Calculation of Aerodynamic Forces Acting on
a Circular Cylinder with Tangential
Injection of Air Immersed in a Uniform Flow*

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The flow region around a circular cylinder with tangential injection of air immersed in a uniform flow is divided into four parts that are parts of curved wall-jet, turbulent boundary layer, laminar boundary layer and wake. The characteristics of each part are calculated by the authors' method on the curved wall-jet, Furuya's method on the turbulent boundary layer and Thwaites' method on the laminar boundary layer.

Based upon these characteristics obtained, the aerodynamic forces are calculated and compared with the authors' experiment.

It was found that, when one parameter among the lift coefficient, the drag coefficient, the front stagnation-point, the separation point of jet and the base-pressure coefficient was given, the rest of them are uniquely determined.

1. Introduction

It has been already known in the latter half of the 19th century that the air jet injected tangentially to a convex wall flows along its surface, and this phenomenon has been called Coanda effect since Coanda got a French patent for it in 1932. The phenomenon attracted many researchers' attention, especially after it had been applied to the aircraft wing for boundary layer control.

Since then this effect has been applied to the fluidics, film-cooling, radial reaction turbine, atomizer and so on⁽¹⁾.

The theoretical investigations on this subject became also flourishing. Glauert⁽²⁾ and Carrier & Eichelbrenner⁽³⁾, for instance, made calculation of the characteristics of the wall-jet on a flat plate in the absence of and in the presence of uniform external flow with an adverse pressure gradient, respectively. Spalding⁽⁴⁾ expressed the velocity profile of wall-jet in a uniform flow by a single relation, and presented a unified theory. Thomas⁽⁵⁾, on the other hand, calculated the jet momentum required to control the separation point of jet by using the momentum thickness only and the slot efficiency experimentally obtained.

All of these theories ignore the curvature of the wall surface. Stratford, Jawor & Goleworthy⁽⁶⁾ showed that Prandtl's mixing length becomes longer when the stream line of jet curves, and experimentally confirmed it. Sawyer⁽⁷⁾ too obtained the same conclusion. Thus the thickness of wall-jet on a circular cylinder grows faster than on a flat plate, and the empirical equation obtained by Newman⁽⁸⁾ on the thickness of wall-jet on a circular cylinder can be derived⁽⁹⁾ by applying the theory of Stratford et al or

Sawyer if the coefficients are assumed. In a former report, the authors calculated the characteristics of wall-jet with an external flow on a circular cylinder taking the curvature effect on the jet thickness into account, and showed the calculated separation point of flow was in good accordance with that of experiments.

In order to apply the authors' calculation method to the boundary-layer control by Coanda effect, the streamwise pressure distribution is required to be known. For instance, the pressure distribution in which the flow separates at the trailing edge of a circular-arc airfoil with tangential injection can rather easily be calculated, so that the required intensity of jet can be calculated by the authors' method. It was also pointed out that the pressure distribution might be theoretically obtained even in the flow around a blunt body, which necessarily includes a separated region. To obtain such a distribution, the stream direction downstream of a separation point in an inviscid flow and the position of the point itself must be given as the first approximation. Since it is almost impossible to satisfy the requirements, it is necessary to determine an empirical relation. Here the questions arise as; (1) whether it is possible to calculate the distribution if experimental data on the dead-water region are available and (2) , if possible, what are the experimental data which can be determined and used easiest? Since the circumferential pressure distribution on a circular cylinder with tangential injection of air immersed in a uniform flow can be approximated by velocity potentials, the lift and drag coefficients as well as the separation point can be calculated if the empirical relation between the separation point can be calculated if the empirical relation between the separation point and the base-pressure coefficient is used. This report presents the method of calculation and the comparison between the calculation and the experiment.

