac{\pi}{2a} \left(z - \frac{a}{2} + i\delta^* \right)} - \ln \frac{\cos \frac{\pi}{2a} \left(z + \frac{a}{2} - i\delta^* \right)}{\cos \frac{\pi}{2a} \left(z + \frac{a}{2} + i\delta^* \right)} \right\} = \phi + i\Psi$$

$$\Psi = \frac{K}{4\pi} \left\{ \ln \frac{\cosh \frac{\pi}{a} (y - \delta^*) - \cos \frac{\pi}{a} \left(x - \frac{a}{2} \right)}{\cosh \frac{\pi}{a} (y + \delta^*) - \cos \frac{\pi}{a} \left(x - \frac{a}{2} \right)} - \ln \frac{\cosh \frac{\pi}{a} (y - \delta^*) + \cos \frac{\pi}{a} \left(x + \frac{a}{2} \right)}{\cosh \frac{\pi}{a} (y + \delta^*) + \cos \frac{\pi}{a} \left(x + \frac{a}{2} \right)} \right\} \quad (19)$$

$$v = \left(\frac{\partial \Psi}{\partial x} \right)_{x=a} = \frac{K}{2a} \left\{ \operatorname{sech} \frac{\pi}{a} (y - \delta^*) - \operatorname{sech} \frac{\pi}{a} (y + \delta^*) \right\} \quad (20)$$

If y is larger than a , Eq. (18) is nearly reduced to Eq. (20). Referring to the author's experimental data, Eq. (20) seems more appropriate than Eq. (18), hence Eq. (20) is adopted in the following procedure.

From Smith's equation

$$d\Gamma = 2vdy - \zeta_1 \frac{\Gamma V_1}{V_\infty^2} dy$$

This gives by integration from y to ∞

$$\Gamma_i - \Gamma = 2 \int_y^\infty vdy - \int_y^\infty \zeta_1 \frac{\Gamma V_1}{V_\infty^2} dy \quad (21)$$

From Eqs. (6) and (7)

$$\zeta_1 \frac{\Gamma V_1}{V_\infty^2} = \frac{4S(\tan \alpha_1 + \tan \alpha_2)}{\{4 + (\tan \alpha_2 - \tan \alpha_1)^2\} \cos \alpha_1} \frac{dV_1}{dy} \quad (22)$$

After substituting Eqs. (20) and (22) into (21), we integrate this under the assumption that $\alpha_2 = \alpha_{2i}$ because the variation of α_2 is small.

$$\Gamma_i - \Gamma = \frac{2K}{\pi} \{ \tan^{-1} e^{\pi(y+\delta^*)/a} - \tan^{-1} e^{\pi(y-\delta^*)/a} \} - \frac{4S(\tan \alpha_1 + \tan \alpha_{2i})(V_{1i} - V_1)}{\{4 + (\tan \alpha_{2i} - \tan \alpha_1)^2\} \cos \alpha_1} \quad (23)$$

Substitution of Eq. (9) into Eq. (17) gives by integration under the assumption that $\alpha_2 = \alpha_{2i}$

$$K = \frac{SV_{1i}}{\cos \alpha_1} \left\{ \sin \alpha_1 \cos \alpha_1 + \sin \alpha_{2i} \cos \alpha_{2i} + \frac{4(\tan \alpha_1 + \tan \alpha_{2i})}{4 + (\tan \alpha_{2i} - \tan \alpha_1)^2} \right\} \quad (24)$$

Eq. (7) gives in the case where y is infinite

$$\Gamma_i = SV_{1i} \cos \alpha_1 (\tan \alpha_1 + \tan \alpha_{2i}) \quad (25)$$

Substitution of Eqs. (7), (24) and (25) into Eq. (23) gives, after some rearrangements,

$$\begin{aligned} \tan \alpha_2 = \tan \alpha_{2i} - \frac{2}{\pi \cos^2 \alpha_1} \{ \tan^{-1} e^{\pi(y+\delta^*)/a} - \tan^{-1} e^{\pi(y-\delta^*)/a} \} & \frac{V_{1i}}{V_1} \left\{ \sin \alpha_1 \cos \alpha_1 + \sin \alpha_{2i} \cos \alpha_{2i} \right. \\ & + \frac{4(\tan \alpha_1 + \tan \alpha_{2i})}{4 + (\tan \alpha_{2i} - \tan \alpha_1)^2} \left. \right\} + (\tan \alpha_1 + \tan \alpha_{2i}) \left[\frac{4}{\{4 + (\tan \alpha_{2i} - \tan \alpha_1)^2\} \cos^2 \alpha_1} + 1 \right] \\ & \times \left(\frac{V_{1i}}{V_1} - 1 \right) \end{aligned} \quad (26)$$

If α_1 , α_{2i} , a and V are known, we can obtain α_2 from Eq. (26).

When the span is finite, we put

$$F(y) = \{ \tan^{-1} e^{\pi(y+\delta^*)/a} - \tan^{-1} e^{\pi(y-\delta^*)/a} \}$$

and replace $F(y) \frac{V_{1i}}{V_1}$ in Eq. (26) with

$$F(y) \frac{V_{1i}}{V_1} + F(h-y) + F(h+y) + F(2h-y) + F(2h+y) + \dots,$$

and then we obtain α_2 for the case of finite span.

According to Smith, it is reasonable to assume that the span is semi-infinite in the case where the span is fairly large.

3. Calculated results

In the velocity profile A of Fig. 3, the boundary layer thickness δ is 20 mm and the displacement thickness is 1.75 mm. The velocity profile coincides with Nikuradse's experiment on the case where the Reynolds Number is very large.

