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On the Flow in a Water Turbine of Axial Flow Type*

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The air test of a model of the propeller turbine was performed to reconsider the methods of designing Kaplan turbines by the usual two-dimensional treatments. The performances of it, the flow conditions at the inlet and outlet of the runner and the pressure distributions on the running blades were measured to compare with the ones of two-dimensional cascades.

1. Introduction

In order to prevent the occurrence of cavitation in Kaplan turbines and to improve their efficiency, the flow in the turbines must be theoretically and experimentally investigated in details. Although many experiments^{(1)~(4)} of the axial flow type turbine were already performed, the hydrodynamical problems of the turbine, especially the methods of designing by the two-dimensional hydrodynamical treatments and the analysis of performance, must be reexamined by experiment and theory. This paper deals with the air test results of a model turbine, which is designed on the two-dimensional theory of the flow in the runner.

2. Test apparatus

Fig. 1 shows a schematic diagram of our test apparatus. The model runner ③ is rotated by the air flow, which is induced by a Sirocco fan ⑧ through the guide vanes and flows out into the atmosphere through a draft tube ④ and a pressure equalizer ⑤. As the pressure in the pressure equalizer corresponds to that on the tail-race level of actual turbines, the difference between the pressure in the pressure equalizer and the atmospheric pressure gives the effective head H of the turbine. The flow quantity of air is measured by a nozzle ⑦ and a screen ⑥ is inserted before it in the duct. The torque generated by the runner is measured by a Prony dynamometer ①.

Fig. 2 shows the main parts of the model tur-

bine and Fig. 3 the runner and its blade form. And then Figs. 4 (a) and (b) are photographs of the test model and the runner respectively. The runner has 4 vanes and is designed on the two-dimensional theory under a specific speed $n_s=800$, and the form of the blade at the mean radius is Göttingen 428. To measure the pressure distribution along the blade surfaces at the four sections A, B, C and D with such radii as shown in Fig. 3, the copper tubes are cast in the aluminum blades and the holes for measuring pressure are drilled on the blade surfaces as far as these tubes. The pressure from the holes is transmitted to the inclined manometers through these tubes and a hollow shaft and a packing box, as shown in Fig. 2. Moreover, the tests of two-dimensional airfoil cascade and an isolated airfoil were performed in the wind tunnel of our laboratory and the characteristics of them were compared with those of airfoils in the model runner. Fig. 5 shows the test section of the cascade wind tunnel. The cascade have the same airfoil sections, stagger angles and pitch-chord ratios as those at the four sections of the model runner and are composed of five airfoils. An airfoil at the center of cascade has about twenty holes at the mid-section of the span, as shown in Fig. 6. Tables 1 and 2 show the forms of the cascade airfoils and the positions of holes in them.

3. Experimental methods and results

The tests of the model turbine were performed under the conditions of various setting angles of guide vanes such as $25^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ$ and 70° . The head H , the flow quantity Q , the number of revolution per minute n and the torque M were

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Fig. 1

Fig. 2

Fig. 3

(a)

(b)

Fig. 4

measured under each condition, and then the characteristic curves of the turbine were plotted.

(i) Characteristic curves

The characteristics are expressed by the following dimensionless forms:

coefficient of the peripheral speed

$$K_u = u_0 / \sqrt{2gH}$$

coefficient of the mean axial velocity

$$K_{\bar{c}_m} = \bar{c}_m / \sqrt{2gH}$$

$$\text{efficiency of the turbine } \eta = \frac{M\omega}{\rho g Q H}$$

where u_0 is the peripheral velocity of the runner, $\bar{c}_m$ is the mean axial velocity in it, ω its angular velocity and ρ the density of fluid.

Fig. 7 shows the characteristic curves, i. e. $K_{\bar{c}_m}$ versus K_u curves and equi-efficiency curves. When

Fig. 5

Fig. 6

the setting angle β of guide vanes is smaller than 50° , the $K_{Lm} \sim K_u$ curves show nearly linear relations between them, as well known about the characteristics of the turbine.