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2. Nomenclature

R	: radius of the circular cylinder
h	: slot width
x	: abscissa taken along the cylinder surface
y	: ordinate perpendicular to x
y_m, y_m^* , and $\frac{1}{2}\delta$	: y at the point where $u=u_m$, $u=(U+u_m)/2$, and $u=U$, respectively.
θ	: angle clockwise measured from the leading edge (See Fig.1)
θ_j	: station of the slot ($\theta_j=0^\circ$ at $\theta=90^\circ$)
θ_{st}	: front stagnation-point
θ_{su} and θ_{sl}	: separation points on the upper and the lower cylinder surfaces, respectively
$(X, X_j, X_{st}, X_{su},$ and $X_{sl})$	: $=\pi/180(\theta, \theta_j + 90^\circ, \theta_{st}, \theta_{su},$ and $\theta_{sl},$ respectively)
$\tilde{h}$	: $=h/R$
θ	: (momentum thickness of the part of boundary layer of jet)/R
Y_m	: y_m/R
Y_m^*	: y_m^*/R
B	: $=2(Y_m^* - Y_m)$
Δ_1 and Δ_2	: (displacement and momentum thicknesses of the boundary layer of the external flow, respectively)/R
u, v	: x, y components of velocity
u_m	: maximum velocity in the jet cross-section
U	: velocity at the outer edge of the mixing region
U_∞	: velocity of the uniform flow
p	: static pressure
p_m	: p at $y = y_m$
p_s	: p at $y = \delta$
τ	: shear stress
$\tilde{U}$	: $=U/U_\infty$
$\tilde{U}_{su}$	: $\tilde{U}$ at $\theta = \theta_{su}$
$\tilde{U}_{sl}$	: $\tilde{U}$ at $\theta = \theta_{sl}$
P_r	: $(p_s + 1/2\rho U^2)/\frac{1}{2}\rho U_\infty^2$
n	: index in the n-th power-law velocity profile
K	: form parameter of Furuya's velocity profile
m	: $= U/u_m$
K and Λ	: see Eq. (14)
ρ	: air density
c_μ	: intensity of jet ($=$ (momentum of jet)/($\rho U_\infty^2 R$))
R_∞	: Reynolds number based on the velocity of the uniform flow
c_p	: pressure coefficient
c_{pb}	: base-pressure coefficient ($=1-\tilde{U}_{su}^2$)
c_{μ_s} and c_{μ_d}	: see Eq.(17)
c_{l_s} and c_{l_d}	: lift and drag coefficients, respectively, obtained from the integration of the static pressure distribution
c_L and c_D	: total lift and drag coefficients, respectively, including the reaction of jet

3. Theory

Since there is no unified theory which can calculate the lift and drag forces acting on a two-dimensional circular cylinder with tangential injection of air, it is necessary to divide the flow field into the fields that the existing calculation methods

can be applied to. In the present theory, the flow field around the circular cylinder is divided into five fields: (1) the laminar boundary layer field from the front stagnation-point to the transition point or to the separation point (III or IV in Fig.1); (2) the turbulent boundary layer field from the transition point to the station of the slot (II); (3) the wall-jet field from the slot to the separation point (I); (4) the separated region (V); (5) potential field (VI). The transition point is assumed to be at $\theta=90^\circ$ in Fig.1. The method of calculation is described below.

Fig.1 Classification of the flow field

3.1 The wall-jet field

The second method of the former three methods of calculation⁽¹⁰⁾ on a wall-jet with external flow is used here in its extended form. At first, the wall-jet was divided into two layers such as the boundary layer closer to the wall and the mixing region closer to the external flow. Furuya's equations⁽¹¹⁾ on a flat plate were modified to those of the turbulent boundary layer of wall-jet on a circular cylinder. The simplified angular momentum equation was applied to the mixing region under the assumption that the jet thickness, y_m , is smaller than the cylinder radius R , thus c_μ is small.

As fairly large value of c_μ appears in the present calculation, the condition of small c_μ is removed in the angular momentum equation. The equation of motion in the circumferential direction and the equation of continuity are given below when the higher terms of y_m/R are ignored;

$$\frac{R}{R+y} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{uv}{R+y} = -\frac{1}{\rho} \frac{R}{R+y} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} + \frac{1}{\rho} \frac{\tau}{R+y} \quad (1),$$

$$\frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \left\{ \left(1 + \frac{y}{R}\right) v \right\} = 0 \quad (2).$$

Multiplying Eq.(1) by $(R+y)^2$ and integrating it from y_m to δ by using Eq.(2) yield the angular momentum equation;

$$\begin{aligned} & \frac{\partial}{\partial x} \int_m^s (u^2 + \frac{p}{\rho})(R+y) dy \\ &= (R+\delta)U \frac{\partial}{\partial x} \int_0^s u dy - (R+y_m)u_m \frac{\partial}{\partial x} \int_0^s u dy + \\ &+ (R+\delta) \frac{p_s}{\rho} \frac{d\delta}{dx} - (R+y_m) \frac{p_s}{\rho} \frac{dy_m}{dx} - \int_m^s \frac{1}{\rho} (1 + \frac{y}{R}) dx \end{aligned} \quad (3).$$

Stratford, Jawor & Golesworthy gave the ratio $\bar{L}$ of the mixing lengths in the flow without and with curvature $1/r$ as follows;

$$\bar{L} = \frac{1}{1 - 2 \frac{u}{r} \frac{1}{(-\frac{\partial u}{\partial y})}} \quad (4).$$