It may be said with generality that in an ordinary cascade wind tunnel the velocity profile seems to coincide with the experiment by Nikuradse, irrespective of boundary layer thickness in the case where no profile generator is used. Because of the lack of available experimental data, the author does not know the velocity profile in an actual turbine exactly but it will probably differ

Fig. 3

Fig. 4

from this profile greatly.

Calculated results from Eq. (26) for the cases where α_{2i} equals to 40° , 50° , and 60° are shown in Figs. 4~6, and in these cases the span is semi-infinite and the spacing is 44 mm. In all these cases the deviations of outlet flow angles $\alpha_2 - \alpha_{2i}$ increase with the increase in the inlet flow angles. The minimum value of α_2 occurs at about the section where y equals to 15 mm. The values of

Fig. 5

Fig. 6

$\alpha_2 - \alpha_{2i}$ at $y = 15$ mm versus outlet angles α_{2i} are shown in Fig. 7 with the inlet angles as parameters. The deviations of outlet flow angles decrease with the increase in α_{2i} generally. The dotted line in Fig. 7 corresponds to the case of impulse blades where $\alpha_1 = \alpha_{2i}$.

The values of $\alpha_2 - \alpha_{2i}$ at $y = 15$ mm versus spacings S are also shown in Fig. 8 for the case where α_{2i} equals to 60° . The deviations of outlet flow angles increase a little with the increase in the spacings, but they are almost constant in spite of the variation of spacing except in the case of impulse blades. The deviations of outlet flow angles versus the distance from the wall are shown in Fig. 9 in the case where the spacings are infinite and α_{2i} equals to 50° . The outlet flow angles α_2 are always larger than α_{2i} in the case where the spacing is infinite. In the case of the velocity profile B in Fig. 3, the boundary layer thickness equals to 40 mm and the displacement thickness δ^* equals to 3.5 mm. The boundary layer thickness in the velocity profile B is twice as thick as the one in A , but the profile shape is similar with A .

Fig. 7

Fig. 8

The outlet flow angle deviations in the case of A and B versus the distance from the end wall are shown in Fig. 10. The minimum outlet flow angle in B does not differ much from the one in A .

4. Comparison of calculated values with experimental ones

The width and the height of the cascade wind tunnel are 28 cm and 53 cm respectively and the

Fig. 9

Fig. 10

maximum velocity of the outlet flow is about 55 m/sec and the axial flow fan is driven by a 55 H.P. induction motor. The wind tunnel has been constructed carefully to keep the variation of inlet flow angle into the cascade within one degree outside the boundary layer. The velocity profile is somewhat "M" shaped. Five-hole pitot tube is mounted so as to be able to rotate about two axes normal to each other and to be traversed to measure three dimensional outlet flow angles. The accuracy of the yawmeter is within ± 0.2 degrees, but the variation of outlet flow angle over one spacing is comparatively large, the mean value of the outlet flow angle at each section is less accurate.

The pitot traversing is made in a plane about 4 cm behind the cascade. The blades are made of wood and their chords are 7 cm and their spans are 28 cm. The experimental results about the nozzle blades ($\alpha_1=0$) are shown in Figs. 11~13. The calculated variations of outlet flow angles outside the boundary layer become smaller compared with the measured ones as α_{2i} increases. The dotted lines show the values calculated from Eq. (26) and the full lines show the revised values obtained in the following manner. In Eq. (26), it is assumed that the passage bounded by two adjacent blades keeps its rectangular form until far downstream, but actually its shape deforms after the fluids have

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

left the trailing edges, hence the secondary vortices behave as if they are in the more wide passage, therefore the mean values between the values calculated from Eq. (26) for the actual spacings and the ones for the infinite spacings are adopted as the revised values. In the case of nozzle blades there seems to be no need to make such revision, but in the case of impulse blades the above procedure seems useful as is shown in Fig. 14 and Fig. 15. In the impulse blades, it is also noticeable that the sections where α_2 is minimum shift toward the midspan.

In the nozzle cascades, the effects of spacings on the variation of outlet flow angles are measured and it is confirmed that the effects of spacings are somewhat greater than expected from calculated values.

The effects of the thickness of the boundary layer on the variation of outlet flow angles are also measured with the artificialy thickend boundary layer by profile generator and it is noticed that the minimum value of α_2 does not vary so largely, but the region of variation extends more widely. In the case of extremely thick boundary layer, the assumption of the semi-infinite span and the concentration of vortices on centroid is unreasonable and the calculated values do not coincide with the measured values.

5. Conclusion

Under the assumption that the span is semi-infinite and the secondary vortices concentrate on their centroid, it is easy to calculate the induced

velocity along the trailing edge, further to integrate the Smith's equation with the assumption of constant outlet angle and to calculate the mean outlet flow angle deviation along the span. As the experimental data are few, we can not conclude definitely, but the calculated values from this approximate theoretical formula seem to coincide fairly well with the experimental values in the case where the boundary layer of inlet flow is comparatively thin. As the author has noticed from the other experiments which are not mentioned here that the blade profile and the boundary layer on the blade surface which are neglected in the present treatment have considerable effects on the mean outlet flow angle, he believes that it is rather desirable to make an empirical formula with the aid of the simple theoretical formula than to try to make a more exact theoretical one.

Acknowledgements

The author wishes to express his thanks to Prof. Hatta and Prof. Okazaki of University of Tokyo for their kind advices, and to Mr. Isogai, Mr. Nakai and others of Mitsubishi Nippon Heavy-Industries Ltd. for their kind cooperation in construction of the wind tunnel.

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