(ii) Flow at the inlet and outlet of the runner

The flows at the inlet and outlet of the runner were measured by cylindrical Pitot tubes at the positions as shown in Fig. 2. By traversing radially the Pitot tubes from boss to tip of the runner, the flow direction, total pressure and velocity distributions were measured with the fixed coefficient K_u .

Fig. 8 (a) shows the circulation distributions in the radial direction at the inlet and outlet of the runner. The ordinates are the dimensionless circulations Γ/Q_1 and the abscissae are the

Fig. 7

Table 1 Airfoil dimensions (Göttingen 428 at the mean radius)

Hydrofoil	x %	0	1.25	2.50	5.00	7.50	10.00	15.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	95.00	100.00	l mm
A	y_u %	0	2.34	3.64	5.47	6.88	7.87	9.31	10.24	10.92	11.13	10.70	9.85	8.50	6.56	3.96	2.35	0.20	100
	y_l %	0	1.42	1.74	1.82	1.82	1.81	1.75	1.62	1.34	1.05	0.73	0.51	0.45	0.53	0.45	0.34	0.24	
B	y_u %	0	1.56	2.62	4.37	5.50	6.21	7.18	7.80	8.21	8.11	7.66	6.90	5.80	4.46	2.85	2.52	0.46	121.1
	y_l %	0	1.44	1.52	1.52	1.44	1.38	1.15	1.12	0.96	0.56	0.31	0.25	0.27	0.31	0.23	0.11	0.07	
C	y_u %	0	2.24	3.30	4.64	5.25	6.16	7.01	7.40	7.71	7.36	7.09	6.34	5.34	4.21	2.93	1.87	0.49	125.5
	y_l %	0	0.86	0.86	0.80	0.74	0.72	0.65	0.65	0.56	0.26	0.07	-0.10	-0.16	0	0.05	0.02	0	
D	y_u %	0	1.69	2.54	3.67	4.43	5.01	5.77	6.20	6.33	6.09	5.71	5.09	4.26	3.26	2.01	1.29	0.45	150.6
	y_l %	0	0.96	1.15	1.24	1.25	1.17	1.13	1.20	1.15	0.93	0.67	0.59	0.61	0.67	0.55	0.38	0.25	

Table 2 Positions of holes for measuring pressure

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	2.9	6.7	13.35	24.5	34.25	44.15	57.60	74.60	87.80	91.80	79.00	72.50	66.00	52.90	37.80	24.30	13.40	5.05	0.20
B	2.31	6.86	15.20	26.60	39.20	54.40	69.40	81.40	85.50	75.60	58.00	41.60	28.40	15.20	9.30	4.50	1.60	—	—
C	2.63	7.10	16.70	28.00	40.60	54.80	70.60	84.80	82.20	68.80	54.00	41.30	29.30	18.40	9.30	4.40	1.00	—	—
D	2.10	6.70	16.20	27.40	43.10	57.70	72.20	86.30	88.80	77.00	63.00	47.20	31.70	18.00	9.80	3.80	1.00	—	—

(a)

(b)

(c)

Fig. 8

dimensionless radial position r/r_0 , where $\Gamma = 2\pi r c_u$, and c_u is the circumferential velocity component at the point of radius r , Q_1 the flow quantity per unit breadth of the guide vanes and r_0 the outer radius of the runner. The experimental results at the inlet of the runner are indicated by the marks I as shown in Fig. 8 (a) and these marks show the scattering range of the measured values under the various conditions of K_u . The results at the same position in the case, when the runner is removed, are shown by fine lines.