Hence ignoring the higher order than a degree of

$$|2 \frac{u}{r} \frac{1}{(-\frac{\partial u}{\partial y})}| < 1$$

yields the relation of eddy kinematic viscosity, ϵ , for $1/r=0$ to ϵ_c for $1/r \neq 0$, i.e.,

where

$$\begin{aligned} a_{31} &= (H_1 + H) \{ (f_8 + 2.5f_7 - 2f_1)B^2\tilde{U}^2 + (2f_4 - 3f_1 + 0.5)B\tilde{U}^2 - (4f_8 - 2f_7)(1 + Y_m)B\tilde{U}^2 - \\ &\quad - 2f_1(1 + Y_m)\tilde{U}^2 + P_T(1 + Y_m + 0.5B) \} + \{ (1 + Y_m + B)H + (1 + Y_m) \frac{H_1}{m} \} \frac{\tilde{U}^2}{m} \\ a_{32} &= (\frac{dH_1}{d\kappa} + \frac{dH}{d\kappa}) \frac{\Theta}{H_1 + H} [a_{31} - \{ (1 + Y_m + B)H + (1 + Y_m) \frac{H_1}{m} \} \frac{\tilde{U}^2}{m}] + \{ (1 + Y_m + B) \frac{dH}{d\kappa} + \\ &\quad + (1 + Y_m) \frac{1}{m} \frac{dH_1}{d\kappa} \} \frac{\tilde{U}^2 \Theta}{m} \\ a_{33} &= (\frac{df_1}{dm} - 1.5 \frac{df_7}{dm}) \frac{B^3\tilde{U}^2}{3} + (\frac{df_1}{dm} - 1.5 \frac{df_4}{dm} + \frac{1}{2m}) B^2\tilde{U}^2 + (\frac{df_8}{dm} - 0.5 \frac{df_7}{dm})(1 + Y_m) B^2\tilde{U}^2 + \\ &\quad + (\frac{df_1}{dm} + \frac{1}{2m})(1 + Y_m) B\tilde{U}^2 + \frac{\Theta H_1}{m^2} \{ (1 + Y_m)(1 - \frac{1}{m}) + B \} \tilde{U}^2 \\ a_{34} &= (2f_1 - 3f_7)B^2\tilde{U}^2 + (4f_1 - 2f_4 - \frac{1}{m})B\tilde{U}^2 + (4f_8 - 2f_7)(1 + Y_m)B\tilde{U}^2 + (2f_1 - \frac{1}{m})(1 + Y_m) \\ &\quad \times \tilde{U}^2 - P_T(1 + Y_m + B) \\ a_{35} &= 0.012(\frac{1}{m} - 1) \{ (1 + Y_m)(\frac{1}{m} - 1) + 2B(\frac{1}{m} + 1) \} B\tilde{U}^2 - [(2f_1 - 3f_7) \frac{B^3\tilde{U}}{3} + (2f_1 - \\ &\quad - f_4 - \frac{1}{2m} - 1)B^2\tilde{U} + (2f_1 - \frac{1}{2m} - 1.5)(1 + Y_m)B\tilde{U} + (2f_8 - f_7)(1 + Y_m)B^2\tilde{U} - \\ &\quad - \frac{H_1}{m} \{ (1 + Y_m)(1 - \frac{1}{m}) + B \} \tilde{U} \} \frac{d\tilde{U}}{dX} \end{aligned} \quad (9),$$

and

$$\begin{aligned} H &= \int_0^s (1 - \frac{u}{u_m}) dy / \int_0^s \frac{u}{u_m} (1 - \frac{u}{u_m}) dx \approx 1.3 + 1.3(0.7 - \kappa) + 3(0.7 - \kappa)^2 \\ H_1 &= 2 + 2/(H - 1), \quad Y_m = \Theta(H_1 + H), \quad B = 2(Y_{m2} - Y_m) \\ f_1 &= \frac{1}{m} + 0.375(\frac{1}{m} - 1)^2 \\ f_4 &= 1 + 1.405(\frac{1}{m} - 1) + 0.5776(\frac{1}{m} - 1)^2 \\ f_7 &= 1 + 0.7974(\frac{1}{m} - 1) + 0.2800(\frac{1}{m} - 1)^2 \\ f_8 &= \frac{1}{m}(\frac{1}{m} - 1)(0.46875 + 0.53125\frac{1}{m}) \end{aligned} \quad (10).$$

This is the extended angular momentum equation in the mixing region. The existing

$$\epsilon_c \approx \epsilon \{ 1 + 4 \frac{u}{r} \frac{1}{(-\frac{\partial u}{\partial y})} \} \quad (5).$$

Therefore, the shear stress, τ , for $1/r(\approx 1/(R+y))$ is given by

$$\tau \approx \rho \epsilon (\frac{\partial u}{\partial y} - 4 \frac{u}{R+y}) \quad (6).$$

