532.58

Experimental Investigation of the Wake of a Wedge*

By Tetsushi OKAMOTO**, Miki YAGITA***
and Kunio OHTSUKA****

The present work is concerned with an experimental investigation of the wake flow behind a two-dimensional wedge. The experiment was performed in the wind tunnel at the Reynolds number $Re = 9.0 \times 10^3$. The pressure distribution on the wedge surface, the vortex-shedding frequency behind the wedge and the distributions of velocities and static pressures in the wake flow were measured. From the surface pressure the drag of wedge was obtained. The velocity distribution and the width of the wake in the similar profile region were compared with the theoretical results obtained by Schlichting and Reichardt, and discussed in comparison with the results for a circular cylinder.

1. Introduction

In recent years increasing attention has been paid to the turbulent wake of a body in relation to wind effects on high-storied buildings and structures, diffusion problems in air-pollution and airplane accidents caused by turbulent air currents behind Mt. Fuji.

A turbulent wake is generally divided into three regions: (1) vortex region, where periodic vortex streets are formed, (2) transition region, where the vortex streets are in the process of breaking down and the flow becomes turbulent and develops into a similar velocity profile, and (3) similar profile region, where velocity profile is similar in each section.

Two theoretical studies were done by Schlichting⁽¹⁾ (1930) and by Reichardt⁽²⁾ (1942) on the similar profile region of the wake of a two-dimensional body. Schlichting shows that the experimental results of a circular cylinder agree well with the theoretical ones. However, the agreement for the bodies other than a circular cylinder has not been examined so far. These theories show that the velocity profile of a wake is related to the drag coefficient of the body. Hence, it seems to be significant to examine the wake of a wedge which has the base drag increasing with the vertex angle.

However, there have been few experimental investigations on the wake of a wedge, except for the studies^{(4),(5)} of vortex-shedding frequency in the vortex region.

In this paper, measurements of surface-pressure were made to obtain the drag coefficients of wedges having various vertex

angles. Furthermore, the wake flows in each region were measured and the results for the similar profile region were discussed in comparison with the theoretical ones. Also, these results were compared with those of a circular cylinder.

Notations

- x, y, z : Cartesian co-ordinates with origin at center of base of wedge (see Fig.1)
 B : width of wedge
 α : vertex angle of wedge
 S : length of wedge slant surface
 ξ : distance along wedge slant surface from vertex
 x_s : length of vortex region
 H : width of vortex region
 f : vortex-shedding frequency
 U_∞ : velocity of uniform free stream
 q : resultant velocity
 u_1 : velocity of wake flow
 u : velocity defect of wake flow $= U_\infty - u_1$
 u_m : maximum velocity defect of wake flow
 b : half width of wake
 $b_{1/2}$: value of y at $u = u_m/2$
 p : static pressure
 p_b : base pressure of wedge
 p_∞ : static pressure of free stream
 C_p : pressure coefficient $= (p - p_\infty) / \frac{1}{2} \rho U_\infty^2$
 C_{pb} : base pressure coefficient $= (p_b - p_\infty) / \frac{1}{2} \rho U_\infty^2$
 σ : $= -C_{pb}$

Fig.1 Notations

* Received 19th July, 1974.

** Professor, College of Science and Engineering, Aoyama Gakuin University, Setagayaku, Tokyo.

*** Research Associate, Tokyo Institute of Technology.

**** Automatic Transmission Design Engineer, Nissan Motor Company.

D : drag
 C_D : drag coefficient $= D / \frac{1}{2} \rho U_\infty^2 S$
 C_{Dp} : pressure drag coefficient
 Re : Reynolds number $= BU_\infty / \nu$
 St : Strouhal number $= Bf / U_\infty$
 St_i : Strouhal number $= Hf / U_\infty$

2. Apparatus and procedure of measurement

The wind tunnel used in this experiment is the N.P.L. blow-down type having a 400mm $\times$ 400mm square working section and being 3000mm in length. The upper wall of working section was adjusted so that there might be no pressure gradient in the flow direction. The velocity distributions of each section of working section were uniform within 0.25 % except in the proximity of the wall.

The wedge made of aluminum was spanned between the side walls of working section at a distance of 200mm from the contraction nozzle (see Fig.2). The width of wedge base was 10mm and kept constant for all wedges.