If the flow from the guide vanes is perfectly two-dimensional, the circulation distribution Γ/Q_1 at the inlet of the runner ought to be uniform according to the free-vortex theory. But the result in the case of $\beta = 30^\circ$ shows that the circulation distribution is not uniform because the flow from guide vanes is not perfectly two-dimensional due to the curved passage between the guide vanes and the runner. Now that the flow at the inlet of the runner is not uniform, the flow at the outlet has some circumferential velocity component even under the normal running condition of the turbine, i.e. $K_u = 1.99$, $K_{cm} = 0.675$ and $\beta = 30^\circ$. The outlet flow at the off-design points also has the circumferential velocity component as shown in this Figure.

Fig. 8 (b) shows the distributions of total pressure p_t in the radial direction at the inlet and outlet of the runner. The ordinate is dimensionless quantity $p_t/\rho g H$. The distributions at the inlet are uniform, but those at the outlet are not uniform except near the normal running condition.

Fig. 8 (c) shows the distributions of axial velocity component $c_m/\bar{c}_m$, where c_m is the axial velocity component. According to the free-vortex theory, the axial velocity is uniform in the radial direction under the normal running condition. The test results also show a nearly uniform distribution.

(iii) Pressure distributions along the runner blades

Figs. 9 (a) and (b) show the pressure distributions along the runner blades at the sections A and C respectively under the normal setting angle of guide vanes $\beta = 30^\circ$. The ordinate is the pressure coefficient C_{p1} referred to the inlet flow conditions: $C_{p1} = (p - p_1)/(\frac{1}{2}\rho w_1^2)$, where p is the pressure

on the blade surfaces, p_1 and w_1 are the pressure and the relative velocity to the runner at the inlet respectively. The abscissa is the positions on the chord of blades. The attack angles α_1 to the relative velocity w_1 are noted in these Figures. The pressure distributions of the cascade and an isolated airfoil near the above attack angles also are

Fig. 9

Fig. 10

plotted with the same pressure coefficients as those of the above model tests.

In Fig. 9, the test results of rotating runner agree well with those of cascade airfoil, but considerably differ from those of an isolated airfoil. Moreover, the results at the sections A and D differ a little from the cascade test results because of the tip clearance and boss effects. But the results at the sections B and C agree well with the latter.

Fig. 10 shows the characteristic curves, i.e. the lift coefficients C_{L1} and the drag coefficients C_{D1} versus the attack angle α_1 . The above-mentioned tendencies are also clearly shown here.

4. Theory of three-dimensional flow in guide vanes

According to the usual methods of designing the axial turbine, the circulation distribution of flow at the inlet of the runner is assumed to be

Fig. 11

uniform in the radial direction, as well known in the free-vortex theory. The test results show that the circulation near the tip of the runner is larger than that near the boss. The flow through the guide vanes is not uniform along the span of vanes due to the curvature of the channel between the guide vanes and the runner. This gives rise to the non-uniform distribution of circulation.

Now the theory of three-dimensional flow in the guide vanes will be dealt with by the following approximate method.

The author already developed the two-dimensional theory of Circular Wing Lattice composed of Thin Airfoils⁽⁵⁾ by means of the conformal representation. The circular wing lattice composed of logarithmic spirals can be conformally represented into a unit circle on the ζ -plane, as well known in the centrifugal pump theory, by the following mapping function;

$$\log z = \frac{e^{i \tan^{-1} \tau / \nu}}{N} \left[e^{i \tan^{-1} \tau / \nu} \log \frac{\zeta + \kappa}{\zeta - \kappa} + e^{i \tan^{-1} \tau / \nu} \log \frac{\kappa \zeta + 1}{\kappa \zeta - 1} - \frac{N}{2} \sqrt{\tau^2 + \nu^2} \right] \dots (1)$$

where N is the number of vanes, τ the angle taken from both ends A and B of the spiral into the center of circular lattice, ν the logarithm of the inner-outer radii ratio, and κ the constant determined by the following functions;