The value of ϵ for $1/r=0$ is expressed by the following equation,

$$\epsilon = 0.024 (y_m - y_m)(u_m - U) \quad (7).$$

When the velocity profiles of the boundary layer and of the mixing region are given by the power law and by the cosine function⁽¹⁰⁾, respectively, and τ being expressed by Eqs. (6) and (7), Eq. (3) is rearranged to

$$a_{31} \frac{d\Theta}{dX} + a_{32} \frac{d\kappa}{dX} + a_{33} \frac{dm}{dX} + a_{34} \frac{dY_m}{dX} = a_{35} \quad (8),$$

equations⁽¹⁰⁾ of the boundary layer and of the growth of the thickness of the mixing region are rewritten as follows;

$$\left. \begin{aligned} a_{11} \frac{d\theta}{dX} + a_{13} \frac{dm}{dX} &= a_{15} \\ a_{22} \frac{d\kappa}{dX} + a_{23} \frac{dm}{dX} &= a_{25} \\ a_{41} \frac{d\theta}{dX} + a_{42} \frac{d\kappa}{dX} + a_{44} \frac{dY_2}{dX} &= a_{45} \end{aligned} \right\} (11),$$

where

$$\left. \begin{aligned} a_{11} &= 1, \quad a_{13} = (H_1 - 2) \frac{\theta}{m} \\ a_{15} &= 0.02949 \kappa^{\frac{1}{4}} m^{\frac{1}{4}} / (R_\infty \tilde{U} \theta)^{\frac{1}{4}} - \{(m^2 - 1) \times (H_1 + H) + (H + 2)\} \frac{\theta}{\tilde{U}} \frac{d\tilde{U}}{dX} \\ a_{22} &= 1, \quad a_{23} = -(1.286 - 0.143 H_1) \frac{\kappa}{m} \\ a_{25} &= 1.46(1 - \kappa^2) \frac{1}{\kappa \tilde{U}} \frac{d\tilde{U}}{dX} + 0.2049(1 - \kappa^2)(0.67 - \kappa) m^{\frac{1}{4}} / (R_\infty \tilde{U} \kappa^{\frac{1}{4}} \theta^{\frac{1}{4}})^{\frac{1}{4}} + (m^2 - 1) \{1.469 - 0.143 \kappa^2 (H_1 + H)\} \frac{1}{\kappa \tilde{U}} \frac{d\tilde{U}}{dX} \\ a_{41} &= (H_1 + H)(1 + m) / [1 - 0.276(X - X_J) + m\{1 + 0.276(X - X_J)\}] \\ a_{42} &= a_{41} \frac{\theta}{H_1 + H} \left(\frac{dH_1}{d\kappa} + \frac{dH}{d\kappa} \right), \quad a_{44} = -1 \\ a_{45} &= -0.115(1 - m)(1 + 2.4 Y_2) / [1 - 0.276(X - X_J) + m\{1 + 0.276(X - X_J)\}] \end{aligned} \right\} (12).$$

Here, the values of $c = 0.115$ and $k = 2.4$ are applied⁽¹²⁾ as the constants in a_{41} and a_{45} . These c and k appear as the constants in the equation of the growth of the thickness of the mixing region of jet without an external flow. Equations (8) and (11) can be solved, as an initial value problem, to give θ, κ, m , and Y_2 when $\tilde{U}$ in X direction is known. The separation point of jet, X_{su} , is defined as the station, X , where κ attains the value of 0.45.

3.2 Turbulent and laminar boundary layer fields(II, III and IV in Fig.1)

In the present knowledge on the transition, it is hard to accurately calculate the transition point from only the Reynolds number. Therefore, the assumptions on the transition point are made by taking it into account that the Reynolds number of the experiment compared with the calculation is about 3×10^5 , such as; the transition point on the upper cylinder surface is at $\theta = 90^\circ$; while the laminar boundary layer on the lower surface remains laminar since the pressure gradient is small there. By these assumptions, the part II in Fig.1 is turbulent boundary layer, the part III and IV being laminar one.

Thwaites' method⁽¹⁴⁾ is applied for the analysis of the laminar boundary layer.

The equations are

$$\Delta_2 = \left(\frac{0.9}{R_\infty} \tilde{U}^{-6} \int_{x_{st}}^x \tilde{U}^5 dX \right)^{\frac{1}{2}} \quad (13),$$

$$K = \frac{1}{2} R_\infty \Delta_2^2 \frac{d\tilde{U}}{dX} \quad (14),$$

$$\Lambda \left\{ \frac{1}{315} \left(37 - \frac{1}{3} \Lambda - \frac{5}{144} \Lambda^2 \right) \right\}^2 = K \quad (15).$$

Furuya's method⁽¹¹⁾ is used to calculate the turbulent boundary layer. The equations are obtained when $m=1$ is substituted in the first and second equations of Eqs.(11). They are

$$\left. \begin{aligned} \frac{d\theta}{dX} &= 0.0295 \kappa^{1.75} (R_\infty \tilde{U} \theta)^{\frac{1}{4}} - (H + 2) \frac{\theta}{\tilde{U}} \frac{d\tilde{U}}{dX} \\ \frac{d\kappa}{dX} &= 1.46(1 - \kappa^2) \frac{1}{\kappa \tilde{U}} \frac{d\tilde{U}}{dX} + 0.2049(1 - \kappa^2)(0.67 - \kappa) / (R_\infty \tilde{U} \kappa^{\frac{1}{4}} \theta^{\frac{1}{4}})^{\frac{1}{4}} \end{aligned} \right\} (16).$$

Both Eqs.(13) and (16) can be solved when the $\tilde{U}$ distribution along X as well as the initial values are known.