① Contraction nozzle ② Wedge
③ Pitot static tube and flow direction-meter ④ Carriage

Fig.2 Working section of wind tunnel

Fig.3 Resultant velocity profile ($\alpha = 30^\circ$)

Fig.4 Static pressure ($\alpha = 30^\circ$)

The vertex angles were varied 15° , 30° , 60° , 90° , 120° and 180° . When the vertex angle was 180° , the wedge became a flat plate. In this case, an aluminum board of 3mm thick was used as the flat plate and its rear edges were pared off at an angle of 45° to make sharp edges. Since the width of wedge was 2.5% of that of the working section, it may be considered there was no wall effect.⁽³⁾

Brass tubes of 1mm diameter were implanted in the slant surface and the base of wedge except the case of $\alpha = 120^\circ$. The brass tube had a number of pressure holes which were plugged with oil-clay except one hole for pressure measurement. The wedge of $\alpha = 120^\circ$ was so thin that brass tubes could not be implanted.

Measurements were done at $U_\infty = 13.0$ m/s corresponding to the Reynolds number of 9.0×10^3 . The intensity of turbulence $\sqrt{u'}/U_\infty$ of the free stream was 0.2% at $U_\infty = 13.0$ m/s. Roshko⁽⁴⁾ showed that Newton's quadratic resistance law held in the range of Reynolds numbers from $Re = 4 \times 10^3$ to 2.2×10^4 for the wedge of $\alpha = 90^\circ$. Hence it may be considered that Newton's quadratic resistance law holds in the present experiment.

The velocity profile and the static pressure were measured by a pitot-static tube of 1mm diameter, and the flow direction was measured by a direction-meter of two-hole type. The vortex-shedding frequency was measured by a hot-wire anemometer of constant-temperature type having a tungsten wire of 0.005mm diameter.

Furthermore, the wake flow of a circular cylinder of 10mm diameter was measured in order to compare with that of a wedge.

3. Results and discussion

3.1. Flow field

By way of example, Figs.3 and 4 show the distributions of velocities and static pressures of the wake flow for the wedge of $\alpha = 30^\circ$. Fig.5 shows the velocity vectors obtained by combining the results of flow direction with the resultant velocity shown in Fig.3. It is found from this figure that, on time-average basis, a closed vortex region is formed behind the wedge. Figs.6 and 7 show the contours of constant velocity and the contours of constant static pressure, respectively.

3.2. Surface-pressure distribution and drag coefficient

Fig.8 shows the pressure distributions on the slant surfaces of wedge. The result for case of $\alpha = 180^\circ$ (flat plate) agrees well with the experimental result by Fafe-Johan-

Fig.5 Velocity vectors in near-wake ($\alpha = 30^\circ$)

sen⁽⁵⁾ Also, it agrees well with the theoretical value by Plesset-Shaffer⁽⁶⁾⁽⁷⁾ In the case of $\alpha=90^\circ$ the present result agrees with the theoretical value over the most part of ξ . But, for $\alpha < 60^\circ$, the theoretical values are lower than the experimental ones.

The base pressure of wedge is practically uniform along the base. Fig.9 shows the variation of base pressure coefficient with α . This figure shows also the values determined by extrapolation from the static pressure of outer flow at $x/B=0$ and those along the center line behind the wedge. Both values agree well with each other. The result by Fage-Johansen for $\alpha=180^\circ$ is lower than the present result. This is due to the fact that the wall effects of wind tunnel are involved in measurements by Fage-Johansen.

The values of $\sigma(=-C_{p_s})$ which were used in analytical computation⁽⁶⁾⁽¹¹⁾ of the pressure distribution on the slant surface and the drag coefficient are shown in Table 1.

The pressure drag coefficient C_{D_p} was obtained from the surface pressure distribution. Then, the drag coefficient C_D was approximately determined by the addition of C_{D_p} to the skin-friction coefficient, which was estimated from Blasius' solution for a laminar flow on a flat plate. C_{D_p} and C_b are shown in Fig.10. Fig.11 shows the variation of C_b with σ , where C_b for $\alpha=120^\circ$ was interpolated from the results for other

Table 1

α	15°	30°	60°	90°	120°	180°
σ	0.73	0.95	1.09	1.16	(1.21)	1.27

Fig.6 Contours of constant velocity ($\alpha=30^\circ$) Fig.7 Contours of constant static pressure ($\alpha=30^\circ$)

vertex angles. In this figure, the present results are compared with the theoretical results of Plesset-Shaffer⁽⁶⁾ and of Wu⁽¹¹⁾ and also with the experimental results of Cox-Clayden⁽⁸⁾ and of Waid⁽⁹⁾. The present results of a flat plate ($\alpha=180^\circ$) agree well with those of Plesset-Shaffer and Wu. The experimental results for other vertex angles agree with the theoretical results for $\sigma < 0.3$, but the discrepancy between them appears as σ increases.