$$\frac{N}{4} = \frac{\nu}{\tau^2 + \nu^2} \tanh^{-1} \frac{2\kappa\nu}{\sqrt{K}} + \frac{\tau}{\tau^2 + \nu^2} \tanh^{-1} \frac{2\kappa\tau}{\sqrt{K}} \quad \dots\dots\dots (2)$$

where

$$K = (1 + \kappa^2)^2 \nu^2 + (1 - \kappa^2)^2 \tau^2 \dots\dots\dots (3)$$

As shown in Fig. 11, a point D on the given airfoil corresponds to a point C on the above basic spiral. Assuming that the camber of the airfoil differs a little from the basic spiral and that the thickness of it is small, the airfoils of a given circular lattice can be expressed as follows, using a parameter θ ;

$$\log r = \frac{2}{N} \frac{\nu}{\tau^2 + \nu^2} \left[\nu \tanh^{-1} \frac{2\kappa \cos \theta}{1 + \kappa^2} + \tau \tanh^{-1} \frac{2\kappa \sin \theta}{1 - \kappa^2} \right] + \frac{\nu}{2} + a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta' + b_n \sin n\theta') \dots\dots\dots (4)$$

$$\varphi = \frac{2}{N} \frac{\nu}{\tau^2 + \nu^2} \left[\nu \tanh^{-1} \frac{2\kappa \cos \theta}{1 + \kappa^2} + \tau \tanh^{-1} \frac{2\kappa \sin \theta}{1 - \kappa^2} \right] + \frac{\tau}{2} + a_0' + \sum_{n=1}^{\infty} (a_n' \cos n\theta' + b_n' \sin n\theta') \dots\dots\dots (5)$$

where $\theta' = \theta - \theta_L$, a_n , b_n , a_n' and b_n' are the small quantities which express the camber and thickness of the airfoil and θ_L is the angle of the point corresponding to the leading edge A in Fig. 11.

Assuming that a given circular cascade can be conformally transformed by the function (1) into a unit circle and, using the boundary condition at the vane surface and the Joukowski's hypothesis, the circulation Γ/N around a guide vane is given as follows:

$$\begin{aligned} \frac{\Gamma}{N} = & -\frac{4\kappa(\tau Q - \nu \Gamma_0)}{N(\sqrt{K} - 2\kappa\nu)} + \frac{\nu Q + \tau \Gamma_0}{\nu} \frac{1 - \kappa^2}{1 + \kappa^2} \frac{\sqrt{K} + 2\kappa\nu}{\sqrt{K}} \frac{\nu^2}{\tau^2 + \nu^2} \\ & \times \left\{ \frac{\tau}{\nu} \sum_{n=0}^{\infty} (-1)^n n a_n - \sum_{n=0}^{\infty} (-1)^n n a_n' \right\} \\ & + \frac{\tau Q - \nu \Gamma_0}{\nu} \frac{1 - \kappa^2}{1 + \kappa^2} \frac{\sqrt{K} + 2\kappa\nu}{\sqrt{K}} \frac{\nu^2}{\tau^2 + \nu^2} \left\{ \frac{\tau}{\nu} \sum_{n=1}^{\infty} (-1)^n n b_n - \sum_{n=1}^{\infty} (-1)^n n b_n' \right\} \\ & + (\tau Q - \nu \Gamma_0) \frac{1 - \kappa^2}{1 + \kappa^2} \frac{2\kappa}{\sqrt{K} - 2\kappa\nu} \frac{\nu^2}{\tau^2 + \nu^2} \left\{ \frac{\tau}{\nu} \sum_{n=1}^{\infty} (-1)^n n b_n - \sum_{n=1}^{\infty} (-1)^n n b_n' \right\} \\ & - (\tau Q - \nu \Gamma_0) \frac{1 - \kappa^2}{1 + \kappa^2} \frac{2\kappa}{\sqrt{K} - 2\kappa\nu} \frac{\nu^2}{\tau^2 + \nu^2} \left\{ \frac{\tau}{\nu} \sum_{n=0}^{\infty} (-1)^n n a_n - \sum_{n=0}^{\infty} (-1)^n n a_n' \right\} \dots\dots\dots (6) \end{aligned}$$

where Γ_0 is the circulation outside the guide vanes, therefore Γ_0/Q_1 gives the flow direction there. The circulation Γ/N given by Eq. (6) is uniform along the span of guide vanes if the flow is two-dimensional. But the actual flow in the model turbine is three dimensional due to the curvature of channel between the guide vanes and the runner.