3.3 Circumferential pressure distribution

The equations described so far can be solved as the initial value problem, when the circumferential pressure distribution, in other words, the velocity distribution, $\tilde{U}$ is given. The pressure distribution around a circular cylinder with tangential injection can be approximated by the following equations;

$$c_p = 1 - \tilde{U}^2 = 1 - \left\{ 2 \sin X + \frac{C_{20}}{2\pi} - \frac{C_{40}}{4\pi} \cot \frac{\pi - X_{st} - X}{2} \right\}^2 \quad (17),$$

and

$$c_p = c_{pb} \quad \text{for } X_{su} \leq X \leq X_{sl} \quad (18).$$

Equation (17) yields $c_p = -\infty$ at $X = \pi - X_{st}$. c_p in the separated region ($X_{su} \leq X \leq X_{sl}$, V in Fig.1), however, is nearly constant in the physical field, so Eq. (18) is successfully applied to this region.

Given c_{20} , c_{40} and X_{st} , it is rather easy to obtain the value of c_p . However, it is fairly tedious to obtain c_{20} , c_{40} and X_{st} when c_p is given. In the latter case, since the calculation becomes less tedious when rather the velocity distribution than c_p is given, Eq.(17) is replaced by

$$\tilde{U} = 2 \sin X + \frac{C_{20}}{2\pi} - \frac{C_{40}}{4\pi} \cot \frac{\pi - X_{st} - X}{2} \quad (19).$$

The relation of the separation velocity $\tilde{U}_{su}$ to the base pressure is then given by

$\tilde{U}_{su} = \sqrt{1 - c_{p_0}}$. In such a case, c_{l_0} , c_{d_0} and X_{st} can be obtained if $\tilde{U}$'s at three different points on a cylinder are given. X_{st} , X_{su} and X_{sl} are chosen as the three points so that we get

$$\left. \begin{aligned} 2\sin X_{su} + \frac{c_{l_0}}{2\pi} - \frac{c_{d_0}}{4\pi} \cot \frac{\pi - X_{st} - X_{su}}{2} &= \tilde{U}_{su} \\ 2\sin X_{sl} + \frac{c_{l_0}}{2\pi} - \frac{c_{d_0}}{4\pi} \cot \frac{\pi - X_{st} - X_{sl}}{2} &= \tilde{U}_{sl} \\ 2\sin X_{st} + \frac{c_{l_0}}{2\pi} - \frac{c_{d_0}}{4\pi} \cot \frac{\pi - 2X_{st}}{2} &= 0 \end{aligned} \right\} (20),$$

where

$$\tilde{U}_{sl} = -\tilde{U}_{su} \quad (21).$$

Now by putting

$$\beta = \tan \frac{X_{st}}{2}, \quad -1 \leq \beta \leq 0 \quad (22),$$

the third equation in Eqs.(20) is reduced to

$$f(\beta) = \beta^4 + a\beta^3 + b\beta - 1 = 0 \quad (23),$$

where

$$a = \frac{c_{d_0} + 8\pi}{c_{l_0}}, \quad b = \frac{c_{d_0} - 8\pi}{c_{l_0}} \quad (24).$$

Eliminating c_{l_0} and c_{d_0} from Eqs. (24), and the first and second equations of Eqs. (20) gives

$$\left. \begin{aligned} a &= 2 \frac{c_{sl} - c_{su} + 2\{g_u(\beta) - g_l(\beta)\}}{c_{sl}g_u(\beta) - c_{su}g_l(\beta)} \\ b &= 2 \frac{c_{sl} - c_{su} + 2\{g_l(\beta) - g_u(\beta)\}}{c_{sl}g_u(\beta) - c_{su}g_l(\beta)} \\ g_u(\beta) &= \frac{c_{tu} + \beta}{1 - c_{tu}\beta}, \quad g_l(\beta) = \frac{c_{tl} + \beta}{1 - c_{tl}\beta} \\ c_{su} &= \tilde{U}_{su} - 2\sin X_{su}, \quad c_{sl} = \tilde{U}_{sl} - 2\sin X_{sl} \\ c_{tu} &= \cot \frac{\pi - X_{su}}{2}, \quad c_{tl} = \cot \frac{\pi - X_{sl}}{2} \end{aligned} \right\} (25).$$

Here, if X_{su} , $\tilde{U}_{su}$ and X_{sl} , $\tilde{U}_{sl}$ are given, a and b are related to only β . Introducing them into Eq.(23) is to give the solution, β . Although the behavior of Eq.(23) is not detailed here, it is obvious that the bi-quadratic equation with respect to β , when it is so regarded, has two or four real roots. The meaningful root among them is in the range of $-1 \leq \beta \leq 0$ and is the one nearer to zero. Since $df(\beta)/d\beta$ does not change its sign in the region between the root and zero, Eq.(23) can be solved by Newton's method. Thus the root β obtained is substituted into Eqs.(22) to give X_{st} , and in turn c_{l_0} and c_{d_0} are obtained from Eqs.(24) and (25). Consequently, if the separation velocities $\tilde{U}_{su}$ and $\tilde{U}_{sl}$ are given at the separation point X_{su} on the upper surface and X_{sl} on the lower surface, respectively, then

the velocity distribution (19) or the pressure distribution (17) are obtained.