Also, the discrepancy between Plesset-Shaffer's result and that of Wu appears as σ increases. This is due to the fact that the wake model of the potential flow theory of Plesset-Shaffer is different from that of Wu. It is found from these results that Wu's result, based on the horizontal far-wake model of Joukowski-Roshko-Epper, is larger than the experimental one for $\sigma > 0.6$. On the other hand, Plesset-Shaffer's result, based on the Riabouchinsky image model, is slightly smaller than the experimental one for wedges of $\alpha < 60^\circ$ and $\sigma > 0.8$.

3.3. Near-wake (vortex region)

It has been found from Fig.5 that a closed vortex region is formed behind the

Fig.8 Surface-pressure of wedge

Fig.9 Base pressure of wedge

wedge. Fig.12 shows the dimensions of the vortex region obtained from its velocity vector chart. The dimensions are reduced as the vertex angle of wedge decreases. The present result for $\alpha=180^\circ$ lies between Fail's result⁽¹²⁾ and that of Arie-Rouse⁽¹⁵⁾, and therefore it may be considered to be appropriate.

The vortex-shedding frequency of wedge is shown in Fig.13 in the form of the Strouhal number St . These results agree well with those of Roshko⁽⁴⁾ for $\alpha=90^\circ$ and 180° and that of Fage-Johansen⁽⁵⁾ for $\alpha=17.8^\circ$. It is found that St or the vortex-shedding frequency increases as the vertex angle decreases. However, the intensity of vortex becomes weak and the decay of vortex accelerates.

If the Strouhal number St is defined by Hf/U_∞ , where H is the width of the vortex region, St is nearly constant for all vertex angles as shown in Fig.13.

3.4. Far-wake

The region from the rear end of the vortex region to a certain value of x/B is the transition region in which the vortices decay and disappear and then the velocity profile approaches a similar one. Roshko⁽⁴⁾ obtained $x/B=48$ as the distance for a circular cylinder in which the wake flow becomes completely turbulent and the energy spectrum of the velocity fluctuations approaches that of isotropic turbulence. Downstream from the point given by Roshko, the velocity profiles in each section of the wake become similar, and this region is called the similar profile region.

Fig.10 Pressure drag coefficient and drag coefficient

Fig.11 Comparison between theoretical values and experimental ones of C_D

By assuming that the mixing length l is constant over the width b and proportional to b , i.e., $l=\beta b$, Schlichting⁽¹⁾ obtained an equation for the similar velocity profile:

$$\frac{u}{U_\infty} = \frac{\sqrt{10}}{18\beta} \left(\frac{x}{C_D B} \right)^{1/2} \left(1 - \left(\frac{y}{b} \right)^2 \right)^{3/2} \quad (1)$$

Also, Reichardt⁽²⁾ obtained the following relation by assuming that the eddy viscosity is expressed as $\epsilon = \kappa u_m b$, which is a constant ϵ_0 because κ is an experimental constant:

$$\frac{u}{U_\infty} = \frac{1}{4/\pi} \frac{\sqrt{U_\infty C_D B}}{\epsilon_0} \left(\frac{x}{C_D B} \right)^{1/2} \exp \left(- \frac{U_\infty}{4 \epsilon_0 x} y^2 \right) \quad (2)$$

Fig.14 shows the variation of the maximum velocity defect u_m at $y=0$ with x . From this figure, the value of $\sqrt{10}/18\beta$ is determined to be 0.869 for wedges and 0.942 for a circular cylinder. Therefore, the values of β are as follows:

$\beta=0.202$ for wedges,

$\beta=0.186$ for a circular cylinder.

The value of β for the circular cylinder agrees approximately with the value $\beta=0.18$ obtained by Schlichting⁽¹⁴⁾.

Fig.15 shows the velocity defect profiles in each section of x for $\alpha=90^\circ$. It is found from this figure that the profiles become already similar at $x/B=55$. Fig.16 shows the variation of u/u_m at $y/b_2=2$ with x . The point where the curve of u/u_m at $y/b_2=2$ becomes parallel to the x -axis is the starting point of the similar profile region. The variation of this starting point x_s with the vertex angle is shown in Fig.17. As a result, it is found that the starting point of the similar profile region advances as the vertex angle increases. The present results for wedges are $x_s/B=45 \sim 57$, which may be seen to be appropriate values in com-

Fig.12 Dimensions of vortex region

Fig.13 Strouhal number

parison with $x_s/B=48$ for a circular cylinder by Roshko.