The breadth Δb of each partial turbine is not constant in the flow direction as shown in Fig. 12. The radial velocity component is given as (See Fig. 13)

Fig. 12

Fig. 13

$$\frac{\Delta Q}{2\pi r} \frac{\Delta b}{\Delta b'} = \frac{\Delta Q}{2\pi r} (1 + \varepsilon) \dots (7)$$

where ΔQ is the flow quantity of each partial turbine, $\Delta b'$ the breadth at the radius r and ε is the small quantity which represents the change of breadth. On the other hand, the tangential velocity component due to the circulation Γ_0 outside the guide vane is assumed not to be affected by the curvature of channel.

The normal and tangential velocity components, Δv_n and Δv_t , at a point on the vane surface added to those at the two-dimensional case are approximately as follows:

$$\Delta v_n = \varepsilon \frac{Q}{2\pi r} \sin \gamma, \quad \Delta v_t = \varepsilon \frac{Q}{2\pi r} \cos \gamma \dots (8)$$

where γ is the angle between the tangent to the basic spiral and the radius (Fig. 13). By means of the above mapping function (1), the additional radial velocity component Δv_r on the unit circle is given as

$$\Delta v_r = \varepsilon \frac{Q}{2\pi} \sin \gamma \frac{1}{N\sqrt{\tau^2 + \nu^2}} \frac{4\kappa\sqrt{K} \sin \theta'}{1 - 2\kappa^2 \cos 2\theta + \kappa^4} \dots (9)$$

Developing the following term in Eq. (9) into Fourier's series:

$$\frac{\varepsilon \sin \theta'}{1 - 2\kappa^2 \cos 2\theta + \kappa^4} = \sum_{n=0}^{\infty} c_n \cos n\theta' + \sum_{n=1}^{\infty} d_n \sin n\theta' \dots (10)$$

and, using the boundary condition on the vane surface and the Joukowski's hypothesis, the circulation around a elementary guide vane must be corrected by the amount $\frac{\Delta \Gamma}{N}$ as follows:

$$\frac{\Delta \Gamma}{\Delta Q} = -\frac{2\kappa\sqrt{K}}{\pi} \frac{1}{\tau^2 + \nu^2} \left[\nu \sum_{n=0}^{\infty} (-1)^n c_n - \tau \sum_{n=1}^{\infty} (-1)^n d_n \right] \dots (11)$$

As a numerical example, the circulation distributions of the model turbine with $\beta = 30^\circ$ were calculated by the two-dimensional theory and the above three-dimensional theory and the results are indicated in Fig. 14 by a broken line and a thick line respectively. The results calculated by the three-dimensional lattice theory agree well with the test results. As the stream lines in Fig. 12

were drawn as the first approximation under the assumption that the flow has no circumferential velocity, the stream lines must be corrected on account of the circumferential velocity which is determined by the above-obtained circulation distribution. These corrected stream lines may be regarded as the second approximation⁽⁶⁾.

If the above processes are repeated, the higher order approximations may be obtained.

5. Conclusion

The air tests of a turbine of axial flow type showed that the cascade tests were very important for designing turbines and that the flow at the inlet of the runner had not uniform distribution of circulation and was affected by the curvature of the channel. The effects of the curvature could be approximately determined by the quasi-two-dimensional flow theory.

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Fig. 14