3.4 Method of calculation

The flow field around a circular cylinder with tangential injection of air immersed in a uniform flow is to be nearly determined if the Reynolds number based on the uniform velocity R_{∞} , the intensity of jet c_{μ} , the slot width h and the station of the slot X_J are given. The authors successfully tried in this report to calculate the lift and drag coefficients acting on the cylinder by giving those values. The method of calculation is described below.

The values of R_{∞} , c_{μ} , h and X_J are assumed to be given. Then first assuming the values of X_{st} , c_{l_0} and c_{d_0} yields $\tilde{U}$ in Eq.(19). Introducing this $\tilde{U}$ and X_{st} into Eqs.(13) and (14), we get the lower separation point X_{sl} , which is defined as the position of $K = -0.1567$. $\tilde{U}_{sl}$ corresponding to this X_{sl} is obtained from Eq.(19). In the same manner, Δ_2 and Δ_1 at $X = \pi/2$ are obtained from Eqs. (13) to (15). When $X_J = \pi/2$ (when the slot is stationed right at the top of the cylinder), the initial values of jet at the slot exit, θ_0 , m_0 , K_0 and Y_{m_0} are obtained by making use of Δ_2 , Δ_1 , $\tilde{U}$ and given values of c_{μ} and h . When $X_J > \pi/2$, the initial values of turbulent boundary layer are $\theta = \Delta_2$ and $\kappa = 0.7$, and integrating Eqs.(16) yields θ as well as K , and then the displacement thickness at $X = X_J$ by considering the first equation of Eqs. (10). Using these values and the given c_{μ} and h together with the assumed value of $\tilde{U}$, we get the initial values of jet at the slot exit when $X > \pi/2$. By making use of these initial values and the assumed value of $\tilde{U}$, Eqs.(8) to (12) are integrated to give the upper separation point X_{su} and $\tilde{U}_{su}$. In this case, X at which $K=0.45$ is defined as X_{su} .

If $\tilde{U}_{su}$ and $-\tilde{U}_{sl}$ thus obtained are different each other, let the velocities at X_{su} and X_{sl} be $(\tilde{U}_{su} - \tilde{U}_{sl})/2$ and $-(\tilde{U}_{su} - \tilde{U}_{sl})/2$, respectively. Making use of these second approximations, we can get the values of X_{st} , c_{l_0} and c_{d_0} by the method of the section 3.3, and the second approximation of $\tilde{U}$ when those X_{st} , c_{l_0} and c_{d_0} are inserted into Eq.(19). Repeating the same procedure will yield the separation points X_{su} and X_{sl} at which $\tilde{U}_{su} = -\tilde{U}_{sl}$. If the calculated $\tilde{U}_{su}$ does not agree with the experimental value, the value of c_{d_0} is varied little by little until the calculated $\tilde{U}_{su}$ agrees with the experimented $\tilde{U}_{su}$ at X_{su} . Finally Eqs.(17) and (18) are integrated to give the lift coefficients c_{l_s} , c_l and the drag coefficients c_{p_s} , c_p .

4. Comparison with the experiment

The results of calculation are compared with the authors' experimental ones⁽¹²⁾. The details of the experiment should be referred to the reference(12), but the outline is as follows: the dimensions of the test circular cylinder were 100mm in diameter $2R$, $0.01(=h)$ in slot width and about four in aspect ratio; the cylinder had end-plates of 360mm high and 726mm long at both ends; the pressure

distribution was measured on the cylinder surface at $R_\infty = 3.2 \times 10^5$, in the range of $c_\mu = 0 \sim 0.46$ and $\theta_J = -40^\circ \sim 30^\circ$. The c_μ 's in the reference (12) were calculated based on the uniform flow velocity and the jet velocity when the compressed air in the cylinder expanded from the pressure inside the cylinder to the static pressure of the uniform flow. The c_μ 's in the present report, however, are based on the uniform flow velocity and the jet velocity when the compressed air inside the cylinder expands to the static pressure at the station of the slot. For it is favorable to have more accurate c_μ 's that determine the initial values at the slot exit. The results of the experiment shown in this report are converted to the values in the two-dimensional flow (the flow around an infinitely long cylinder) by the method described in the reference (12). The equations were numerically solved by using the electronic computer of Nagoya University for the values of $R_\infty = 3 \times 10^5$, $\tilde{h} = 0.011$, $\theta_J = 0^\circ, 10^\circ, 20^\circ$ and $c_\mu = 0.1 \sim 0.5$. The systems of the differential equations were integrated by Runge-Kutta-Gill's method and Simpson's method was applied to obtain the lift and drag coefficients from the pressure distribution. The reasons why c_μ 's were limited to the greater value than 0.1 are because an unacceptable condition of $m > 1$ takes place and because the $\tilde{U}_{su} - \theta_{su}$ relation becomes dependent on θ_J , when c_μ is too small.