Fig.18 shows the velocity defect profiles in the similar profile region for $\alpha=30^\circ$, 90° and 180° . The results for a two-dimensional circular cylinder are also shown in this figure to compare with those of wedges. The results for wedges agree well with those of the circular cylinder. The profiles of u/u_m do not vary with the vertex angle and agree well with the theoretical value of Reichardt for $y/b_{1/2} < 1$ and that of Schlichting for $y/b_{1/2} > 1$.

Fig.19 shows the spread of $b_{1/2}$ of the wake. Schlichting⁽¹⁴⁾ obtained the following relation by the theoretical consideration:

$$b_{1/2} = 0.44 \sqrt{10\beta} (x C_D B)^{1/2}. \quad (3)$$

And he obtained $\beta=0.18$ from experiments with a circular cylinder and hence,

$$b_{1/2} = 1/4 (x C_D B)^{1/2}. \quad (4)$$

As the value of β is 0.186 for the circular cylinder and 0.202 for wedges by the present experiments, it follows that

$$b_{1/2} = 0.259 (x C_D B)^{1/2} \text{ for the circular cylinder,} \quad (5)$$

$$b_{1/2} = 0.282 (x C_D B)^{1/2} \text{ for wedges.} \quad (6)$$

Fig.14 Similarity of maximum velocity defect of wake flow

Fig.15 Velocity defect profile of wake flow ($\alpha=90^\circ$)

Fig.16 Variation of u/u_m at $y/b_{1/2}=2$ with x

The comparison of the experimental results with these relations is made in Fig.19.

4. Conclusions

The results of this study are summarized as follows:

(1) The surface-pressure distribution and the base-pressure coefficient C_{pb} (or $-\sigma$) of wedge vary with the vertex angle α . The pressure distribution along the slant surface of wedge agree well with the theoretical value of Plesset-Shaffer for $\alpha \geq 90^\circ$ but it does not agree with that for $\alpha \leq 60^\circ$. Also, C_{pb} increases with an increasing α .

(2) The drag coefficient C_D measured agrees well with the theoretical values of Plesset-Shaffer and Wu for $\sigma < 0.3$ independently of α . But, the discrepancy between them appears for $0.6 < \sigma < 1.2$. The results of Plesset-Shaffer are in better agreement with those of the present experiments than with those of Wu over a wide range of σ . Also, C_D increases with α .

(3) The dimensions of the vortex region behind the wedge increase with α .

(4) The Strouhal number $St = Bf/U_\infty$ decreases as α increases. On the other hand, $St = Hf/U_\infty$ is nearly constant for various α , where H is

Fig.17 Boundary point x_s of similar region

Fig.18 Velocity profiles in similar region

Fig.19 Similarity of width of wake

width of the vortex region.

(5) The velocity profiles in the similar profile region of the wake are independent of α and agree with those of a circular cylinder.

(6) The starting point of the similar profile region shifts forward as α increases.

(7) The value of β in the relation of $l = \beta b$, where l is the mixing length and b the width of the wake, is 0.186 for the circular cylinder and it nearly agrees with the result of Schlichting. β is equal to 0.202 for wedges and differs from the value of the circular cylinder.

The authors would like to thank Assistant Professor M.Kiya of Hokkaido University for his advice.

References

- (1) Schlichting, H., Ing. Arch., Vol.1, (1930), p.533.
- (2) Reichardt, H., VDI Forsch.-h. 414 (1942).
- (3) Okamoto, T. and Takeuchi, M., Bull. JSME, Vol.18, No.123 (1975-9), p.1011.
- (4) Roshko, A., NACA TN 3169 (1954).
- (5) Fage, A. and Johansen, F.C., Phil. Mag., Vol.5, No.28 (1928), p.417.
- (6) Plesset, M.C. and Shaffer, P.A., J. Appl. Phys., Vol.19 (1948), p.934.
- (7) Plesset, M.C. and Shaffer, P.A., Rev. Mod. Phys., Vol.20 (1948), p.228.
- (8) Cox, A. and Clayden, W., J. Fluid Mech., Vol.3 (1958), p.615.
- (9) Waid, R., Calif. Inst. Tech., Hydrodyn. Lab. Rep. E-73.6 (1957).
- (10) Wu, T.Y., J. Fluid Mech., Vol.13 (1962), p.161.
- (11) Wu, T.Y., J. Fluid Mech., Vol.18 (1964), p.65.
- (12) Fail, R., et al., Brit. ARC., R & M, No.3120 (1957).
- (13) Roshko, A., NACA TN 2913 (1953).
- (14) Schlichting, H., Boundary-Layer Theory, 6th ed. (1968), p.693, McGRAW-HILL.
- (15) Arie, M. and Rouse, H., J. Fluid Mech. Vol.1 (1956), p.129.