The relation of the separation point θ_{su} to the base-pressure coefficient c_{pb} ($= 1 - \tilde{U}_{su}^2$) shown in Fig.2 is almost independent of the slot station θ_J . The figure shows the experimental values in the range of $\theta_J = -40^\circ \sim 30^\circ$ and $c_\mu = 0 \sim 0.57$. It is obvious that c_{pb} is uniquely related to θ_{su} independently of θ_J , when θ_{su} or c_μ is not so small. The solid line is the curve to be used for calculation. Fig.3 compares the calculated result (solid line) with the experimental one (symbol $\odot$) of the circumferential pressure distribution. The calculation is performed for $c_\mu = 0.3$ and $\theta_J = 0^\circ$, while the experiment for $c_\mu = 0.2998$ and $\theta_J = 0^\circ$. The experimental values in Fig. 3, however, are shifted down by an induced angle of attack 8.9° in the θ -direction. The figure clearly shows that the calculated result well agrees with the experimental one.

Fig.2 Relation between base pressure c_{pb} and upper separation point θ_{su}

Fig.3 Circumferential pressure distribution c_p

Fig.4~5 indicate the lift coefficient c_L and the drag coefficient c_D , respectively, against the jet intensity c_μ at various values of θ_J as a parameter. Here lines show the calculated results and the symbols $\odot$ those of experiment.

Fig.4 Relation between jet intensity c_μ and lift coefficient c_L

Fig.5 Relation between jet intensity c_μ and drag coefficient c_D

It is seen in Fig.4 that the calculated results at $\theta_J = 0^\circ$ and 10° well agree with the measured ones, while the former at $\theta_J = 20^\circ$ is a little larger than the latter, especially at larger values of c_μ . According to the authors' experiment⁽¹²⁾, c_L does no

longer increase even if θ_j is increased, while c_μ is kept constant when c_μ is large and $\theta_j \geq 10^\circ \sim 20^\circ$. This phenomenon seems to be relevant to the suction effect of jet on the flow upstream of the slot. Such an effect was not taken into consideration in the equations mentioned previously, which in turn seems to have given larger calculated values of c_l than the experimental counterparts at $\theta_j = 20^\circ$. Since the accuracy of the conversion of the experimental coefficient c_p to the two-dimensional counterpart is not good enough when c_l is large, the comparison between the calculated result and the measured one does not necessarily reveal so much on the accuracy of calculation. The calculated result, however, is seen in Fig.5 to agree with that of measurement, when c_μ is small, namely when the accuracy of the conversion is good enough. The calculated result clearly shows that the larger θ_j is, the smaller c_p is at equal c_μ although it is not so obvious in experiment. It is also seen from the curves in Fig.5 that the thrust force acts on the cylinder when $c_\mu > 0.2 \sim 0.3$.

Fig.6 Relation between lift coefficient c_{Ls} and front stagnation-point θ_{st} (solid line is due to calculation)

Fig 6 shows the relation between the lift coefficient c_{Ls} (to exclude the jet reaction) and the front stagnation-point θ_{st} . The experimental values (symbols $\odot$ etc.) are well coincident with the calculated values (solid line). The c_{Ls} is obviously seen to be linear to θ_{st} .

Fig.7 Relation between lift coefficient c_{Ls} and upper separation point θ_{su} (solid line is due to calculation)

The experimented θ_{su} are plotted against those of $\sqrt{c_{Ls}}$ in Fig.7 in which the solid line is the calculated result. θ_{su} is shown to be nearly linear to $\sqrt{c_{Ls}}$ in both experiment and calculation although the experimental values are a little scattered at the larger values of c_{Ls} . The relation of c_{Ls} to c_{Ds} , where c_{Ls} and c_{Ds} do not include the jet reaction, is shown in Fig.8. Although the experiments are a little scattered when c_{Ls} is large, and the conversion of c_{Ds} to the two-dimensional values becomes less accurate at larger values of c_{Ls} , the calculated result can be said to well agree with those of experiment.

Fig.8 Relation between lift coefficient c_{Ls} and drag coefficient c_{Ds} (solid line is due to calculation)

5. Discussion

The equations on the wall-jet in the present calculation were extended from those of the former second method⁽¹⁰⁾, possibly to make a calculation at large c_μ . In order to inspect the equations, the characteristics of the wall-jet were calculated by using the measured static pressure distributions when $\theta_j = 0^\circ$, and $c_\mu = 0.151, 0.408$, and 0.661 . The separation points calculated were $151^\circ, 181^\circ$, and 199° , while those points measured $147^\circ, 180^\circ$, and 187° , the calculated results by the former second method $144^\circ, 165^\circ$, and 176° , respectively. The considerable difference at $c_\mu = 0.661$ seems to have come from neglecting the fluid compressibility. Anyway, the equations are useful in the range of c_μ up to 0.5 , as the agreement of the theory with the experiment implicitly shows above. Under the same restrictions as in the former report⁽¹⁰⁾, or in other words, when c_μ is small, the difference between the present calculation and the former one is negligible and at most below one to a few degrees of angle. It is therefore obvious that the present results of calculation agree with those of experiment not only when c_μ is small but also when c_μ is large ($c_\mu \leq 0.5$).

Figs. 2, 6 ~ 8 indicate the relation among $\theta_{su}(X_{su})$, $c_{pb}(=1-\tilde{U}_{su}^2)$, $\theta_{st}(X_{st})$, c_{ls} and c_{ps} such as the other values are uniquely determined when one of them is given. The calculated results reveal this more confirmatory. It is extremely difficult to analytically find the relation, to analytically solve, for instance, Eqs. (8) and (11).

It is, however, possible to qualitatively show the relation. First, it is assumed that Eqs. (8) and (11) are solved to give the relation as shown in Fig. 2 as

$$\tilde{U}_{su} = \tilde{U}_{su}(X_{su}) \quad (26).$$

On the other hand, the first equation of Eqs. (14) together with Eqs. (13) and (19) is rewritten as follows;

$$K = K(X, X_{st}, \tilde{U}) \quad (14a),$$

$$\tilde{U} = \tilde{U}(X, X_{st}, c_{le}, c_{do}) \quad (19a).$$

Substituting $X = X_{su}$, $\tilde{U} = \tilde{U}_{su}$ and $X = X_{st}$, $\tilde{U} = 0$ into Eq. (19a) yields $c_{le} = c_{le}(X_{st}, X_{su})$ and $c_{do} = c_{do}(X_{st}, X_{su})$. Hence $\tilde{U} = \tilde{U}(X, X_{st}, X_{su})$. Since $\tilde{U}_{sl} = -\tilde{U}_{su} = -\tilde{U}_{su}(X_{su})$, we get

$$X_{sl} = X_{sl}(X_{st}, X_{su}) \quad (27).$$

Putting $X = X_{sl}$, $K = -0.1567$ and $\tilde{U}_{sl} = -\tilde{U}(X_{sl})$ into Eq. (14a), we have

$$X_{sl} = X_{sl}(X_{sl}, X_{su}) \quad (28).$$

From Eqs. (27) and (28)

$$X_{sl} = X_{sl}(X_{su}) \quad (29).$$

That is, X_{sl} is uniquely determined if X_{su} is given.

Fig. 9 Calculated relation between the upper and the lower separation points, θ_{su} , θ_{sl}

Fig. 9 shows the relation of θ_{sl} to θ_{su} . Once X_{su} and X_{sl} are determined, X_{st} , c_{le} and c_{do} are all obtained from the relation mentioned above, which in turn gives the corresponding distribution of $\tilde{U}$. Integration of $\tilde{U}$ results in c_{ls} and c_{ps} . Namely, the above mentioned relation among X_{su} , c_{pb} , X_{st} , c_{ls} , and c_{ps} is qualitatively shown.

Although the empirical relation between $\theta_{su}(X_{su})$ and $c_{pb}(=1-\tilde{U}_{su}^2)$ is used in this report, it is obvious from the preceding discussion that any relation of two quantities among θ_{su} , c_{pb} , θ_{st} , c_{ls} and c_{ps} can be replaced by the relation. However, it is thought a fair

judgement to use the θ_{su} - c_{pb} relation, because the relation is least laborious to experimentally determine and the problem to obtain the pressure distribution is most directly connected to the shape of free stream lines and to the static pressure as well as the velocity profile in the wake.

6. Conclusion

Giving the values of c_{μ} , $\tilde{h}$, R_{∞} and θ_j , and assuming that the empirical relation of θ_{su} to $\tilde{U}_{su}$ is known, we calculated the aerodynamic forces acting on a circular cylinder with tangential injection of air, and obtained the following conclusions:

- (1) It is possible to calculate the lift and drag coefficients by the present theory in the range of $c_{\mu} = 0.1 \sim 0.5$ and $\theta_j = 0^\circ \sim 10^\circ$.
- (2) in this case, the larger c_{μ} as well as θ_j is, the smaller (larger) the drag (lift) coefficient is. The thrust force acts on the cylinder when $c_{\mu} > 0.2 \sim 0.3$ in the range of calculation.
- (3) In the range where the boundary layer thickness and the slot station can be assumed not to affect the external stream field as in the present theory, one of θ_{su} , c_{pb} , θ_{st} , c_{ls} and c_{ps} will uniquely determine the others.
- (4) The present theory on the wall-jet are valid up to a large value of c_{μ} ($c_{\mu} \leq 0.5$).

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